SYDNEY BOYS H.S. - 2004

YEAR 11 EXTENSION 1 FUNCTIONS & GRAPHS

MAy 2004

NAME Daniel Ong

1. State the domains and ranges of the following functions.

(a) $\dfrac{1}{x+1}$ (b) x^3+2x+1 (c) $\sqrt{7-x}$ (d) $\dfrac{-2\sqrt{x^2+4}}{3}$ **8**

2. Sketch the region indicated by $x^2+y^2 > 25$ **3**

3. Consider the function $y = f(x) = \dfrac{x^2}{x^2-1}$

(a) What is the domain of the function? **1**

(b) Show that the function passes through the origin. **1**

(c) Determine if the function is odd or even or neither. Justify your answer. **2**

(d) What happens to the value of y as x approaches infinity? Justify your answer. **1**

(e) Sketch the curve showing important features including asymptote(s). **2**

(f) What is the range of the function. **1**

(g) From the graph, determine the values of x for which the curve is increasing. **1**

4. Sketch the region indicated by:

$y < 5+4x-x^2$ and $y \geq 2 - x$ **5**

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YEAR 11 EXTENSION 1 TRIGONOMETRY
MAY 2004
NAME

SHOW WORKING - NO MARKS FOR ANSWERS ONLY

1. (a) Find the value of x if $\sin 20° = \cos(18+x)°$.

   (b) Find the exact value of

   (i) $\cot^2 120° - \operatorname{cosec}^2 120° + \sec^2 180°$

   (ii) $\sin^2(-210°) + \cos^2 690°$

2. Find all the solutions for $\cos \phi = \dfrac{1}{\sqrt{2}}$ for $0 \leq \phi \leq 360°$.

3. Show that

   (a) $\sin^4 x - \cos^4 x = 1 - 2\cos^2 x$

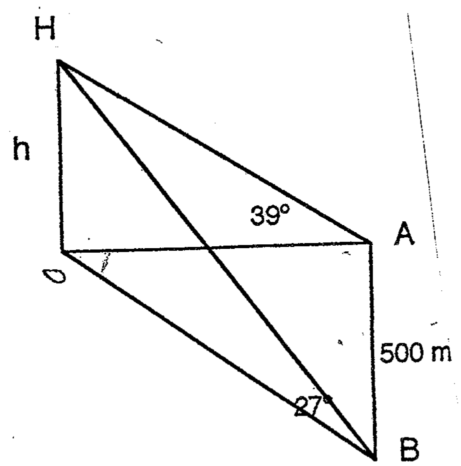
   (b) $\tan 75° = 2 + \sqrt{3}$

   (c) $\dfrac{\sin(x+y)}{\cos(x-y)} = \dfrac{\tan x + \tan y}{1 + \tan x \tan y}$

4. solve for $0° \leq \phi \leq 360°$

   $2\sin\phi - 4\cos\phi = 3$

5. The elevation of a hill at a place A due east of it is 39°. At a place B due south of A, the elevation is 27°. If the distance from A to B is 500 m, find the height of the hill.



Average : 14 Ong $\frac{20}{25}$ Good

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1. a) $\frac{1}{x+1}$

Domain: All real x, $x \neq -1$

Range: All real y, $y \neq 0$

b) x^3+2x+1

Domain: All real x

Range: All real y

c) $\sqrt{7-x}$

Domain: $x \leq 7$

Range: $y \geq 0$

d) $\frac{-2\sqrt{x^2+4}}{3}$

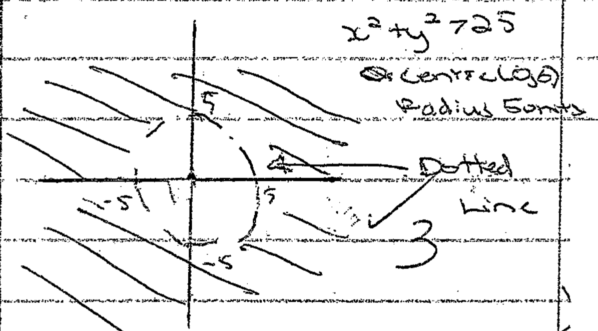
Domain: All real x

Range: $y \leq -4/3$

2. $x^2+y^2 > 25$

Centre (0,0)

Radius 5 units



3

3. $y = f(x) = \frac{x^2}{x^2-1}$

a) Domain: All real x,

b) Sub in $x=0$, $y=0$.

$0 = \frac{(0)^2}{(0)^2-1}$

$\therefore$ RHS = LHS

$\therefore$ $f(x)$ passes through (0,0)

c) $f(x) = \frac{x^2}{x^2-1}$

$f(-x) = \frac{(-x)^2}{(-x)^2-1}$

$= \frac{x^2}{x^2-1}$

$\therefore f(-x) = f(x)$

$\therefore f(x)$ is EVEN

2

d) As x approaches infinity, y approaches 1, never touching 1

Sub in $x=100$

$y = \frac{100^2}{100^2-1}$

$= 1\frac{1}{9999}$

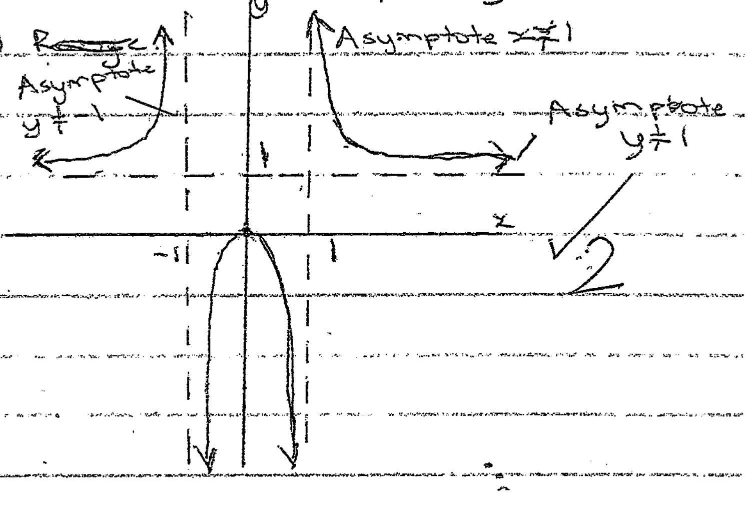
Sub in $x=2$

$y = \frac{2^2}{2^2-1}$

$= 1\frac{1}{3}$

$\therefore$ as $x \to \infty$, the $y \to 1$

e) ~~Range~~



2

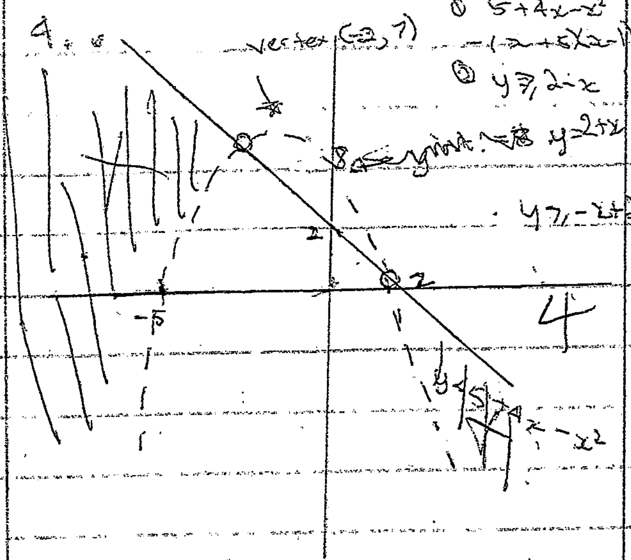
(f) R : $y > 1$ or $y \leq 0$

(g) For increasing function

$x < 0$ except $x \neq -1$

Vertex (-2, -7)

Average : 12.   Ong   Good $\frac{19}{23}$



$5+4x-x^2 = -x+2$

Trigonometry

1. a. $\sin 20 = \cos(18+x)^\circ$

$\cos 70 = \cos(18+x)^\circ$

$\therefore x = 70-18$

$x = 52^\circ$ ✓ ✓

b. $\cot^2 120 - \csc^2 120^\circ + \sec^2 180$

$\left(\frac{1}{-\sqrt{3}}\right)^2 - \left(\frac{1}{\frac{\sqrt{3}}{2}}\right)^2 + \left(\frac{1}{-1}\right)^2$

$\frac{1}{3} - \frac{4}{3} + 1$ ✓✓

$= -\frac{2}{3}$

c) $\sin^2(-210^\circ) + \cos^2 690.$

$\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2$

$= \frac{1}{4} + \frac{3}{4}$

$= 1$ ✓ ✓

2. $\cos\theta = \frac{1}{\sqrt{2}}$

$\theta = 45^\circ, 315^\circ$ ✓ ✓

S A
T C

(3)(a) $L.H.S. = (\sin^2 x)^2 - (\cos^2 x)^2$

$= -(\cos^2 x + \sin^2 x)(\cos^2 x - \sin^2 x)$

$= -(1)(\cos^2 x - (1-\cos^2 x))$

$= -(2\cos^2 x - 1)$

$= 1 - 2\cos^2 x$

$=$ R.H.S. as required.

b) $\tan 75 = 2+\sqrt{3}$

$\tan(45+30) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$

$\tan 75 = \frac{\tan 45 + \tan 30}{1 - \tan 45 \tan 30}$

$= \frac{1 + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}}$

$= \frac{1+\frac{1}{\sqrt{3}}}{1-\frac{1}{\sqrt{3}}} \times \frac{1+\frac{1}{\sqrt{3}}}{1+\frac{1}{\sqrt{3}}}$

$= \frac{1\frac{1}{3} + \sqrt{3}}{\frac{2}{3}}$ ✓ 3

$= 2+\sqrt{3}$

$=$ RHS

c. $\frac{\sin(x+y)}{\cos(x-y)} = \frac{\tan x + \tan y}{1 + \tan x \tan y}$

$\frac{\sin x \cos y + \sin y \cos x}{\cos x \cos y + \sin x \sin y}$

Divide by $\cos x \cos y$

$\frac{\frac{\sin x \cos y + \sin y \cos x}{\cos x \cos y}}{\frac{\cos x \cos y + \sin x \sin y}{\cos x \cos y}}$ ✓

$= \frac{\tan x + \tan y}{1 + \tan x \tan y}$

$=$ RHS.

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Note : $0^\circ \le \theta \le 360^\circ$

4. $2\sin\theta - 4\cos\theta = 3.$ $\Rightarrow$ $R\sin(\theta - \alpha) = 3$

$2\sin\theta = 3 + 4\cos\theta$ where $R = \sqrt{2^2+4^2}$ $\tan\alpha = \frac{4}{2}$

$\cancel{\theta = \frac{3+4\cos\theta}{\cos\theta}}$ $= \sqrt{20} = 2\sqrt{5}$ $\alpha = 63^\circ 26'$

~~2NS.~~ $\therefore 2\sqrt{5}\sin(\theta - 63^\circ 26') = 3$

$2\sin^2\theta - 4\cos^2\theta = 9$ $\frac{1}{2}$ $\therefore \sin(\theta - 63^\circ 26') = 0.6708$

$\cancel{2 = 4}$ $=$ $\therefore \theta - 63^\circ 26' = 42^\circ 8'$ or $137^\circ 52'$

or using 't' result for $-63^\circ 26' \le \theta - 63^\circ 26' \le 296^\circ 34'$

$\therefore \theta = 105^\circ 34'$ or $\cancel{163^\circ 52'}$ $201^\circ 18'$

5. $AO = \tan\frac{h}{39}$

$BO = \tan\frac{h}{27}$

$BO = h\cot 27$

$AO = h\cot 39.$ $1\frac{1}{2}$

$\cancel{500^2 = h^2\cot^2 27 + h^2\cot^2 39 -}$

$\cancel{h^2 = \frac{\cot^2 27 + \cot^2 39 - 2}{500^2}}$ X

Using Pythagoras' Theorem in ΔOAB

$$OB^2 = OA^2 + AB^2$$

$$h^2\cot^2 27^\circ = h^2\cot^2 39^\circ + 500^2$$

$$\therefore h^2(\cot^2 27^\circ - \cot^2 39^\circ) = 500^2$$

$$\therefore h^2 = \frac{500^2}{\cot^2 27^\circ - \cot^2 39^\circ}$$

$$\therefore h = \underline{327.78\ m} \text{ (to 2 d.p.)}$$