SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2008

Year 10

Yearly Examination

Mathematics

General Instructions
- Working time – 90 minutes
- Write using black or blue pen. Pencil maybe used for diagrams.
- Board approved calculators may be used.
- All *necessary* working should be shown in every question if full marks are to be awarded.
- Marks may **NOT** be awarded for messy or badly arranged work.
- If more space is required, clearly write the number of the QUESTION on one of the back pages and answer it there. Indicate that you have done so.
- Clearly indicate your class by placing an **X**, next to your class.
- Answer in simplest exact form unless otherwise instructed.

NAME:

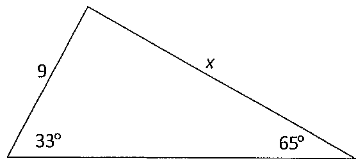
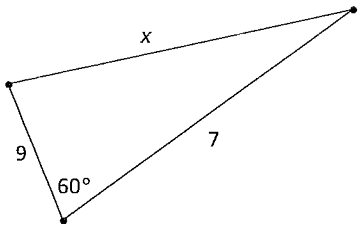
Examiner: *C.Kourtesis*

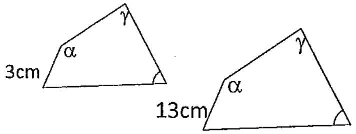
Class	Teacher	
10A	Mr. Fuller	
10B	Mr. McQuillan	
10C	Mr. Choy	
10D	Ms. Ward	
10E	Ms. Nesbitt	
10F	Mr. Boros	

Question	Mark	
1		/20
2		/16
3		/15
4		/16
5		/15
6		/18
Total		/100

Question One (20 Marks)		Answers	Marks
A	Find 18% of $640.		
B	Simplify $\frac{a}{4}+\frac{2a}{3}$		
C	Simplify $\frac{12a-4}{4}$		
D	If $\sqrt{12}+\sqrt{3}=\sqrt{b}$ find the value of b.		
E	Solve the inequality $5-3x<10$		
F	The volume of a cube is 64cm^3. What is its surface area?		
G	If $a -\ \ 3$ and $b=5$, evaluate ba^2.		
H	Express $\sqrt{1.6\times10^9}$ in standard (scientific) notation.		

I	Simplify $2(a+b)-(2a-b)$		
J	If $\sin\theta = \frac{\sqrt{3}}{2}$ where $0^\circ \le \theta \le 180^\circ$, find θ.		
K	Express with h as the subject of the equation $d = 25\sqrt{\frac{h}{2}}$		
L	On separate diagrams sketch the graphs of: (i) $y = x^2$ (ii) $xy = 1$ (iii) $x^2 + y^2 = 100$		
M	If $k = \frac{4}{a}$ find k^{-3} (answer in positive index form)		

Question Two (16 Marks)		Answer	Marks
A	 Find the value of x, correct to 2 decimal places		
B	 Use the Cosine Rule to find the value of x. (Leave your answer in Surd form)		
C	Ronald has a jar containing 120 jelly beans. Each jelly bean is either red, yellow or black. The ratio of red to yellow to black is 4 : 5 : 3. Ronald chooses a jelly bean at random. Find the probability it is: (i) Black (ii) Not Yellow		

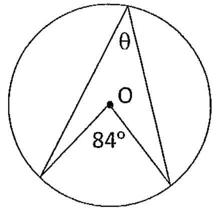
<table>
<tr><td>D</td><td>The graph of $y - 4 \quad kx^2$ passes through the point (-5, 2). Find the value of k.</td><td></td><td></td></tr>
<tr><td>E</td><td>Consider the polygons:


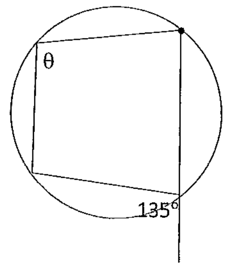
(i) Find the ratio of their areas.

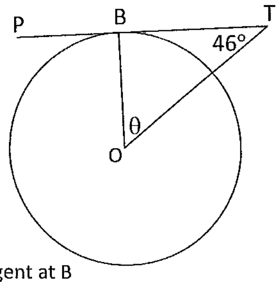
(ii) If the area of the smaller polygon is 30cm², find the area of the larger.</td><td></td><td></td></tr>
<tr><td>F</td><td>The equation of a parabola is given by $y = x^2 - 4x + 3$
(i) Find the x and y intercepts.

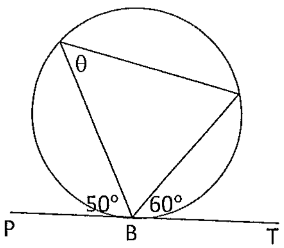
(ii) Find the coordinates of the vertex.

(iii) Hence, sketch the graph of the parabola.</td><td></td><td></td></tr>
</table>

<table>
<tr><th>Question Three (15 Marks)</th><th></th><th>Answers</th><th>Marks</th></tr>
<tr><td>A</td><td>Find the value of θ in each case. You are not required to give reasons. O is the centre of the circle:
(i)


(ii)


(iii)


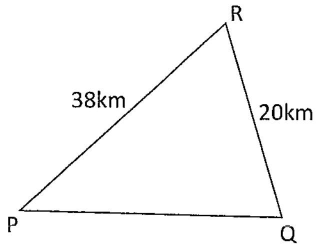
PT is a tangent at B
(iv)


PT is a tangent at B</td><td></td><td></td></tr>
</table>

B In 1954 a total of 6527mm of rain fell at Sprinkling Tarn and this set a UK record for annual rainfall. The tarn has a surface area of 23450m². How many litres of water fell on Sprinkling Tarn in 1954?

C Factorise $A^2-(B+C)^2$

D



In the diagram, the point Q is due east of P. The point R is 38km from P and 20km from Q. The bearing of R from Q is 325°.

(i) What is the size of $\angle PQR$?

(ii) What is the bearing of R from P?

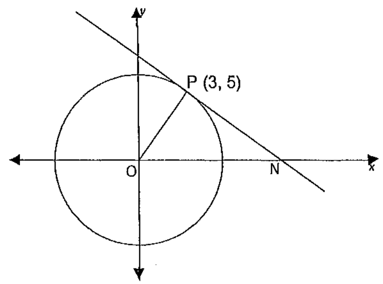
E

Test	Kim's Mark	Class Mean	Class Standard Deviation
A	79	60	20
B	70	60	10

Indicate, giving reasons, in which test Kim performed better.

Question Four (16 Marks)	Answers	Marks

A



OP is a radius of the circle. PN is a tangent.

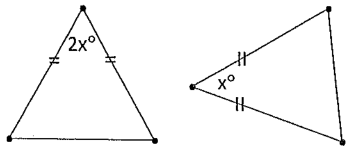
(i) Calculate the gradient of OP.

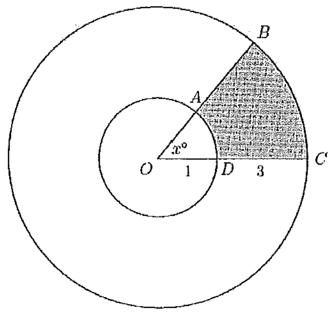
(ii) Show that the equation of PN is $3x+5y+24=0$.

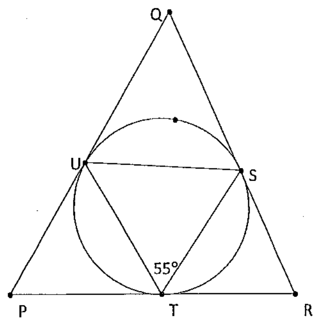
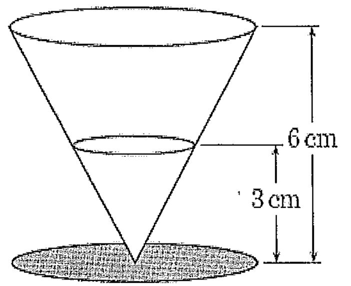
(iii) Find the coordinates of N.

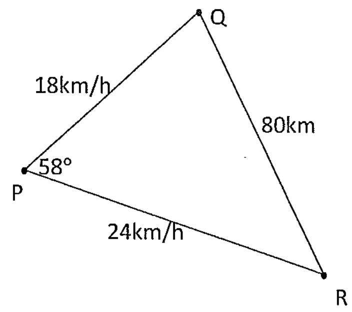
(iv) Write down the equation of the circle.

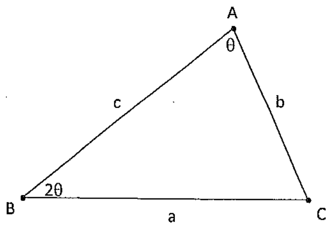
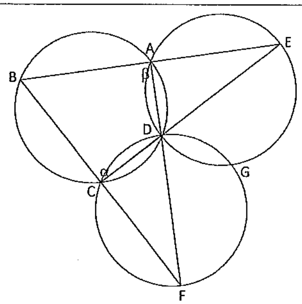
B A 20cm by 5cm by 6cm block of lead is melted and cast into identical spherical fishing sinkers each of radius 1cm.
How many (whole) sinkers can be made?

C	 The two triangles have equal areas and the four lengths are equal. What is the value of x?		
D	The equation of a circle is $x^2 + y^2 - 2x + y = 0$. (i) Express this in the form: $(x-a)^2 + (y-b)^2 = r^2$ (ii) Write down the coordinates of the centre and the length of the diameter.		

Question Five (15 Marks)		Answers	Marks
A	On separate diagrams sketch the graphs of the following, indicating the x and y intercepts in each case: (i) $y = (x+3)(x-1)(x-4)$ (ii) $y = (x+1)^2(x-3)$ (iii) $y - 1 \quad (x \quad 1)^4$		
B	O is the centre of both circles with radii 1cm and 4cm.  (i) Show that the shaded are A is given by $A = \frac{\pi x}{24}$ (ii) If the shaded area is one sixth of the area of the outer circle find the value of x		

C	 The largest circle which it is possible to draw inside triangle PQR touches the triangle at S, T and U. If $\angle STU = 55°$, find the size of $\angle PQR$. (Do Not Give Reasons).		
D	 A medicine glass in the shape of a cone has a height of 6cm. 3mL of liquid fills the cone to a height of 3cm. How many more mL of liquid is required to fill the cone to a height of 6cm?		

Question Six (18 Marks)		**Answers**	**Marks**
A	 Two straight roads PQ and PR are inclined to each other at 58°. Two bike riders begin simultaneously from P and travel along the roads at 18km/h and 24km/h respectively. After t hours they are 80km apart in a direct line. i) Show that $t = \frac{80}{\sqrt{(900 - 864\cos 58°)}}$ ii) Find the value of t (correct to 2 decimal places)		
B	Two regular polygons have N and (N – 5) number of sides. The number of degrees of each of their angles differ by 1. (i) Show that $N^2 \quad 5N \quad 1800 - 0$ (ii) Find the possible value(s) of N.		

C	 Consider the triangle ABC. i) Given the fact that $\sin 2\theta = 2\sin\theta\cos\theta$, use the sine rule to show that $\cos\theta = \frac{b}{2a}$ ii) Hence prove that: $b^2 = a(a + c)$ where $a \neq c$.		
D	 Let $\angle BCD = \alpha$ and $\angle BAD = \beta$ ABCD is a cyclic quadrilateral. BA and CD are both produced and intersect at E. BC and AD produced intersect at F. The circles EAD, FCD intersect at G as well as at D. Prove that the points E, G and F are collinear.		

Year 10 Yearly 2008

Question 1

A) $\$115.20$ ①

B) $\frac{3a+8a}{12} = \frac{11a}{12}$ ①

C) $\frac{4(3a-1)}{4} = 3a-1$ ①

D) $\sqrt{12} = 2\sqrt{3}$

$2\sqrt{3} + \sqrt{3} = 3\sqrt{3}$

$= \sqrt{27}$

$b = 27$ ②

E) $5 - 3x < 10$

$-3x < 5$

$x > -\frac{5}{3}$ ①

F) $SA = 6x^2$

$V = x^3 = 64$

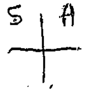
$x = 4$

$SA = 6 \times 4^2 = 96\ cm^2$ ②

G) $5\times(-3)^2 = 5\times 9 = 45$ ①

H) 4.0×10^4 or $(16\times 10^8)^{\frac{1}{2}} = 4\times 10^4$ etc. ①

I) $2a + 2b - 2a + b = 3b$ ①

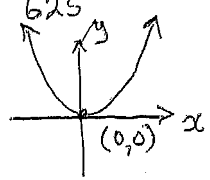
J)  $\theta = 60^\circ, 120^\circ$ ②

K) $d = 25\sqrt{\frac{h}{2}}$

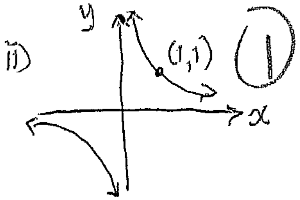
$\frac{d}{25} = \sqrt{\frac{h}{2}}$

$\frac{d^2}{625} = \frac{h}{2}$

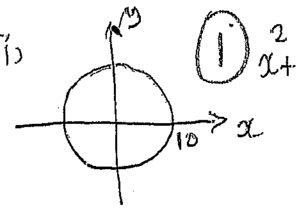
$h = \frac{2d^2}{625}$ ②

L) (i) 

$y = x^2$ ①

(ii) 

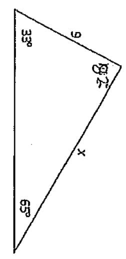
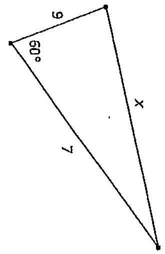
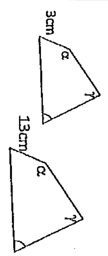
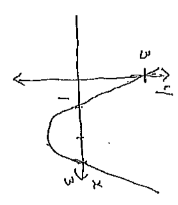
$y = \frac{1}{x}$ ①

(iii) 

$x^2 + y^2 = 100$ ①

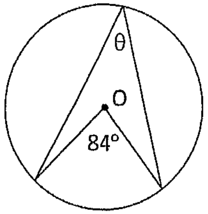
M) $k^{-3} = \frac{1}{k^3} = \frac{1}{\frac{4^3}{a^3}} = 1 \div \frac{64}{a^3}$

$= 1 \times \frac{a^3}{64} = \frac{a^3}{64}$ ②

Question Two (16 Marks)		Answer	Marks
A	 Find the value of x, correct to 2 decimal places	$\frac{x}{\sin 33} = \frac{9}{\sin 65}$; $x = \frac{9\sin 33}{\sin 65}$; $x = 5.408$; $x = 5.41$ (2 dec pl)	
B	 Use the Cosine Rule to find the value of x. (Leave your answer in Surd form)	$x^2 = 7^2 + 9^2 - 2\times7\times9\cos 60$; $x = \sqrt{67}$	
C	Ronald has a jar containing 120 jelly beans. Each jelly bean is either red, yellow or black. The ratio of red to yellow to black is 4 : 5 : 3. Ronald chooses a jelly bean at random. Find the probability it is: (i) Black (ii) Not Yellow	120 jelly beans, 12 parts; Red $\frac{4}{12}\times120 = 40$; Yell or $\frac{5}{12}\times120 = 50$; Black $\frac{3}{12}\times120 = 30$; $P(B) = \frac{30}{120} = \frac{1}{4}$; $P(\bar{Y}) = \frac{70}{120} = \frac{7}{12}$	
D	The graph of $y = 4 - kx^2$ passes through the point (-5, 2). Find the value of k.	$2 = 4 - k\times 25$; $25k = 2$; $k = \frac{2}{25}$	
E	Consider the polygons:  (i) Find the ratio of their areas. (ii) If the area of the smaller polygon is $30cm^2$, find the area of the larger.	ratio of areas $= \frac{9}{169}$; 9:169; Larger area $= 30\times\frac{169}{9}$ $= 563\frac{1}{3}$	
F	The equation of a parabola is given by $y = x^2 - 4x + 3$ (i) Find the x and y intercepts. (ii) Find the coordinates of the vertex. (iii) Hence, sketch the graph of the parabola.	$x = 0$, $y = 3$ (0,3) (y int); $y = 0$ $(x-3)(x-1) = 0$, $x = 1, 3$ (1,0) (3,0) (x intercepts); vertex $x = 2$ $y = -1$; 	

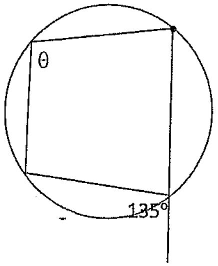
Question Three (15 Marks)		Answers	Marks
A	Find the value of θ in each case. You are not required to give reasons. O is the centre of the circle:		

(i)



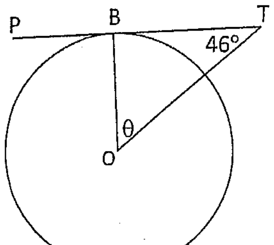
Answer: $\theta = 42^\circ$

(ii)



Answer: $\theta = 135^\circ$

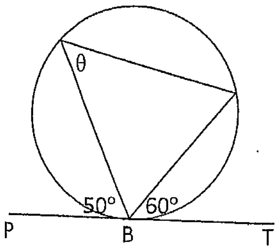
(iii)



PT is a tangent at B

Answer: $\theta = 44^\circ$

(iv)



PT is a tangent at B

Answer: $\theta = 60^\circ$

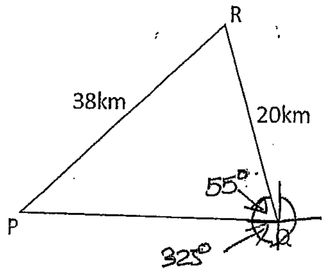
B In 1954 a total of 6527mm of rain fell at Sprinkling Tarn and this set a UK record for annual rainfall. The tarn has a surface area of 23450m². How many litres of water fell on Sprinkling Tarn in 1954?

Answer: 153,058,150 L.

C Factorise $A^2-(B+C)^2$

Answer: $(A-(B+C))(A+B+C)$
$=(A-B-C)(A+B+C)$

D



In the diagram, the point Q is due east of P. The point R is 38km from P and 20km from Q. The bearing of R from Q is 325°.

(i) What is the size of $\angle PQR$?

Answer: 55°.

(ii) What is the bearing of R from P?

Answer: $\sin P = \frac{20 \sin 55^\circ}{38} = 25^\circ 32'$.
R from P $= 90 - 25^\circ 32' = 64^\circ 28'$

E

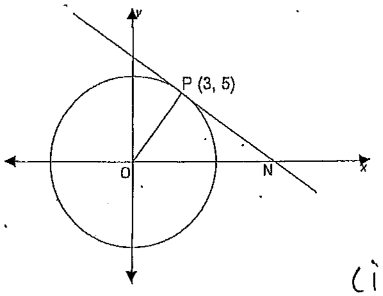
Test	Kim's Mark	Class Mean	Class Standard Deviation
A	79	60	20
B	70	60	10

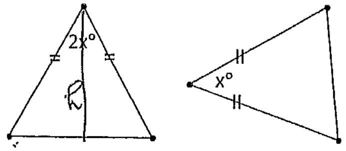
Indicate, giving reasons, in which test Kim performed better.

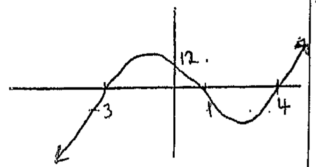
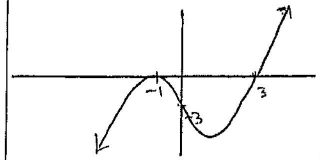
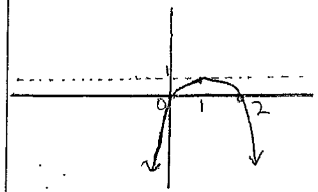
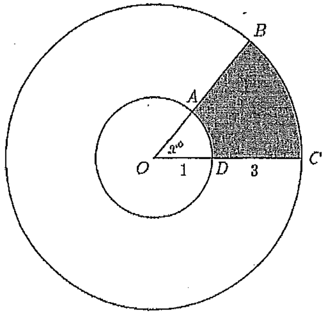
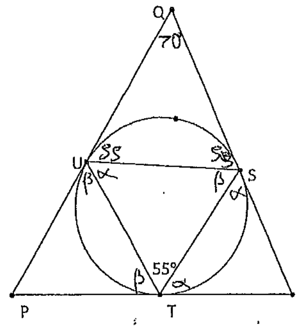
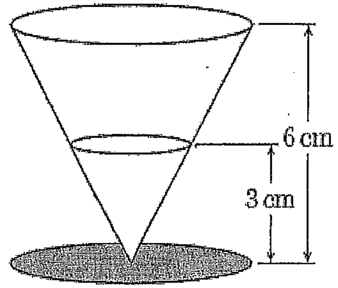
Test A, kim is 19 away from mean and almost 1 standard deviation

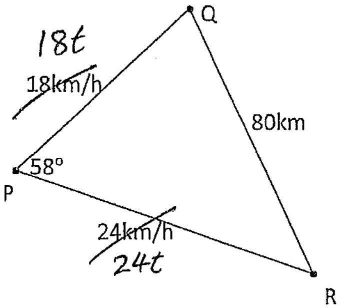
Test B, kim is 10 away & exactly 1 standard deviation.

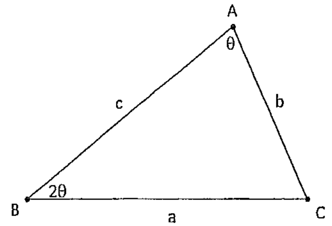
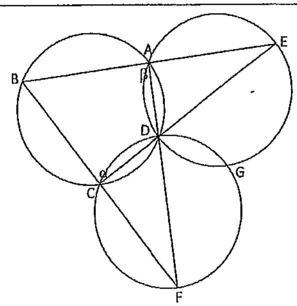
Did better in TEST B as his standard deviation is further away from mean

Question Four (16 Marks)		Answers	Marks
A	 OP is a radius of the circle. PN is a tangent. (i) Calculate the gradient of OP. (ii) Show that the equation of PN is $3x+5y-34=0$. (iii) Find the coordinates of N. (iv) Write down the equation of the circle.	(i) $m_{OP} = 5/3$. $m_{PN} = -3/5$ $y-5 = -\frac{3}{5}(x-3)$ $5y-25 = -3x+9$ (ii) $\therefore 3x+5y-34=0$. (iii) When $y=0$ $x = \frac{34}{3}$ $\therefore N\left(\frac{34}{3}, 0\right)$. $\lvert OP \rvert = \sqrt{9+25}$ $= 34$ (iv) $\therefore x^2+y^2=34$.	
B	A 20cm by 5cm by 6cm block of lead is melted and cast into identical spherical fishing sinkers each of radius 1cm. How many (whole) sinkers can be made?	$V = 20 \times 30$ $= 600$ $V = \frac{4}{3}\pi r^3 = \frac{4\pi}{3}$ $= 143$ whole	

C	 The two triangles have equal areas and the four lengths are equal. What is the value of x? $\frac{1}{2}x^2 \sin 2x = \frac{1}{2}x^2 \sin x$ $\sin 2x = \sin x$ $\therefore x = 60^0$		
D	The equation of a circle is $x^2+y^2-2x+y=0$. (i) Express this in the form: $(x-a)^2+(y-b)^2=r^2$ (ii) Write down the coordinates of the centre and the length of the diameter. $(x^2-2x+1)+(y^2+y+\frac{1}{4}) = \frac{5}{4}$ $(x-1)^2+(y+\frac{1}{2})^2 = \frac{5}{4}$ $\therefore C = (2, -\frac{1}{2})$ $r = \frac{\sqrt{5}}{2}$ $\therefore$ diameter $= \sqrt{5}$		

	Question Five (15 Marks)	Answers	Marks
A	On separate diagrams sketch the graphs of the following, indicating the x and y intercepts in each case:		
	(i) $y=(x+3)(x-1)(x-4)$		2
	(ii) $y=(x+1)^2(x-3)$		2
	(iii) $y=1-(x-1)^4$		2
B	O is the centre of both circles with radii 1cm and 4cm.  (i) Show that the shaded are A is given by $A=\frac{\pi x}{24}$	(i) $\frac{x}{360}(4^2\pi-1^2\pi)$ $=\frac{x}{360}(15\pi)$ $=\frac{\pi x}{24}$.	1
	(ii) If the shaded area is one sixth of the area of the outer circle find the value of x	(ii) $6\times\frac{\pi x}{24}=16\pi$ $x=64°$.	2
C	 The largest circle which it is possible to draw inside triangle PQR touches the triangle at S, T and U. If $\angle STU=55°$, find the size of $\angle PQR$. (Do Not Give Reasons).	$\angle PQR=70°$.	3
D	 A medicine glass in the shape of a cone has a height of 6cm. 3mL of liquid fills the cone to a height of 3cm. How many more mL of liquid is required to fill the cone to a height of 6cm?	1:2. lengths 1:8 volumes. 24mL. i.e. 21mL more.	3

Question Six (18 Marks)		Answers	Marks
A	 Two straight roads PQ and PR are inclined to each other at 58°. Two bike riders begin simultaneously from P and travel along the roads at 18km/h and 24km/h respectively. After t hours they are 80km apart in a direct line. i) Show that $t=\frac{80}{\sqrt{(900-864\cos 58^\circ)}}$ ii) Find the value of t (correct to 2 decimal places)	(i) $80^2=(18t)^2+(24t)^2-2(18t)(24t)\cos 58$ $80^2=324t^2+576t^2-864t^2\cos 58$ $t^2(900-864\cos 58)=80^2$ $t^2=\frac{80^2}{900-864\cos 58}$ $t=\pm\sqrt{\frac{80^2}{900-864\cos 58}}$ $=\frac{80}{\sqrt{(900-864\cos 58^\circ)}}$ since $t>0$ (ii) $t=3.80$ hours.	(3) (1)
B	Two regular polygons have N and (N – 5) number of sides. The number of degrees of each of their angles differ by 1. (i) Show that $N^2-5N-1800=0$ (ii) Find the possible value(s) of N.	(i) $\frac{(N-2)180}{N}-\frac{(N-7)180}{N-5}=1$ $(N-2)(N-5)180-N(N-7)180=N(N-5)$ $(N^2-7N+10)180-(N^2-7N)180=N^2-5N$ $(N^2-7N)180+1800-(N^2-7N)180=N^2-5N$ $N^2-5N-1800=0$ (ii) $(N-45)(N+40)=0$ $N=45, -40$ only possible value of N is 45 (since $N>0$)	(3) (2)

C	 Consider the triangle ABC. i) Given the fact that Sin2θ = 2SinθCosθ, use the sine rule to show that $\cos\theta=\frac{b}{2a}$ ii) Hence prove that: $b^2=a(a+c)$ where $a\neq c$.	(i) $\frac{\sin 2\theta}{b}=\frac{\sin\theta}{a}$ $\frac{2\sin\theta\cos\theta}{b}=\frac{\sin\theta}{a}$ $\cos\theta=\frac{b}{2a}$ (ii) $\cos\theta=\frac{b^2+c^2-a^2}{2bc}$ $\frac{b}{2a}=\frac{b^2+c^2-a^2}{2bc}$ $\frac{2b^2c}{2a}=b^2+c^2-a^2$ $b^2c=b^2a+a(c^2-a^2)$ $b^2(c-a)=a(c-a)(c+a)$ $b^2=\frac{a(c-a)(c+a)}{(c-a)}$, $a\neq c$ $b^2=a(c+a)$	(2) (3)
D	 Let ∠BCD = α and ∠BAD = β ABCD is a cyclic quadrilateral. BA and CD are both produced and intersect at E. BC and AD produced intersect at F. The circles EAD, FCD intersect at G as well as at D. Prove that the points E, G and F are collinear.	Note $\alpha+\beta=180^\circ$ (opp. ∠'s of cyclic quad. ABCD) $\angle EGD=\beta$ (ext. ∠ of cyclic quad AEGD equal to opp. int. ∠) $\angle DGF=\alpha$ (ext. ∠ of cyclic quad. DGFC equal to opp. int. ∠) $\angle FGE=\alpha+\beta$ $=180^\circ$ ∴ E, G and F are collinear	(4)