(S.B.H.S) YEAR 11 EXTENSION1 TRIGONOMETRY TEST

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SHOW ALL WORKING IN EACH QUESTION

1. If $t = \tan\frac{\phi}{2}$,

   (a) Express $\frac{1-\cos\phi}{\sin\phi}$ in terms of t.

   (b) Show that $\frac{1+\sin\phi+\cos\phi}{1+\sin\phi-\cos\phi} = \frac{1}{t}$

2. Solve the following to the nearest degree by writing in the form:

   $R(\cos x \pm \alpha)$ or

   $R(\sin x \pm \alpha)$

   (a) $\sqrt{2}\sin x - \sqrt{2}\cos x = 1 \quad 0 \le x \le 360°$

   (b) $9\cos x + 12\sin x = 7.5 \quad 0 \le x \le 360°$

3. Find the general solution of $\sin\phi = 0$

Aver. 14. 25/26 well Done!

Daniel Og

Trig emometry.

1a. $\frac{1-\cos\theta}{\sin\theta}$ $\quad \cos = \frac{1-t^2}{1+t^2}$ $\quad \sin = \frac{2t}{1+t^2}$ $\quad \tan = \frac{2t}{1-t^2}$

$= \frac{1-\left(\frac{1-t^2}{1+t^2}\right)}{\frac{2t}{1+t^2}}$

$= \frac{\frac{1+t^2}{1+t^2} - \frac{1-t^2}{1+t^2}}{\frac{2t}{1+t^2}}$

$= \frac{\frac{2t^2}{1+t^2}}{\frac{2t}{1+t^2}}$

$= \frac{2t^2}{2t}$ ✓ 3

$= \frac{t}{1} = t$

b. $\frac{1+\sin\theta+\cos\theta}{1+\sin\theta-\cos\theta} = \frac{1}{t}$

$LHS = \frac{1+\frac{2t}{1+t^2}+\frac{1-t^2}{1+t^2}}{1+\frac{2t}{1+t^2}-\frac{1-t^2}{1+t^2}}$

$= \frac{\frac{1+t^2+2t+1-t^2}{1+t^2}}{\frac{1+t^2+2t-1+t^2}{1+t^2}}$

$= \frac{\frac{2t+2}{1+t^2}}{\frac{2t^2+2t}{1+t^2}}$ 5

$= \frac{2(t+1)}{2t(t+1)} \times \frac{1+t^2}{1+t^2}$ ✓

$= \frac{1}{t}$

$= RHS.$

2a. $\sqrt{3}\sin x - \sqrt{2}\cos x = 1 \quad (0 \le x \le 2\pi)$ $\quad r^2 = \sqrt{3}^2 + \sqrt{2}^2$

$r = 2$

$r\sin(x-\alpha) = 1$ $\quad \tan\alpha = \frac{\sqrt{2}}{\sqrt{3}}$

$2\sin(x-45) = 1$ $\quad \tan\alpha = 1$

$\sin(x-45) = \frac{1}{2}$ $\quad \alpha = 45.$

$(x-45) = 30, 150.$ $\quad (-\frac{\pi}{4} \le x \le \frac{7\pi}{4})$

$x = 75, 195$ ✓ $\quad (0 \le x \le 2\pi)$ 7

Domain ↑

PTO

2.b. $9\cos x + 12\sin x = 7\frac{1}{2} \quad 0 \le x \le 2\pi$

$r^2 = 9^2 + 12^2$ ✓ $\quad \tan\alpha = \frac{12}{9}$

$r = 15$ ✓ $\quad = 53°8'$

$15\cos(x - 53°8') = 7\frac{1}{2}$ 7

Domain $(-53°8' \le x \le 2\pi - 53°8')$ ⇒ $(0 < x \le 2\pi)$

$\cos(x - 53°8') = \frac{1}{2}$

$(x - 53°8') = 60°, 300°$

$x = 113°8', 353°8'$ ✓

3. $\sin\theta = 0.$

$\theta = 0, 180, 360 \ldots$ ✓

$\theta = -180, -360 \ldots$ ✓

General Solution 3

$= 180n + (-1)^n \alpha$

$= 180n + (-1)^n \times 0.$ ✓

$= 180n. \quad \#0.$

$= 180n.$ where n is an integer