NAME		Maths Class	

SYDNEY BOYS' HIGH SCHOOL

HALF-YEARLY EXAMINATION 2001

MATHEMATICS

YEAR 10 ADVANCED

Time allowed — $1\frac{1}{2}$ hours

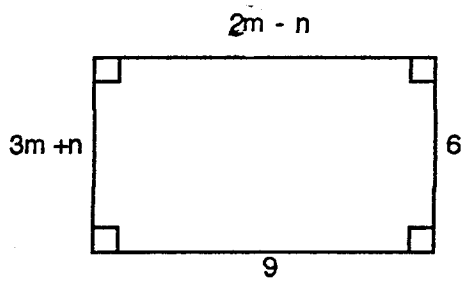
Examiner: F.Nesbitt

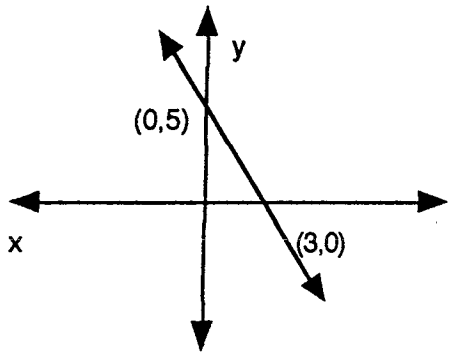
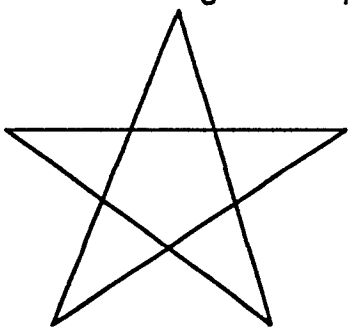
DIRECTIONS TO CANDIDATES

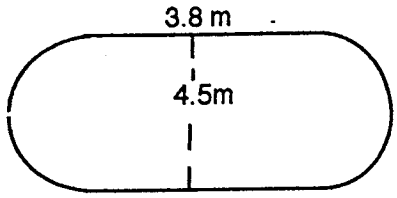
- Answers are to be written in the spaces provided.
- *ALL* questions may be attempted.
- All necessary working should be shown in every question. Full marks may not be awarded for careless or badly arranged work.
- Use the backs of the sheets for extra working space, if necessary.
- Approved calculators may be used.

Staff use only	Mark
Question 1	
Question 2	
Question 3	
Question 4	
Question 5	
Question 6	
Total	

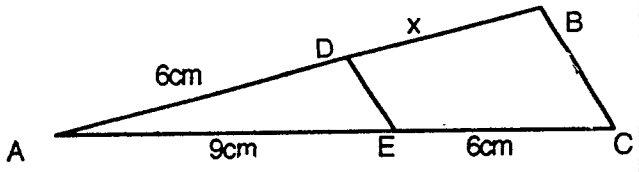
SECTION A
(20 marks)

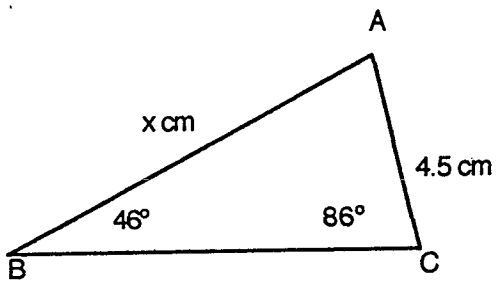
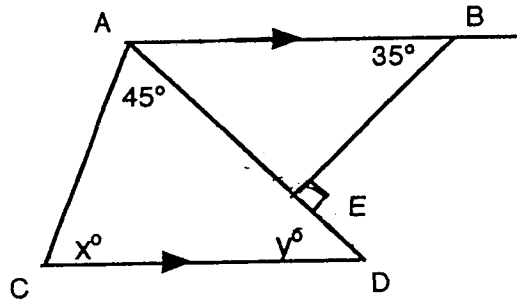
		Answers
1.	Write 2458 in scientific notation.	
2.	Factorise fully: $3x^2 - 12y^2$	
3.	Find the median from this stem and leaf plot Stem | Leaves 14 | 04 15 | 022333456 16 | 123444556789 17 | 1112257899 18 | 3577 19 | 27	
4.	Find the value of m in the diagram below. 	
5.	Write the equation of the line parallel with the x axis and going through the point (2,-3).	
6.	What is the gradient of the line in question 5 ?	
7.	Convert 8 m / s to km / h.	
8.	Expand and simplify fully. $(2-\sqrt{2})(1+\sqrt{2})$	
9.	Write in simplest form: $\dfrac{1}{\left(\frac{3}{4}\right)^{-2}}$	

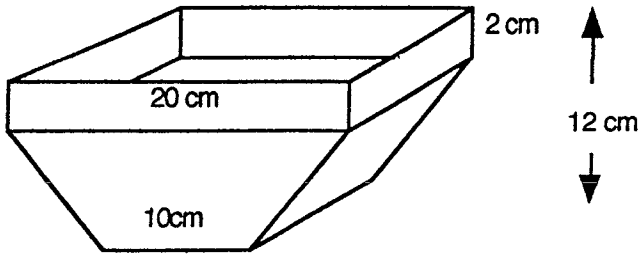
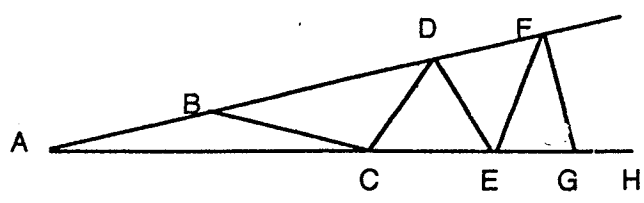
		Answers
10.	Write the equation of this line.  	
11.	Solve the equation: $5\,b^2 - 17\,b + 6$	
12.	Find the midpoint of A(- 1,5) and B(2,-6).	
13.	The sides of a regular pentagon have been produced to form a star.What is the size of the angle at the point of each star? 	
14.	A volume of 2.5 cubic metres of concrete is poured onto a rectangular area measuring 4m long by 3.5m wide. Find the depth of the concrete in cm. (nearest cm).	
15.	Divide \$270 in the ratio 1 : 3 : 5	
16.	If 18% of an amount of money is \$36, find the amount.	
17.	Write down the exact value of cos 30°.	

		Answers
18.	 Find the area of this garden bed. It has semi-circular ends.	
19.	Find the simple interest on $2500 at 9.5% p.a. for four years.	
20.	Simplify ,correct to 2 decimal places: $\frac{\sqrt{64.4-11.3}}{15.2+9.5}$	

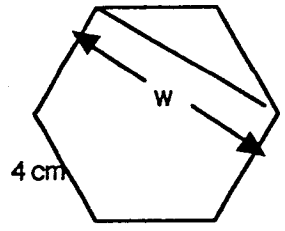
SECTION B

1.	 In the figure above DE and BC are parallel. (i) Name two similar triangles, giving reasons for your answer. (ii) Find x, the length of DB.	
2.	How much will an investment of $15 000 at 6 % p.a. amount to after 2 years if the interest is compounded monthly?	

		Answers
3.	 Find the value of x in the above triangle.	2
4.	 In the above figure the angle AEB=90°. Find the values of x and y.	2
5.	In a Maths test the mean was 68. Two new students arrived and sat for the test. The new mean was 67. If the new students had marks of 66 and 50 ,how many students were in the class?(including the new students)	2
6.	Jim paid $25 000 for a new car.If cars depreciate at a rate of 11% p.a. on average, find the expected value of the car after 4 years.	2

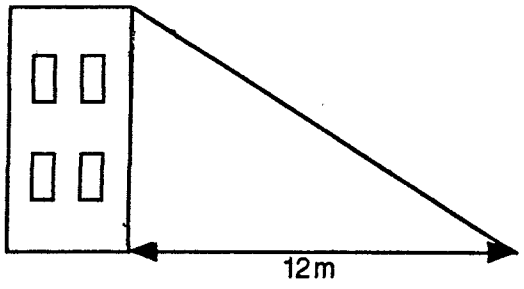
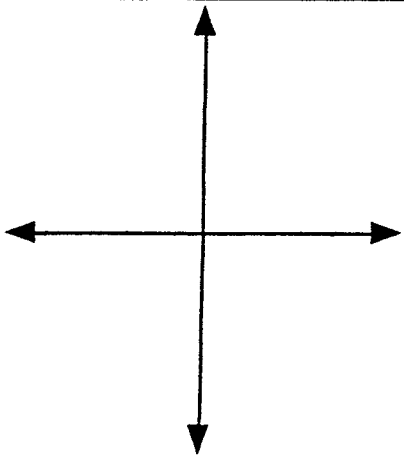
		Answers
4.	 A planter box is in the shape of a square pyramid (with the end cut off), topped by a square prism of height 2cm. The width of the base is 10 cm and the top is 20 cm wide. The total height is 12 cm. For the above planter box , find: (i) the volume, (ii) the capacity in litres to 2 dec.pl., (ii) the external surface area, including the base to 2 dec pl..	6
5.	In the diagram below, angle FGH=120° AB=BC=CD=DE=EF=FG  Find the size of the angle BAC. Show working.	2

SECTION D

		Answers
1.	An \$8 000 loan is taken out at 15% p.a. reducible interest compounded monthly. The monthly repayment is \$290. (i) How much interest is charged in the first month? (ii) How much is owing on the loan after the second repayment?	4
2.	Simplify: $\frac{\left(a^2b^2\right)^{\frac{3}{2}}}{ab}$ ab>0	2
3.	 In this regular hexagon with sides of 4 cm, find the width w between opposite sides.	3

		Answers
4.	A ship sailed from a port (P) 100 nautical miles on a bearing of 080° to an Island (L). It then turned due North and sailed 190 nautical miles to a resort (R). (i) Draw a diagram in the space below to represent this information. (ii) Find the distance from R to P. (iii) Find the bearing of R from P	6

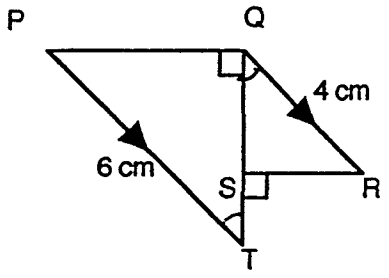
SECTION E

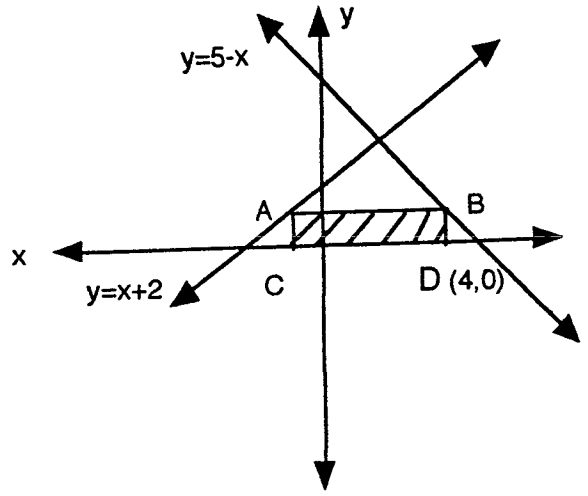
1.	From a point on the ground 12 m away, the angle of elevation to the top of a building was 43°.  Find the height of the building to the nearest metre. 2 .	2
2.	On the set of axes on the right, shade the region described by the inequality $2x + y \geq 4$.	3 

		Answers
3.	$7500 is to be invested for 5 years. There is a choice between a simple interest rate of 15.5% p.a. or a compound rate of 10.25% compounded yearly. Which is the better option and by how much?	3
4.	Write as a single fraction: $\frac{x}{x+2} - \frac{3x}{4x-1}$	3

		Answers
5.	After 3 years of depreciation averaging 20%p.a., a painting is worth $3584. How much was the painting worth three years ago?	2

SECTION F

1.	 In the diagram above PT and QR are parallel. The angles PQT and RST are right angles. (i) Name another pair of equal angles. (ii) Explain why the triangles PQT and QSR are similar. (iii) If the area of PQT= 8.9 cm^2, find the area of the triangle QSR.	4
2.	What is the exact value of cos 135°	1
3.	Find all possible angles ø, given $0° \leq ø \leq 180°$ if $\sin \phi = \frac{\sqrt{3}}{2}$	2

		Answers
4.	 Find the area of the shaded rectangle ABCD Show all working.	3
5.	Mrs Chen was 20 when her eldest child was born. Two years later another child was born and four years after that her youngest child was born. Now the average of their four ages is 39. How old are Mrs Chen and her three children?	2

SOLUTIONS

NAME		Maths Class	

SYDNEY BOYS' HIGH SCHOOL

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MATHEMATICS

YEAR 10 ADVANCED

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DIRECTIONS TO CANDIDATES

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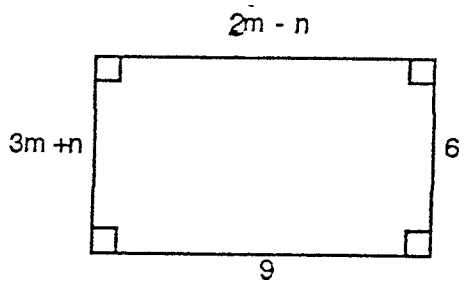
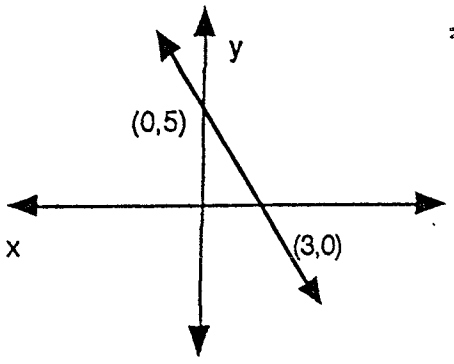
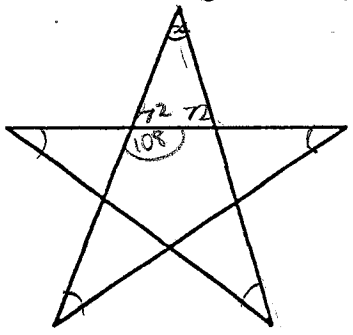
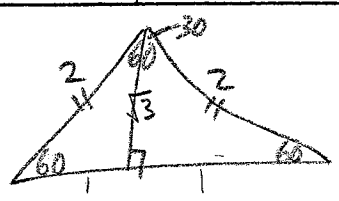
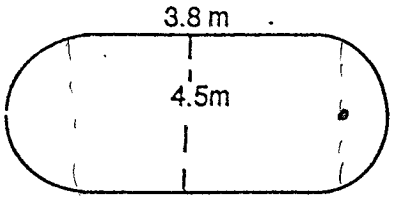
Staff use only	Mark
Question 1	
Question 2	
Question 3	
Question 4	
Question 5	
Question 6	
Total	

SECTION A
(20 marks)

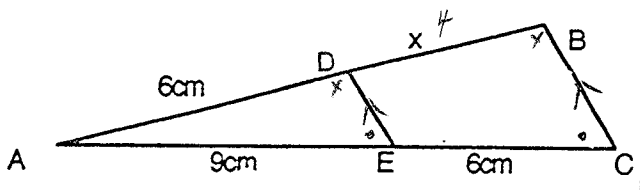
		Answers
1.	Write 2458 in scientific notation.	2.458×10^3. ✓
2.	Factorise fully: $3x^2 - 12y^2$	$3(x^2 - 4y^2) = 3(x+2y)(x-2y)$
3.	Find the median from this stem and leaf plot Stem | Leaves 14 | 04 15 | 022333456 16 | 123444556789 17 | 1112257899 18 | 3577 19 | 27 19.5.	165.5. ✓
4.	Find the value of m in the diagram below. 	$2m-n=9$ —① ✓ $3m+n=6$ —② ①+②: $5m=15$ $\therefore m=3$. ✓
5.	Write the equation of the line parallel with the x axis and going through the point (2,-3).	$y=-3$. ✓
6.	What is the gradient of the line in question 5 ?	0. ✓
7.	Convert 8 m / s to km / h. $8m/s = 0.008 km/s =$ 0.008 km/	$= \frac{0.008\ km}{\frac{1}{60\times60}\ hr} = 0.008\times3600\ km/hr = 28.8\ km/hr.$
8.	Expand and simplify fully: $(2-\sqrt{2})(1+\sqrt{2})$ $2+2\sqrt{2}-\sqrt{2}-2=$	$\sqrt{2}$. ✓
9.	Write in simplest form: $\frac{1}{\left(\frac{3}{4}\right)^{-2}} = \frac{1}{\left(\frac{4}{3}\right)^2} = \frac{1}{\frac{16}{9}} = \frac{9}{16}$.	$\frac{19}{16}$. ✓

	(0,5) (3,0).	Answers
0.	Write the equation of this line.  $m = \frac{y_1-y_2}{x_1-x_2} = \frac{5-0}{0-3} = \frac{5}{-3}$	eg^{y}: $y-y_1 = m(x-x_1)$ $y-5 = \frac{5}{-3}(x-0)$ ✓ $y-5 = -\frac{5}{3}x$ $\therefore y = -\frac{5}{3}x+5$. ✓
1.	Solve the equation: $5b^2 - 17b + 6$ $5b^2-15b-2b+6$ $5b(b-3)-2(b-3)=0$. $(5b-2)(b-3)=0$.	$(5b-2)(b-3)=0$ ✓ $b = \frac{2}{5}$ or 3
2.	Find the midpoint of A(-1,5) and B(2,-6). $M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{1}{2}, -\frac{1}{2}\right)$	$M = \left(\frac{1}{2}, -\frac{1}{2}\right)$. ✓
3.	The sides of a regular pentagon have been produced to form a star. What is the size of the angle at the point of each star?  $180(n-2)$ $= 180 \times 3$ $= 540$.	36°. ✓
4.	A volume of 2.5 cubic metres of concrete is poured onto a rectangular area measuring 4m long by 3.5m wide. Find the depth of the concrete in cm. (nearest cm). 	$4 \times 3.5 \times d = 2.5$ $\therefore 14d = 2.5$ ✓ $\therefore d = 0.17857..$ m $= 18$cm (to nst cm).
5.	Divide \$270 in the ratio 1 : 3 : 5 $\frac{1}{9} \times 270$	\$30 : \$90 : \$150 ✓
6.	If 18% of an amount of money is \$36, find the amount. $\frac{18x}{100} = 36$ $\frac{9x}{50} = 36$ $9x = 1800$	\$200. ✓
7.	Write down the exact value of cos 30°.	$\frac{\sqrt{3}}{2}$. ✓



		Answers
3.	 πr^2 $2.25 = r$. Find the area of this garden bed. It has semi-circular ends.	$3.8 \times 4.5 + 15.9043\ldots$ $= 33\ m^2$ (to nst m). ✓
).	Find the simple interest on \$2500 at 9.5% p.a. for four years.	$I = PRN$ $= \$2500 \times 0.095 \times 4$ $= \$950$. ✓
).	Simplify ,correct to 2 decimal places: $\frac{\sqrt{64.4 - 11.3}}{15.2 + 9.5} = \frac{\sqrt{53.1}}{24.7} =$	0.30 (to 2dp). ✓

SECTION B



In the figure above DE and BC are parallel.

(i) Name two similar triangles, giving reasons for your answer.

(ii) Find x, the length of DB.

(1) In $\triangle ADE$ and $\triangle ABC$: ✓
① $\hat{A}$ is common
② $A\hat{D}E = A\hat{B}C$ (corres. $\angle$'s = DE // BC)
③ $A\hat{E}D = A\hat{C}B$ (")
$\therefore \triangle ADE \,///\, \triangle ABC$ (equiangular). ✓

(2) $\frac{6+x}{9+6} = \frac{6}{9}$ (corres. sides in ratio in /// $\triangle$'s). ✓

$\therefore \frac{6+x}{15} = \frac{6}{9}$

$6 + x = 10$ ✓

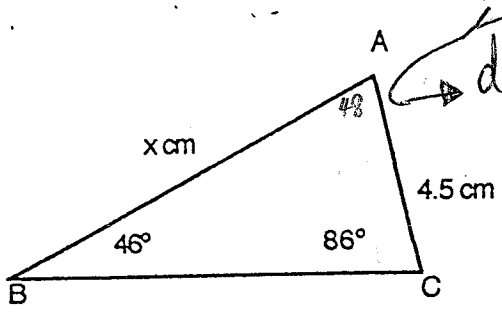
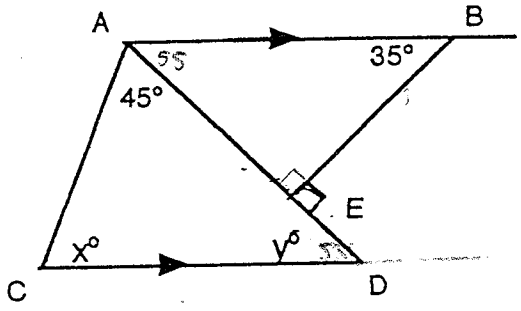
$\therefore x = 4$.

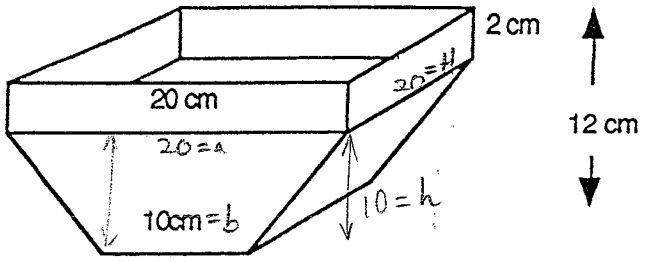
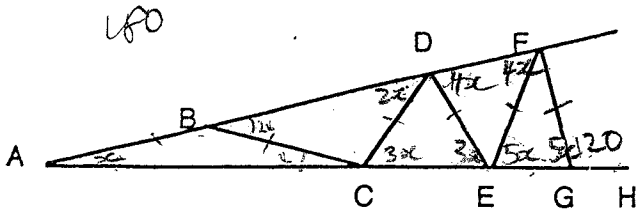
$\therefore DB = 4$ cm ✓

How much will an investment of \$15 000 at 6 % p.a. amount to after 2 years if the interest is compounded monthly?

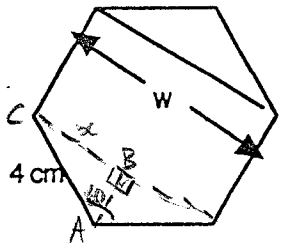
$\frac{6}{12} = 0.5$

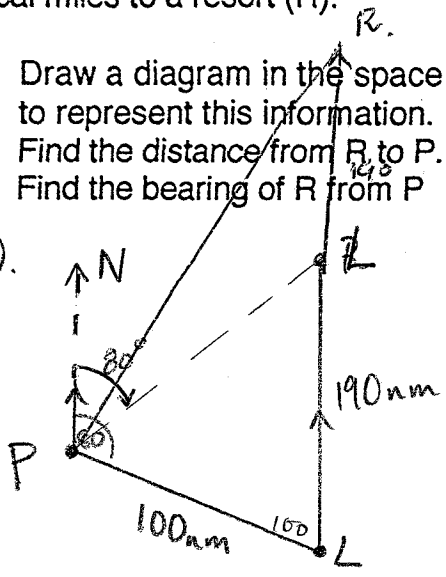
$A = P(1+r)^n$
$= \$15\,000\,(1.005)^{24}$ ✓
$= \$16\,907.40$ ✓

		Answers
3.	 cosine rule. don't know yet. Find the value of x in the above triangle.	$\frac{x}{\sin 86^\circ} = \frac{4.5}{\sin 46^\circ}$ 2
4.	 In the above figure the angle AEB=90°. Find the values of x and y.	$B\hat{A}E = 55^\circ$ (∠ sum Δ is 180). 2 $\therefore y = 55$ (alt ∠'s = AB // CD) $\therefore x = 80$ (∠ sum Δ is 180) ✓
5.	In a Maths test the mean was 68. Two new students arrived and sat for the test. The new mean was 67. If the new students had marks of 66 and 50 ,how many students were in the class?(including the new students)	let x be no. of original students 2 $\therefore 67 = \frac{68x + 116}{x+2}$ ✓ $\therefore 67x + 134 = 68x + 116$ $\therefore x = 18$ ∴ no. of students in class is 20. ✓
6.	Jim paid $25 000 for a new car. If cars depreciate at a rate of 11% p.a. on average, find the expected value of the car after 4 years.	$A = P(1-r)^n$ $= 25\,000(0,89)^4$ ✓ 2 $= \$15\,685.56$. ✓ ∴ value after four years is $15 685.56.

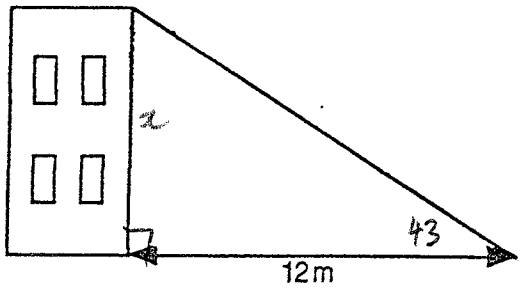
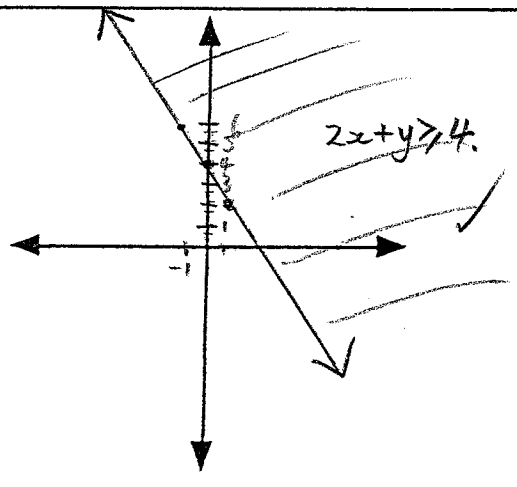
		Answers
4.	 A planter box is in the shape of a square pyramid (with the end cut off), topped by a square prism of height 2cm. The width of the base is 10 cm and the top is 20 cm wide. The total height is 12 cm. For the above planter box , find: (i) the volume, (ii) the capacity in litres to 2 dec.pl., (ii) the external surface area, including the base to 2 dec pl..	(1) volume of square prism + vol. of square pyramid 6 $= AH + \frac{1}{2}(a+b)h \times h$ $= 20 \times 2 \times 20 + \frac{1}{2}(20+10) \times 10 \times 10$ $= 800 + 1500$ $= 2300\ cm^3$ ∴ Vol. is $2300\ cm^3$ (2) $V = 2300\ cm^3$ Capacity $= 2300\ mL$ $= 2.30\ L$ (to 2dp) ∴ capacity is 2.30 L. (3) SA = $20 \times 2 \times 4 + 10 \times 10 +$ $\frac{1}{2}(20+10) \times 10 \times 4$ $= 160 + 100 + 600$ $= 860\ cm^2$. ∴ external surface area is $860\ cm^2$.
.	In the diagram below, angle FGH=120° AB=BC=CD=DE=EF=FG  Find the size of the angle BAC. Show working.	$B\hat{C}A = x$ (base ∠'s = in isos. Δ's) $D\hat{B}C = 2x$ (ext. ∠ Δ = sum 2 int. opp. ∠'s) ∴ $B\hat{D}C = 2x$ (base ∠'s isos Δ) ∴ $D\hat{C}E = 3x$ (ext. ∠ Δ ...) ∴ $D\hat{E}C = 3x$ (base ∠'s ...) ∴ $F\hat{D}E = 4x$ (ext. ∠ Δ ...) ∴ $D\hat{F}E = 4x$ (base ∠'s isos Δ ...) ∴ $F\hat{E}G = 5x$ (ext. ∠ Δ ...) ∴ $E\hat{G}F = 5x$ (base ∠'s ... ∴ $120 = x + 4x + (180 - 10x)$ $120 = 5x + 180 - 10x$ $120 = -5x + 180$ $5x = 60$ ∴ $x = 12°$

SECTION D

	Answers
1. An \$8 000 loan is taken out at 15% p.a. reducible interest compounded monthly. The monthly repayment is \$290. (i) How much interest is charged in the first month? (ii) How much is owing on the loan after the second repayment?	4 (1) $A = P(1+r)^n$ $= 8000(1.0125)$ $= \$8100$ $\therefore$ interest $= \$100$. $\therefore$ tot. $= \$8100 - 290 = \$7810 = P_1$ (2) $A_1 = P(1.0125)$ $= 7810 \times 1.0125$ $= \$7907.63 - 290$ $= \$7617.63$ $\therefore$ amount owing is \$7617.63.
2. Simplify: $\dfrac{(a^2b^2)^{\frac{3}{2}}}{ab}$ ab>0	2 $= \dfrac{a^3b^3}{ab}$ $= a^2b^2$.
3. 180(n-2)  In this regular hexagon with sides of 4 cm, find the width w between opposite sides.	3 angle sum $= 180(n-2)$ $= 180(6-2)$ $= 720$ $\therefore$ each angle $= 120$ $C\hat{A}B = 120 \div 2 = 60$ Let $CB = x$ $\therefore \sin 60 = \frac{x}{4}$ $\therefore 4\sin 60 = x$ $\therefore w = 2(4\sin 60)$ $= 6.93$ cm (to 2dp).

		Answers
4.	A ship sailed from a port (P) 100 nautical miles on a bearing of 080° to an Island (L). It then turned due North and sailed 190 nautical miles to a resort (R). (i) Draw a diagram in the space below to represent this information. (ii) Find the distance from R to P. (iii) Find the bearing of R from P (i). 	(ii) Sine rule. 6

SECTION E

1.	From a point on the ground 12 m away, the angle of elevation to the top of a building was 43°.  Find the height of the building to the nearest metre. 2.	let height be x. 2 $\therefore \tan 43 = \frac{x}{12}$ ✓ $\therefore x = 12 \tan 43$ $\therefore x = 11.1901...$ $= 11$ m (to nst m) ✓ $\therefore$ height of building is 11m.
.	On the set of axes on the right, shade the region described by the inequality $2x + y \geq 4$. $y = -2x + 4$ x: −1, 0, 1 y: 6, 4, 2 test (0,0): $0 \geq 4$ FALSE.	3 

		Answers
3.	\$7500 is to be invested for 5 years. There is a choice between a simple interest rate of 15.5% p.a. or a compound rate of 10.25% compounded yearly. Which is the better option and by how much?	3

simp. interest:

$I = PRN$

$= 7500 \times 0.155 \times 5$

$= \$5812.50$ ✓

compound int:

$A = P(1+r)^n$

$= 7500(1.1025)^5$

$= \$12216.71$

$\therefore$ int $= \$4716.71$ ✓

$\therefore$ simple interest option is better by \$1095.79.

4. check.	Write as a single fraction: $\frac{x}{x+2} - \frac{3x}{4x-1}$	3

$\frac{x}{(x+2)} \times \frac{(4x-1)}{4x-1} - \frac{3x}{(4x-1)} \times \frac{(x+2)}{(x+2)}$ ✓

$= \frac{4x^2 - x - (3x^2 + 6x)}{(x+2)(4x-1)}$

$= \frac{4x^2 - x - 3x^2 - 6x}{(x+2)(4x-1)}$

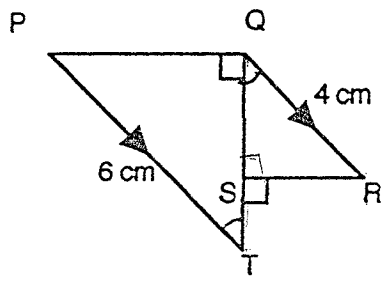
$= \frac{x^2 - 7x}{(x+2)(4x-1)}$ ✓

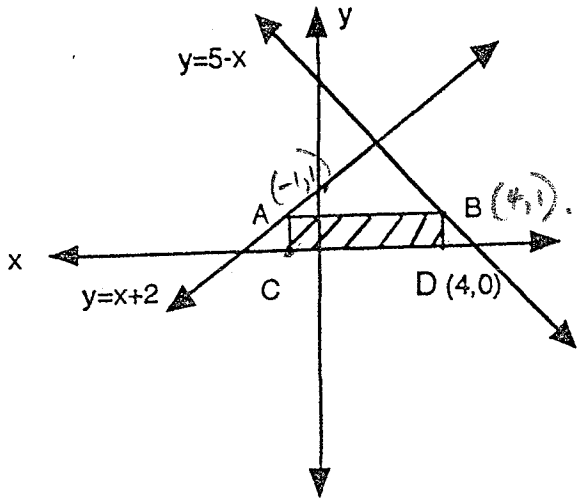
$= \frac{x(x-7)}{(x+2)(4x-1)}$ ✓

$\frac{x(x-7)}{(x+2)(4x-1)}$

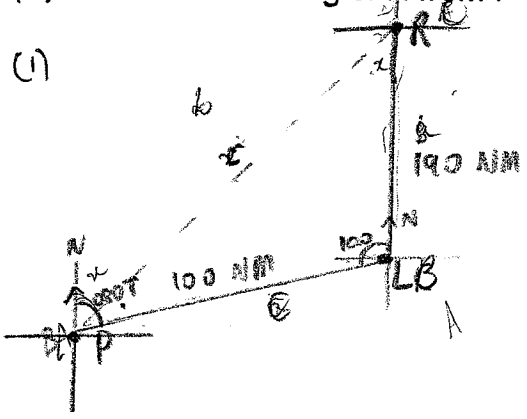
		Answers	
5.	After 3 years of depreciation averaging 20%p.a., a painting is worth \$3584. How much was the painting worth three years ago?	$A = P(1-r)^n$ $\$3584 = P(0.8)^3$ ✓ $3584 = 0.512P$ $\therefore P = \$7000$. ✓ ∴ painting was worth \$7000 3 years ago.	2

SECTION F

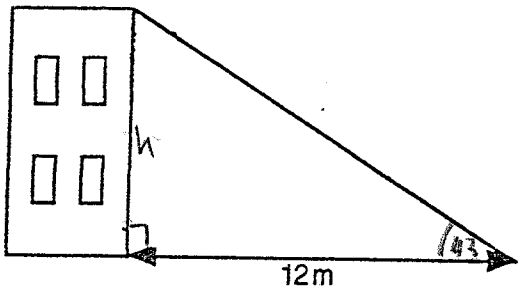
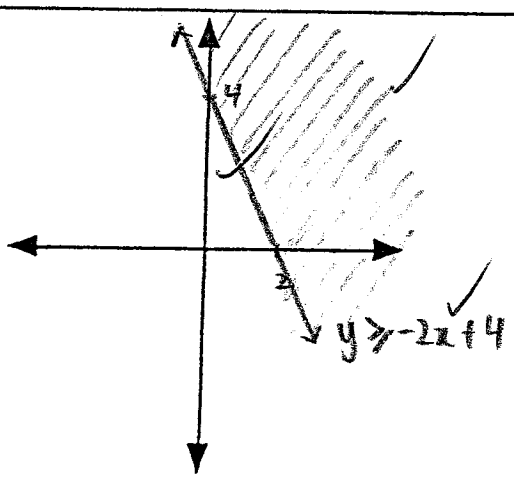
1.	 In the diagram above PT and QR are parallel. The angles PQT and RST are right angles. (i) Name another pair of equal angles. (ii) Explain why the triangles PQT and QSR are similar. (iii) If the area of PQT= 8.9 cm², find the area of the triangle QSR.	(1) $R\hat{Q}T = P\hat{T}Q$ ✓ (2) In $\triangle PQT$ and $\triangle QSR$: ① $P\hat{Q}T = R\hat{S}T = 90$ (given) ② $R\hat{Q}T = P\hat{T}Q$ (given). ✓ ∴ $\triangle PQT \,///\, \triangle QSR$ (equiangular). (3) $\frac{4}{6} = \frac{2}{3} = 2:3$ ratio of sides is 2:3 ✓ let area of $\triangle QSR$ be x ∴ $\frac{2^2}{3^2} = \frac{x}{8.9}$ $\therefore \frac{4}{9} = \frac{x}{8.9}$ $x = 3.95$ $= 3.96$ (to 2 dp) ∴ Area of $\triangle QSR$ is 3.9 cm².	4
2.	What is the exact value of cos 135°	$-\frac{1}{\sqrt{2}}$	1
3.	Find all possible angles ø, given $0° \le ø \le 180°$ if $\sin \phi = \frac{\sqrt{3}}{2}$ ∴ $\theta = \sin^{-1}(\sqrt{3} \div 2)$ $= 60°$	$\theta = 60°, 120°$ ✓	2

		Answers
4.	 Find the area of the shaded rectangle ABCD Show all working.	$y = 5 - x$ Sub $x = 4$: $\therefore y = 1$. $\therefore B = (4,1)$ $y = x + 2$ sub $y = 1$: $\therefore 1 = x + 2$ $\therefore x = -1$. $\therefore A = (-1,1)$ dis $A(-1,1)$ $B(4,1)$ $= \sqrt{25} = 5$. dis. $B(4,1)$ $D(4,0)$ $= \sqrt{1} = 1$ $\therefore$ Area $ABCD = AB \times BD = 5 \times 1 = 5$ units^2.
5.	Mrs Chen was 20 when her eldest child was born. Two years later another child was born and four years after that her youngest child was born. Now the average of their four ages is 39. How old are Mrs Chen and her three children?	Mrs Chen is 56, her eldest child is 36, second eldest child is 34, and her youngest child is 30.

$39 \times 4 = 156$
$\therefore$ sum of all ages $= 156$.
Let eldest child's age be x.
$\therefore (20 + x) + x + (x - 2) + (x - 6) = 156$.
$\therefore 20 + x + x + x - 2 + x - 6 = 156$
$\therefore 12 + 4x = 156$
$\therefore 4x = 144$.
$\therefore x = 36$.

		Answers
4.	A ship sailed from a port (P) 100 nautical miles on a bearing of 080° to an Island (L). It then turned due North and sailed 190 nautical miles to a resort (R). (i) Draw a diagram in the space below to represent this information. (ii) Find the distance from R to P. (iii) Find the bearing of R from P (i) 	(ii) $a^2 = b^2 + c^2 - 2bc\cos A$ 6 $a^2 = 190^2 + 100^2 - 2\times 190\times 100\times \cos 100$ $a = \sqrt{52698.63}$ $= 229.56$ Nautical miles (iii) $\cos RPL = \frac{b^2+c^2-a^2}{2bc}$ (nearest 2 dp) $= \frac{229.56^2+100^2-190^2}{2\times 229.56\times 100}$ $= 54°36'$ The bearing of R from P is 025°T

SECTION E

.	From a point on the ground 12 m away, the angle of elevation to the top of a building was 43°.  Find the height of the building to the nearest metre. 2 .	2 $\tan 43 = \frac{h}{12}$ $h = 12\tan 43°$ $= 11.19$ $= 11$ m (to the nearest m)
.	On the set of axes on the right, shade the region described by the inequality $2x + y \geq 4$. $y = -2x + 4$ $y \geq -2x + 4$	3 

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