



SYDNEY BOYS HIGH
MOORE PARK, SURRY HILLS

2005
YEAR 10 YEARLY EXAMINATION

Advanced Mathematics

Directions to Candidates:

- Answer all questions in the spaces provided in this question booklet.
- Full marks may not be awarded for careless or badly arranged work.
- Use black or blue pen for written answers, but pencil for diagrams and graphs.

- If additional working space is needed, use the spare pages at the end of the booklet. Show clearly which question you are continuing.
- Board-approved calculators may be used.

Time allowed: 120 minutes
Examiner: D.M.Hespe

Name: _____

Your Mathematics Class (Tick the box)	
10MaA	Mr Choy
10MaB	Ms Kourtesis
10MaC	Ms Ward
10MaD	Mr Gainford
10MaE	Mr Parker
10MaF	Mr Boros

Markers' Use Only	
Question 1	/20
Question 2	/20
Question 3	/15
Question 4	/15
Question 5	/15
Question 6	/15
Total	/100

Question 1 (20 marks)

Answers

Marks

(a) Express $\frac{2}{15}$ as a recurring decimal.

1

(b) Write $\frac{3}{8} + \frac{5}{6}$ as a single fraction.

1

(c) What is the value of $0.04 \times (0.3)^2$?

1

(d) Find 19% of \$38 000 000.

1

(e) Simplify $-6p + 2q + p - 5q$.

1

(f) Factorise $5 - 10y$.

1

(g) Write as an algebraic expression:
"Half of the sum of x and y ."

1

(h) Find correct to two decimal places $\frac{5 \cdot 28}{57 \cdot 3 \times 13}$.

1

(i) Write down the value of $\sin 34^\circ 15'$ (correct to 3 significant figures).

1

(j) Convert $S15^\circ E$ to a True bearing.

1

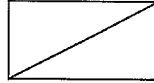
(k) Evaluate $\sqrt{17^2 - 15^2}$.

1

(l) Write $2^3 \times 2^6$ in simplest index notation.

1

(m) A farm gate is 2 m high and 3 m wide.
Find the exact length of the diagonal strut.



NOT TO SCALE

1

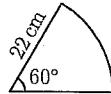
(n) Simplify $(x^2y)^3 \div (x^3y)^2$.

1

(o) Express $\frac{4}{\sqrt{8}}$ with a rational denominator.

1

(p) Find the area of the sector in terms of π .



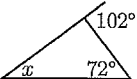
1

(q) Solve $2 - x = -5$.

1

(r) If $v^2 = u^2 + 2aS$, find v when $u = 2$,
 $a = 10$, and $S = 3$.

1

(s)  Find x .

1

(t) My *Edvest* account pays 6% p.a. simple interest.
If I invest \$1200 for two years, what will my
investment be worth?

1

Question 2 (20 marks)

Answers

(a) Fully factorise the following:

(i) $t^2 - 10t + 16$

1

(ii) $4x^2 - 4$

2

(b) Find the smallest part when \$19 is divided in
the ratio 2 : 3 : 5.

2

(c) With the aid of a diagram, explain what is
wrong with the following proof:

In Δ s ABC , PQR ,
 $\angle ABC = \angle QRP = 65^\circ$,
 $\angle BCA = \angle QPR = 35^\circ$,
 $BC = QP = 15$ cm.
 $\therefore \Delta ABC \equiv \Delta PQR$ (AAS).

4

(d) Given the points $A(3, 6)$ and $B(-3, -2)$, write down the
(i) midpoint of AB ,

1

(ii) length of the interval AB ,

1

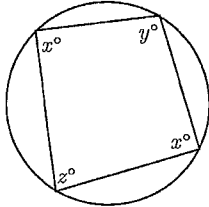
(iii) gradient of the line AB .

1

(e) Find x if $5\sqrt{7} - \sqrt{x} = 0$.

2

(f)



(i) Find the value of x , giving reasons.

2

(ii) Write an equation connecting z and y .

1

(g) Solve simultaneously, the system of equations

$$\begin{aligned}4x - 5y &= 2 \\3x - 2y &= -2\end{aligned}$$

3

Question 3 (15 marks)

Answers

(a) Evaluate the pronumerals to the nearest whole number or whole degree, as appropriate.

(i) $b = \frac{33.8 \sin 117^\circ}{\sin 38^\circ}$

1

(ii) $c^2 = 36^2 + 23^2 - 2 \times 36 \times 23 \cos 28^\circ$

1

(iii) $\sin A = \frac{407 \sin 71^\circ}{586}$

1

(b) Find the volume and total surface area of a solid cone of radius 5 cm and slant height 13 cm.

4

(c) Find all possible values of θ , given $0^\circ \leq \theta \leq 360^\circ$.

(i) $\cos \theta = \frac{1}{\sqrt{2}}$

1

(ii) $\tan \theta = -\sqrt{3}$

1

(d) Three coins are tossed simultaneously.

(i) Use a tree diagram to write out the sample space.

3

Find the probability of obtaining

(ii) three heads,

1

(iii) at least one head,

1

(iv) at most one head.

1

Question 4 (15 marks)

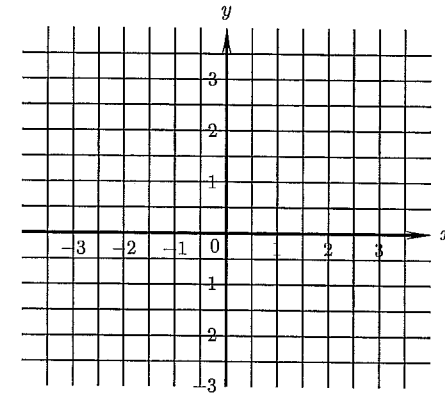
Answers

(a) Find (to the nearest dollar) the compound interest earned if \$96 000 is invested for 5 years at 6.5% p.a.

2

(b) Sketch $y = x^2 - 2x$ on the grid below, showing clearly any intercepts and maximum or minimum points.

3



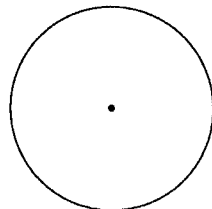
(c) Solve $x^2 = 3x + 7$ by the method of completing the square. Give your answer as a surd in simplest form and as a decimal approximation correct to 2 decimal places.

3

(d) The probability that a certain event will happen is $\frac{1}{x}$. Which of the following is the probability that the event will not happen?

$\frac{x}{1}$, $\frac{1-x}{x}$, $\frac{x+1}{x}$, $\frac{x-1}{x}$, $\frac{x}{x-1}$

(e) Prove that equal chords subtend equal angles at the centre of a circle.



(f) When blowing up a balloon, the first hearty breath gives a diameter of 7 cm. By the time it is fully blown up, its diameter is 21 cm. Find in simplest form:

(i) the ratio of these diameters,

(ii) the ratio of the respective surface areas,

(iii) the ratio of the respective volumes.

1

3

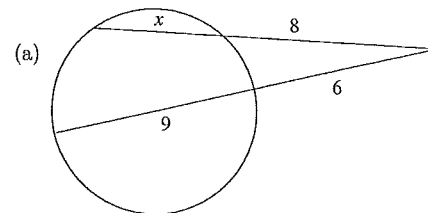
1

1

1

Question 5 (15 marks)

Answers



Find x .

(b) If $\frac{3}{4}$ of the boys in a junior class use a backpack, and $\frac{2}{5}$ are not in full uniform, find the probability that a boy chosen by lot would be in uniform and using a backpack.

(c) The numbers 30, 67, x , 2, 24, 9, 7, 83 have a mean of 32.

(i) Calculate the value of x .

(ii) Find the median.

(iii) Use your calculator to find the standard deviation correct to three significant figures.

2

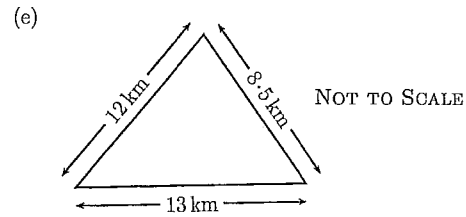
3

2

1

1

- (d) A is a wireless station 35 nautical miles East of another station at B . A ship in a fog discovers by wireless direction finder that she is $S20^\circ E$ of B and $S55^\circ W$ of A . How far is she from B (to the nearest nautical mile).



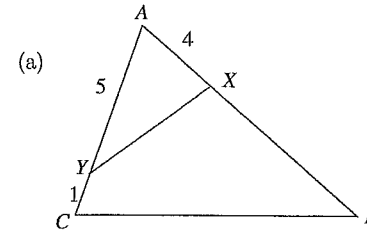
- (i) Calculate the size of the smallest angle in this triangle correct to the nearest minute.

- (ii) Hence find its area to the nearest square metre.

3

Question 6 (15 marks)

Answers



$$\begin{aligned}
 AX &= 4, \\
 AY &= 5, \\
 YC &= 1, \\
 \angle AXY &= \angle ACB.
 \end{aligned}$$

Calculate, giving reasons, the value of the following ratios:

- (i) $XY : CB$

3

- (ii) Area $\triangle AXY$: area $\triangle ACB$

2

- (iii) Area $\triangle AXY$: area $\triangle AXC$

2

1

(b) If $\frac{1}{x} = \frac{1}{a} + \frac{1}{b}$,

(i) show that $x = \frac{ab}{a+b}$.

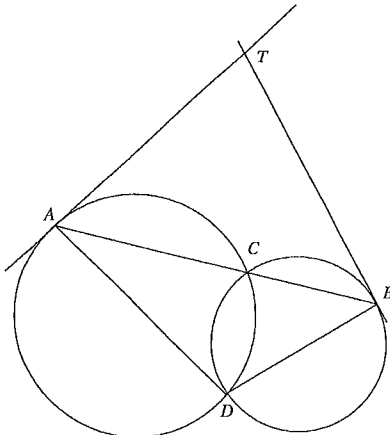
1

(ii) Hence prove that $\sqrt{\frac{a-x}{b-x}} = \frac{a}{b}$ where a, b are positive.

2

(c)

5



ACB is a straight line.
 TA and TB are tangents to the circles.
 Prove that $TADB$ is a cyclic quadrilateral.

End of Paper



SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

Year 10

Yearly Examination 2005

Advanced Mathematics

Sample Solutions

Question	Marker
A	RB
B	PSP
C	AMG
D	AW
E	CK
F	EC

Question 1 (20 marks)

Answers

Marks

(a) Express $\frac{2}{15}$ as a recurring decimal.

$$0.\overline{13}$$

1

(b) Write $\frac{3}{8} + \frac{5}{6}$ as a single fraction.

$$1\frac{5}{24} = \frac{29}{24}$$

1

(c) What is the value of $0.04 \times (0.3)^2$?

$$0.0036$$

1

(d) Find 19% of \$38 000 000.

$$\$7\,220\,000$$

1

(e) Simplify $-6p + 2q + p - 5q$.

$$-5p - 3q$$

1

(f) Factorise $5 - 10y$.

$$5(1 - 2y)$$

1

(g) Write as an algebraic expression: "Half of the sum of x and y ." $\frac{(x+y)}{2}$; $(x+y) \div 2$ etc.

1

(h) Find correct to two decimal places $\frac{5.28}{57.3 \times 13}$.

$$0.01$$

1

(i) Write down the value of $\sin 34^\circ 15'$ (correct to 3 significant figures).

$$0.563$$

1

(j) Convert $S15^\circ E$ to a True bearing.

$$165^\circ T$$

1

(k) Evaluate $\sqrt{17^2 - 15^2}$.

8

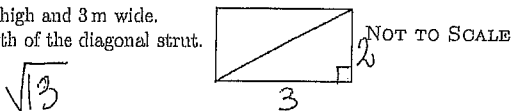
1

(l) Write $2^3 \times 2^6$ in simplest index notation.

2^9

1

(m) A farm gate is 2 m high and 3 m wide. Find the exact length of the diagonal strut.



1

(n) Simplify $(x^2y)^3 \div (x^3y)^2$.

$\frac{x^6 y^3}{x^6 y^2} = y$

1

(o) Express $\frac{4}{\sqrt{8}}$ with a rational denominator.

$\frac{4}{\sqrt{8}} \times \frac{\sqrt{8}}{\sqrt{8}} = \frac{4\sqrt{8}}{8} = \frac{\sqrt{8}}{2} = \frac{2\sqrt{2}}{2} = \sqrt{2}$

1

(p) Find the area of the sector in terms of π .

$A = \frac{60}{360} \times \pi \times 22 \times 22 = \frac{\pi}{3} \times 22 \times 22 = \frac{242\pi}{3} \text{ cm}^2$

1

(q) Solve $2 - x = -5$.

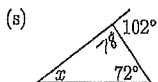
$-x = -7 \quad x = 7$

1

(r) If $v^2 = u^2 + 2aS$, find v when $u = 2$, $a = 10$, and $S = 3$.

$v^2 = 4 + 2 \times 10 \times 3 = 64$
 $v = \pm 8$

1



Find x .

30

1

(s) My *Edvest* account pays 6% p.a. simple interest. If I invest \$1200 for two years, what will my investment be worth?

$S.I = 1200 \times \frac{6}{100} \times 2 = 144$
worth \$1344

1

Question 2 (20 marks)

Answers

(a) Fully factorise the following:

(i) $t^2 - 10t + 16$

$(t-2)(t-8)$

1

(ii) $4x^2 - 4 = 4(x^2 - 1)$

$= 4(x-1)(x+1)$

2

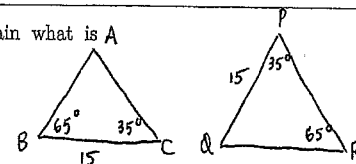
(b) Find the smallest part when \$19 is divided in the ratio 2 : 3 : 5.

$\frac{2}{10} \times \$19 = \3.80

2

(c) With the aid of a diagram, explain what is A wrong with the following proof:

In Δ s ABC , PQR ,
 $\angle ABC = \angle QRP = 65^\circ$,
 $\angle BCA = \angle QPR = 35^\circ$,
 $BC = QP = 15 \text{ cm}$.
 $\therefore \Delta ABC \equiv \Delta PQR$ (AAS).



4

BC and QP are not corresponding sides

(d) Given the points $A(3, 6)$ and $B(-3, -2)$, write down the

(i) midpoint of AB , $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = (0, 2)$

1

(ii) length of the interval AB , $d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$
 $= \sqrt{(-6)^2 + (-8)^2}$
 $= 10$

1

(iii) gradient of the line AB . $m = \frac{y_2-y_1}{x_2-x_1} = \frac{-2-6}{-3-3} = \frac{-8}{-6} = \frac{4}{3}$

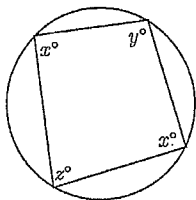
1

(e) Find x if $5\sqrt{7} - \sqrt{x} = 0$.

2

$$\begin{aligned}\sqrt{x} &= 5\sqrt{7} = \sqrt{25 \times 7} \\ &= \sqrt{175} \quad \therefore x = 175\end{aligned}$$

(f)



(i) Find the value of x , giving reasons.

2

$$x + x = 180 \quad (\text{opposite angles are supplementary})$$

$$\therefore 2x = 180$$

$$\therefore x = 90$$

(ii) Write an equation connecting z and y .

1

$$z + y = 180$$

(g) Solve simultaneously, the system of equations

3

$$\begin{array}{l} 4x - 5y = 2 \quad \text{--- (1)} \\ 3x - 2y = -2 \quad \text{--- (2) } \times 2.5 \quad \text{--- (3)} \\ 7.5x - 5y = -5 \quad \text{--- (3)} \end{array} \quad \text{--- (1) - (3)}$$

$$3.5x = -7$$

$$\therefore x = -2$$

sub into (1)

$$-8 - 5y = 2$$

$$\therefore -5y = 10$$

$$y = -2$$

$$\therefore \boxed{x = -2, y = -2}$$

Question 3 (15 marks)

Answers

(a) Evaluate the pronumerals to the nearest whole number or whole degree, as appropriate.

1

$$(i) b = \frac{33.8 \sin 117^\circ}{\sin 38^\circ}$$

$$49 \\ (48.916...)$$

$$(ii) c^2 = 36^2 + 23^2 - 2 \times 36 \times 23 \cos 28^\circ$$

$$\approx 362.839$$

$$c \approx 19$$

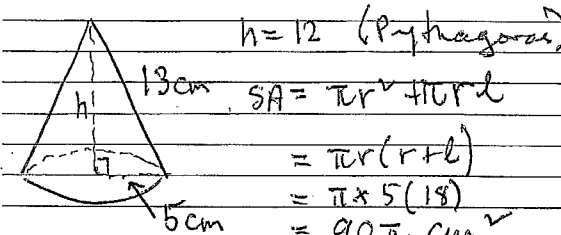
$$(iii) \sin A = \frac{407 \sin 71^\circ}{536}$$

1

$$A = 41^\circ$$

(b) Find the volume and total surface area of a solid cone of radius 5 cm and slant height 13 cm.

4



$$h = 12 \quad (\text{Pythagoras})$$

$$SA = \pi r^2 + \pi r l$$

$$= \pi r(r+l)$$

$$= \pi \times 5(18)$$

$$= 90\pi \text{ cm}^2$$

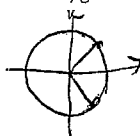
$$V = \frac{1}{3} \pi r^2 h$$

$$= \frac{1}{3} \pi (25) \times 12$$

$$= 100\pi \text{ cm}^3$$

(c) Find all possible values of θ , given $0^\circ \leq \theta \leq 360^\circ$.

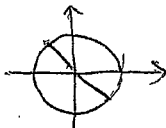
(i) $\cos \theta = \frac{1}{\sqrt{2}}$



$\theta = 45^\circ, 315^\circ$

1

(ii) $\tan \theta = -\sqrt{3}$

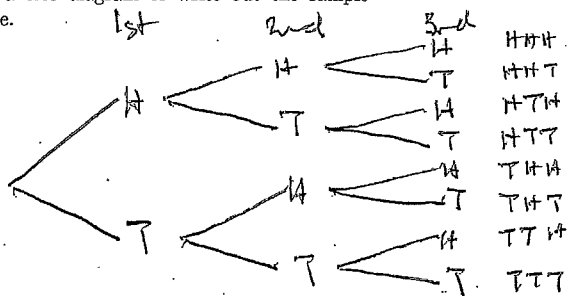


$\theta = 120^\circ, 300^\circ$

1

(d) Three coins are tossed simultaneously.

(i) Use a tree diagram to write out the sample space.



3

Find the probability of obtaining

(ii) three heads,

$P(\text{HHH}) = \frac{1}{8}$

1

(iii) at least one head,

$P(\text{at least 1 head}) = 1 - P(\text{TTT})$
 $= \frac{7}{8}$

1

(iv) at most one head.

$P(\text{at most 1 H}) = P(\text{TTT}) + P(\text{1 H})$
 $= \frac{1}{8} + \frac{3}{8} = \frac{1}{2}$

1

Question 4 (15 marks)

Answers

(a) Find (to the nearest dollar) the compound interest earned if \$96 000 is invested for 5 years at 6.5% p.a.

2

$A = P(1+r)^n - P \quad n=5, r=0.065, P=96000$

$A = 96000(1+0.065)^5 - 96000$

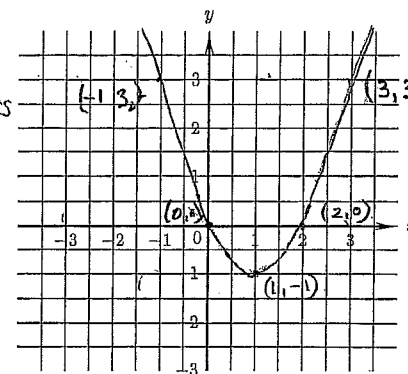
$= \$35,528$

18131

35528

(b) Sketch $y = x^2 - 2x$ on the grid below, showing clearly any intercepts and maximum or minimum points.

3



2 = no labels.
 2 1/2 labels but careless

vertex = $-\frac{b}{2a}$
 $x = \frac{2}{2} = 1$
 $\therefore y = -1$
 $x=0 \quad y=0$
 $y=0 \quad x=2$

$x=0 \quad y=0 \quad y=0 \quad 0=x^2-2x \quad 0=(x)(x-2) \quad x=0 \quad x=2$

(c) Solve $x^2 = 3x + 7$ by the method of completing the square. Give your answer as a surd in simplest form and as a decimal approximation correct to 2 decimal places.

3

$x^2 = 3x + 7$

$x^2 - 3x + (\frac{3}{2})^2 = 7 + \frac{9}{4}$

$(x - \frac{3}{2})^2 = \frac{37}{4}$

$x = \frac{3}{2} \pm \frac{\sqrt{37}}{2}$

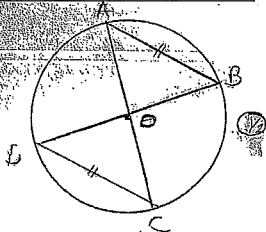
$2 \frac{1}{2}$ for no decimal $x = \frac{3 \pm \sqrt{37}}{2}$ or $x = 4.54$ or $x = -1.54$
 2 for only 1 value.

- (d) The probability that a certain event will happen is $\frac{1}{x}$. Which of the following is the probability that the event will not happen?

$\frac{x}{1}$, $\frac{1-x}{x}$, $\frac{x+1}{x}$, $\frac{x-1}{x}$

$1 - \frac{1}{x} = \frac{x-1}{x}$

- (e) Prove that equal chords subtend equal angles at the centre of a circle.



In $\triangle AOB$ & $\triangle COD$.

Let $AB = CD$ (equal chords)
 $OA = OC$ (equal radii)
 $OB = OD$ (similarly)
 $AB = CD$ (given)

} Congruence. (1 1/2)

by SSS $\triangle OAB \cong \triangle OCD$. SOLUTION (1)
 $\therefore \angle AOB = \angle COD$ (Corresp. \angle 's of congruent \triangle 's)

- ① alt \angle 's.
 ② vert opp.

- (f) When blowing up a balloon, the first hearty breath gives a diameter of 7 cm. By the time it is fully blown up, its diameter is 21 cm. Find in simplest form:

- (i) the ratio of these diameters,

$7:21 = 1:3$

- (ii) the ratio of the respective surface areas,

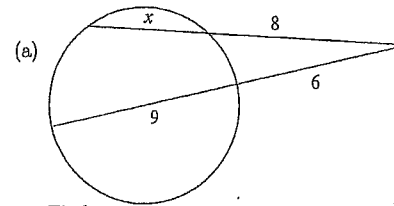
$1:9$

- (iii) the ratio of the respective volumes.

$1:27$

Question 5 (15 marks)

Answers



Find x .

$(x+8)8 = (9+6) \cdot 6$
 $8x+64 = 90$

$x = 3\frac{1}{4}$

- (b) If $\frac{3}{4}$ of the boys in a junior class use a backpack, and $\frac{2}{5}$ are not in full uniform, find the probability that a boy chosen by lot would be in uniform and using a backpack.

$\frac{3}{4} \times \frac{2}{5} = \frac{9}{20}$

- (c) The numbers 30, 67, x , 2, 24, 9, 7, 83 have a mean of 32.

- (i) Calculate the value of x .

$\frac{222+x}{8} = 32$

$x = 34$

- (ii) Find the median.

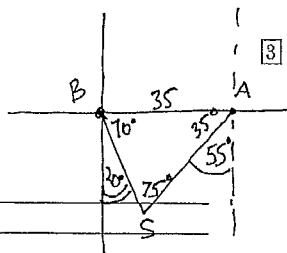
2, 7, 9, 24, 30, 32, 67, 83

\therefore median is $\frac{24+30}{2} = 27$

- (iii) Use your calculator to find the standard deviation correct to three significant figures.

27.3

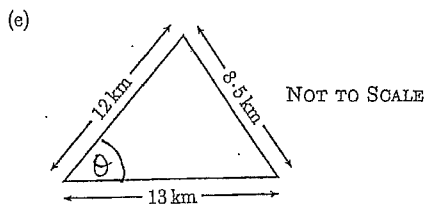
(d) A is a wireless station 35 nautical miles East of another station at B. A ship in a fog discovers by wireless direction finder that she is S20°E of B and S55°W of A. How far is she from B (to the nearest nautical mile).



By the Sine Rule

$$\frac{35}{\sin 75^\circ} = \frac{BS}{\sin 35^\circ}$$

$$\Rightarrow BS = \frac{35 \sin 35^\circ}{\sin 75^\circ} \approx 20.78 \quad \therefore \underline{21 \text{ n. miles}}$$



(i) Calculate the size of the smallest angle in this triangle correct to the nearest minute.

Smallest angle is opposite the shortest side. ($\therefore \theta$)

$$\cos \theta = \frac{12^2 + 13^2 - 8.5^2}{2 \cdot 12 \cdot 13}$$

$$\theta = 39^\circ 30'$$

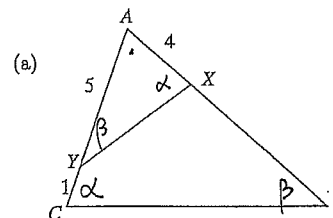
(ii) Hence find its area to the nearest square metre.

$$\text{Area} = \frac{1}{2} \cdot 12 \cdot 13 \cdot \sin 39^\circ 30' = 49.6 \text{ km}^2$$

ANS: 49.6 km²

Question 6 (15 marks)

Answers



$$\begin{aligned} AX &= 4, \\ AY &= 5, \\ YC &= 1, \\ \angle AXY &= \angle ACB. \end{aligned}$$

Calculate, giving reasons, the value of the following ratios:

(i) $XY : CB$

$$\triangle AXY \parallel \triangle ACB$$

(equiangular).

$$\therefore \frac{XY}{CB} = \frac{4}{6} = \frac{2}{3}$$

(ratio of corresponding sides are equal).

(ii) Area $\triangle AXY$: area $\triangle ACB$

$$A_1 : A_2 = l_1^2 : l_2^2$$

$$= \frac{4}{9} \quad 2$$

(iii) Area $\triangle AXY$: area $\triangle AXC$

$$A = \frac{1}{2} ab \sin C$$

$$\text{Area of } \triangle AXY = \frac{1}{2} \times 4 \times 5 \times \sin A$$

$$\frac{1}{2} \times 4 \times 6 \times \sin A$$

$$\therefore \frac{\triangle AXY}{\triangle AXC} = \frac{5}{6} \quad 2$$

(b) If $\frac{1}{x} = \frac{1}{a} + \frac{1}{b}$,

(i) show that $x = \frac{ab}{a+b}$.

$$\frac{1}{x} = \frac{a+b}{ab}$$

$$\therefore x = \frac{ab}{a+b}$$

1

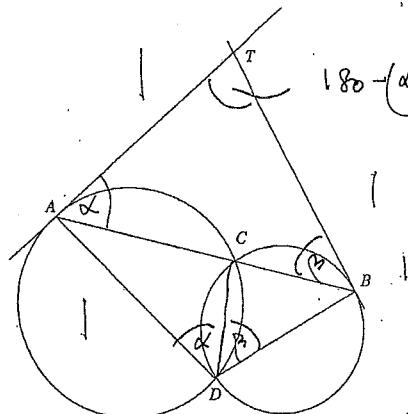
(ii) Hence prove that $\sqrt{\frac{a-x}{b-x}} = \frac{a}{b}$ where a, b are positive.

2

$$\sqrt{\frac{a - \frac{ab}{a+b}}{b - \frac{ab}{a+b}}} = \sqrt{\frac{\frac{a^2+ab-ab}{a+b}}{\frac{b^2+ab-ab}{a+b}}} = \sqrt{\frac{a^2}{b^2}} = \frac{a}{b} \quad a, b > 0$$

(c)

5



ACB is a straight line.
TA and TB are tangents to the circles.
Prove that TADB is a cyclic quadrilateral.

Join CD.
Let $\angle TAB = \alpha$
 $\angle TBA = \beta$.

$\therefore \angle CDA = \alpha$.
(alternate segment theorem)

Similarly, $\angle CDB = \beta$.

In $\triangle TAB$, $\angle ATB = 180 - (\alpha + \beta)$
(angle sum of $\triangle TAB$).

In quad ATBD, $\angle ATB + \angle ADB = 180$.

End of Paper

\therefore TADB is a cyclic quad.