

(S.B.M.S) YEAR 11 EXTENSION 1 TRIGONOMETRY TEST

JULY 2004

NAME: Daniel Ong

SHOW ALL WORKING IN EACH QUESTION

1. If  $t = \tan \frac{\phi}{2}$ ,

(a) Express  $\frac{1 - \cos \phi}{\sin \phi}$  in terms of  $t$ .

(b) Show that  $\frac{1 + \sin \phi + \cos \phi}{1 + \sin \phi - \cos \phi} = \frac{1}{t}$

2. Solve the following to the nearest degree by writing in the form:

$R(\cos x \pm \alpha)$  or

$R(\sin x \pm \alpha)$

(a)  $\sqrt{2} \sin x - \sqrt{2} \cos x = 1 \quad 0 \leq x \leq 360^\circ$

(b)  $9 \cos x + 12 \sin x = 7.5 \quad 0 \leq x \leq 360^\circ$

3. Find the general solution of  $\sin \phi = 0$

Daniel Ong

Trigonometry

$$1. a. \frac{1 - \cos \theta}{\sin \theta} \quad \cos = \frac{1-t^2}{1+t^2} \quad \tan = \frac{t}{1-t^2} \quad \frac{2t}{1-t^2}$$

$$= \frac{1 - \left( \frac{1-t^2}{1+t^2} \right)}{\frac{2t}{1+t^2}}$$

$$= \frac{\frac{1+t^2}{1+t^2} - \frac{1-t^2}{1+t^2}}{\frac{2t}{1+t^2}}$$

$$= \frac{\frac{2t^2}{1+t^2}}{\frac{2t}{1+t^2}}$$

$$= \frac{2t^2}{1+t^2} \quad \cancel{t} \quad \cancel{1+t^2} \quad \cancel{2t} \quad 3$$

$$= \frac{t}{1} = t$$

$$b. \frac{1 + \sin \theta + \cos \theta}{1 + \sin \theta - \cos \theta} = \frac{1}{t}$$

$$\text{LHS} = \frac{1 + \frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2}}{1 + \frac{2t}{1+t^2} - \frac{1-t^2}{1+t^2}}$$

$$= \frac{1+t^2 + 2t + 1-t^2}{1+t^2}$$

$$= \frac{2t^2 + 2t}{1+t^2}$$

$$= \frac{2t+2}{1+t^2}$$

$$= \frac{2(t+1)}{1+t^2} \times \frac{1+t^2}{2t(t+1)}$$

$$= \frac{1}{t}$$

= RHS.

$$2. a. \sqrt{3} \sin x - \sqrt{2} \cos x = 1 \quad (0^\circ \leq x \leq 2\pi)$$

$$r^2 = \sqrt{3}^2 + \sqrt{2}^2$$

$$r = \sqrt{13}$$

$$\theta = \tan^{-1} \frac{\sqrt{2}}{\sqrt{3}} \quad \theta = 45^\circ$$

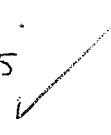
$$(-\frac{\pi}{4} \leq x \leq \frac{7\pi}{4}) \quad \theta = 45^\circ$$

$$(0^\circ \leq x \leq 2\pi) \quad 7$$

$$\sin(x-45^\circ) = \frac{1}{\sqrt{13}}$$

$$\sin(x-45^\circ) = \frac{1}{\sqrt{13}} \quad 150^\circ$$

$$\sin(x-45^\circ) = \frac{1}{\sqrt{13}} \quad 195^\circ$$



Domain

[P TO]

Q. b.  $9 \cos x + 12 \sin x = 7\sqrt{2} \quad 0 \leq x \leq 2\pi$

$$r^2 = 9^2 + 12^2 \quad \tan \alpha = \frac{12}{9} = \frac{4}{3} \Rightarrow \alpha = 53^\circ 8'$$

$$r = 15$$

$$15 \cos(x - 53^\circ 8') = 7\sqrt{2}$$

$$\cos(x - 53^\circ 8') = \frac{7\sqrt{2}}{15}$$

Domain  
 $(-53^\circ 8' \leq x \leq 2\pi + 53^\circ 8')$   $\Rightarrow (x - 53^\circ 8') = 60^\circ 300$   
 $(0^\circ < x \leq 2\pi)$   $x = 113^\circ 9' 353^\circ 8' \checkmark$

3.  $\sin \theta = 0.$

$$\sin \theta = 0, 180, 360, \dots \checkmark$$

$$\theta = -180, -360, \dots$$

General Solution

$$= 180n + (-1)^n \alpha$$

$$= 180n + (-1)^n \times 0.$$

$$= 180n, \# 0.$$

=  $180n$  where  $n$  is an integer

3