SYDNEY BOYS' HIGH SCHOOL

March 2000

11G (ungraded) Class Test #1

MATHEMATICS

TIME ALLOWED: 1 period

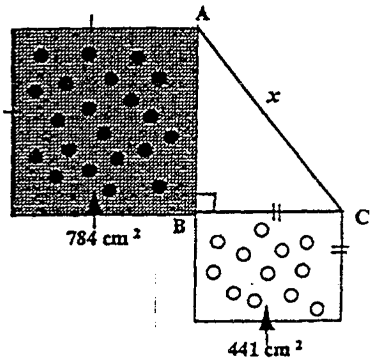
INSTRUCTIONS:

* All questions may be attempted.
* Marks may be deducted for careless or badly arranged work
* Necessary working out should be shown.
* All working and answers are to be written in this test booklet. The back of pages may be used if clearly marked.
* Approved calculators may be used.

NAME:

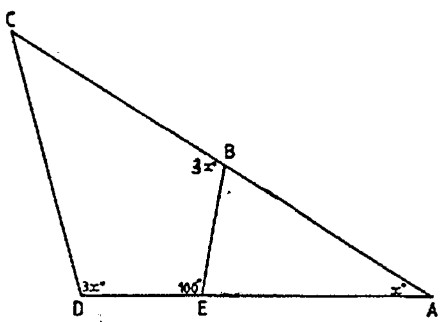
1) Given that $\Delta ABC ||| \Delta FED$, then $\angle ACB = \angle EDF$. True or false, giving a reason.

(2) Find the value of x in the diagram below, which has two squares drawn on its sides.

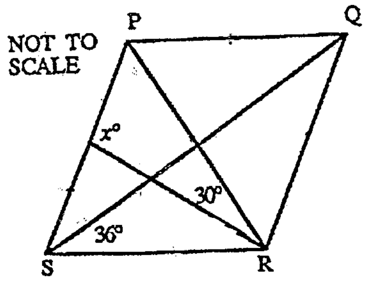


(3) As shown in the figure (which is not to scale), B, E lie on the sides AC, DA respectively of ΔACD.

Use the information shown on the figure to find the value of $A\hat{C}D$.



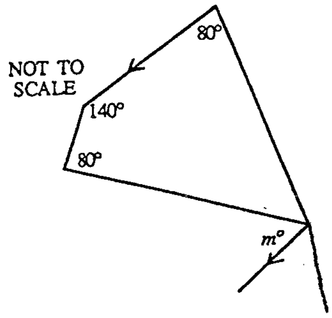
(4) *PQRS* is a rhombus. Find the value of x



(5) Which of the following is **NOT** sufficient to ensure that a quadrilateral is a parallelogram?

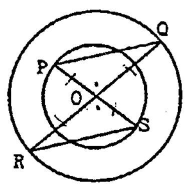
(A) One pair of opposite sides are equal and parallel.
(B) The diagonals are of different lengths.
(C) The diagonals bisect one another.
(D) All four sides are equal.

(6) Find the value of m?

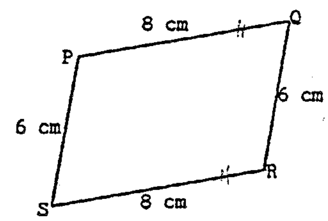


(7) In which diagram is PQ **NOT** necessarily parallel to RS?

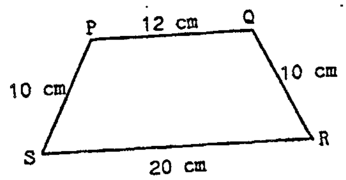
(A) This diagram shows two circles centre O. PS and RQ are diameters.



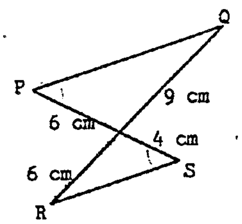
(B)



(C)

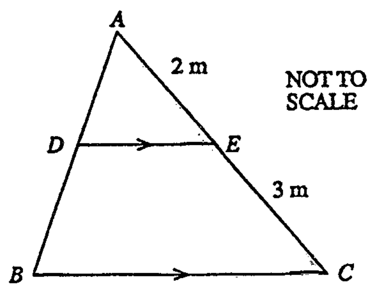


(D)



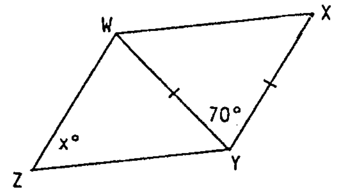
(8) $\Delta\ ADE$ is similar to $\Delta\ ABC$.
$\Delta\ ADE$ has an area of 16 m^2.

Calculate the area of $\Delta\ ABC$.

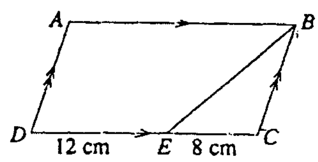


(9) *WXYZ* is a parallelogram.
$WY = XY$.

Find the value of x.

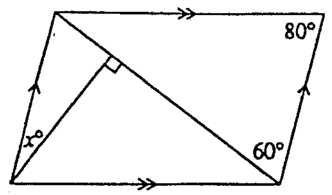


(10) *ABCD* is a paralleogram.
The area of triangle *BCE* is $32\ cm^2$.
Calculate the area of *ABED*.

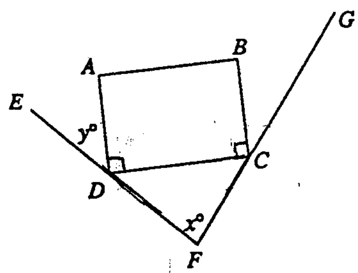


(11)

$x = ?$

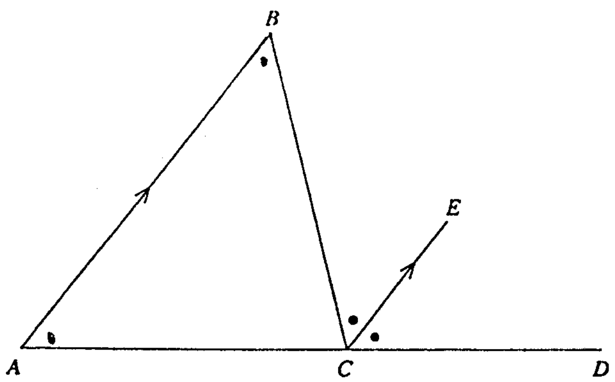


(12) *ABCD* is a rectangle.
EDF and *FCG* are straight lines.
$\angle EDA = y^\circ$ and $\angle DFC = x^\circ$
$\angle BCG = ?$

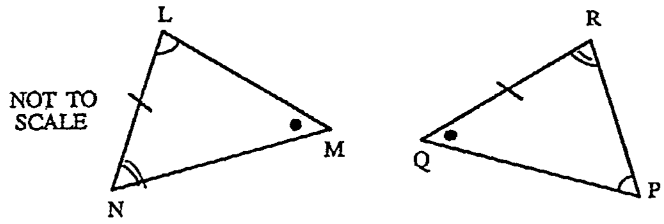


(13) Given $AB \parallel CE$ and CE bisects $\angle BCD$

Prove $AC = BC$.



(14)



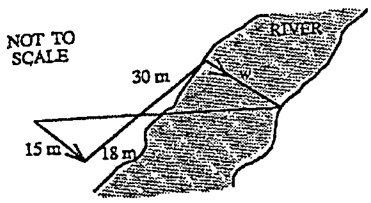
Consider these statements:

I. Triangles *LMN* and *PQR* and congruent.
II. Triangles *LMN* and *PQR* are similar.

Which is always correct?

(A) I only (B) II only (C) Both I and II (D) Neither I nor II

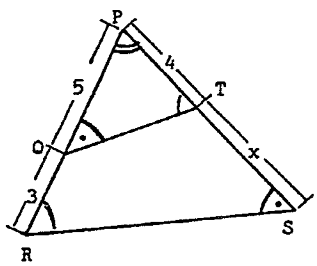
(15)



A student uses the diagram to find the width of the river, w.

Find the value of w.

(16)

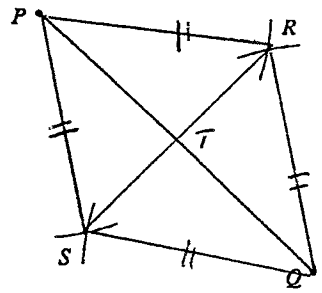


(a) Write down an equation that you could use to find x.

(b) $x = ?$

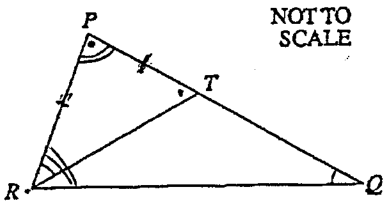
(17) Two arcs of equal radius with centres P and Q. These arcs intersect at R and S.

Prove that RS is a *perpendicular bisector* of PQ. (ie cuts in half at right angles)



(Hint: Join PR, QR etc and find some congruent triangles)

(18)



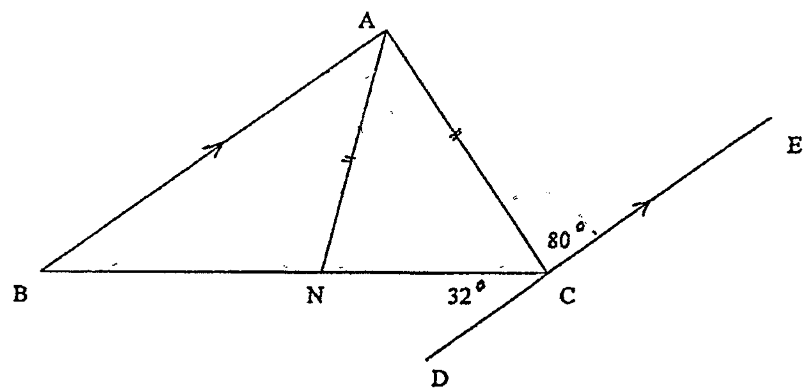
ΔPQR is similar to ΔPRT where $\angle PQR = \angle PRT$

Then $\dfrac{QR}{RT} =$

(A) $\dfrac{PQ}{PT}$ (B) $\dfrac{PR}{PT}$

(C) $\dfrac{PT}{PR}$ (D) $\dfrac{RT}{PT}$

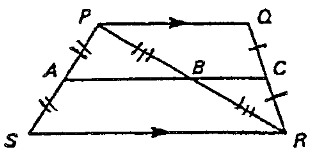
(19)



In the diagram above $AB \parallel ED$, $AN = AC$, $\angle NCD = 32°$ and $\angle ACE = 80°$.

Find $\angle NAB$, giving reasons.

(20)



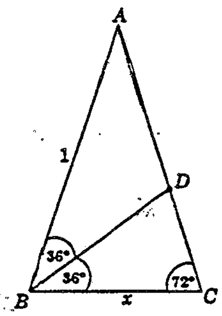
PQRS is a trapezium with $PQ \parallel SR$.
If *A, B* and *C* are midpoints of *SP, PR* and *QR* respectively prove that:

(a) $AB \parallel SR$

(b) $BC \parallel PQ$

(c) the points *A, B* and *C* are collinear.

(21)



In the diagram *ABC* is an isosceles triangle where $\angle ABC = \angle BCA = 72^\circ$ and $AB = AC = 1$.

$\angle ABC$ is bisected by *BD*, and $BC = x$.

(i) Explain why $BD = AD = x$

(ii) Show that $\Delta ABC \parallel\!| \Delta BCD$

(iii) By using (i) & (ii) show that $x = \dfrac{-1+\sqrt{5}}{2}$

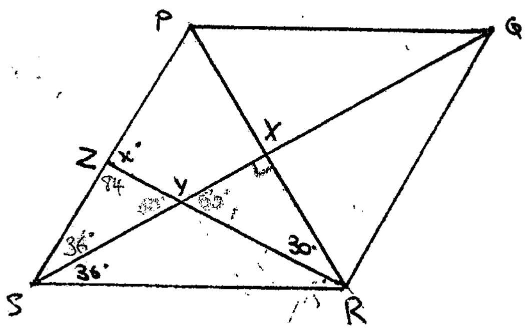
Sarah F.

See corrections!

SBHS Class Test 1 2000

1. True - similar triangles are equiangular ✓ (matching angles)
2. $x^2 = (\sqrt{784})^2 + (\sqrt{441})^2$ ✓
   $= 1225$
   $\therefore x = 35cm.$ ✓
3. $100° = x° + (180 - 3x°)$ ✓ (ext. ∠ of Δ is the sum of the two opposite interior angles)
   $2x = 80°$
   $x = 40°$ ✓
   $\angle ACD = 180 - 3x - x$
   $= 180 - 4x$
   $= 180 - 4(40)$
   $= 20°$ ✓

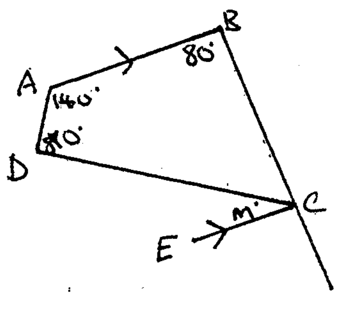
4



$\angle YXR = 90°$ (diagonals of a rhombus ~~cross~~ bisect at right angles)
$\therefore \angle XYR = 60°$ (∠ sum of Δ)
$\angle ZYS = 60°$ (vert. opp. ∠'s) ✓
$\angle ZSY = 36°$ (diagonals of a rhombus bisect the angles through which they pass)
$x° = 36 + 60°$ (ext. ∠ of Δ is the sum of the two opposite interior angles)
$= 96°$ ✓

5. B - the diagonals are of different lengths.

6.



$\angle BCD = 360° - (80 + 140 + 80)$
$= 60°$ ✓
$m° = 180 - 80 - 60°$
$= 40°$ ✓

7.6. C ✓

8 $\Delta ADE \,|||\, \Delta ABC$ (given)

$\therefore \frac{\Delta ADE}{\Delta ABC} = \frac{16}{x} = \frac{2}{5}$

$2x = 80m^2$

$x = 40m^2$ ✗

Area of $\Delta ABC = 40m^2$

Note: Area of $\Delta ADE = \frac{1}{2} \times DE \times h_1$

" " $\Delta ABC = \frac{1}{2} \times BC \times h_2$

but $\frac{DE}{BC} = \frac{h_1}{h_2} = \frac{2}{5}$

$\therefore \frac{A_{\Delta ADE}}{A_{\Delta ABC}} = \frac{4}{25} = \frac{16}{A_{\Delta ABC}}$

$\therefore A_{\Delta ABC} = \frac{16 \times 25}{4}$

$= 100\ m^2$

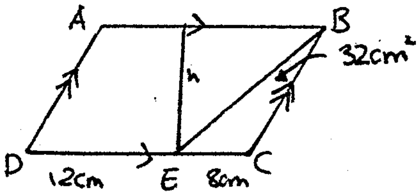
9. $\angle WXY = (180 - 70) \div 2$ ($\angle$ sum of isosceles Δ) ✓

$= 55^\circ$.

$\angle WXY = \angle WZX = x^\circ$ (opposite $\angle$'s of parallelogram are equal) ✓

$x^\circ = 55^\circ$

10



$32 = \frac{8}{2} \times h$

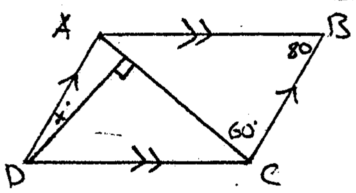
$\therefore h = 8.$ ✓

Area of ABED $= \frac{1}{2} \times 8[(12+8)+12]$

$= 4 \times 32$ ✓

$= 128cm^2$ ✓

11



$\angle CAD = 60^\circ$ (alt $\angle$'s, $AD \parallel BC$)

$x = 180 - (60 + 90)$ ($\angle$ sum of Δ)

$= 30^\circ$ ✓

12. ~~$\angle BCG$~~ $\angle DCF = 180 - (90 - y) - x$

$= 180 - 90 + y - x$

$= 90 + y - x.$

$\therefore \angle BCG = 90 - (90 + y - x)$

$= x - y.$

13. ∠BCE = ∠ABC (alt. ∠'s, AB || CE)
∠BAC = ∠ECD (—— " ——) (Corresponding ∠s AB // CE)
∠BCE = ∠ECD (given)
∴ △ABC is isoceles
∴ AC = BC. (opp. sides of an isoceles △ are equal)

14. B, II only ✓

15 The triangles are similar (equiangular)

$\therefore \frac{30}{18} = \frac{w}{15}$ ✓

18w = 450

w = 25m ✓

16. a) $\frac{x+4}{4} = \frac{8}{5}$

b) 5(x+4) = 32
5x+20 = 32
5x = 12
x = 2.4u.

$\frac{x+4}{5} = 2$
x + 4 = 10
x = 6

17. PR = PS (equal radius of the arc)
SQ = RQ (")
∴ PRSQ is a rhombus.
∴ RS is the perpendicular bisector of PQ (diagonals of a rhombus bisect ~~rigt~~ at right angles.

Firstly prove that ~~PRSQ~~ △PRT ≡ △QRT (using PRQS is a llegram) ~~therefore △PRT ≡ △~~

This is stating a property!

18 B ✓

19. ∠ACN = ∠ANC (base angles of an isoceles △ are equal)
∠ANC = 180 − (80 + 32) (∠ sum of straight ∠)
= 68° ✓
∠ABC = ∠BCD (alt ∠'s, AB || ED)
= 32° ✓
∠BAN = ∠ANC − ∠ABC (ext. ∠ of △ equals the sum of the interior opposite ∠'s)
= 68° − 32°
= 36° ✓

20 a) In Δ's PAB and PSR:

$\angle SPR$ is common ✓

~~$\angle PAB = \angle PSR$ (corresp. ∠'s~~,

$\frac{PA}{PS} = \frac{PB}{PR} = \frac{1}{2}$ ✓

$\therefore \Delta PAB ||| \Delta PSR$ (sides in the same ratio)

$\therefore \angle PAB = \angle PSR$ (corresp. ∠'s, similar triangles)

$\therefore AB \parallel SR$ (corresp ∠'s are equal) ✓

b) In Δ's RBC and RPQ:

$\frac{BR}{PR} = \frac{CR}{CQ} = \frac{1}{2}$ and $\angle PRQ$ is common

$\therefore \Delta RBC ||| \Delta RPQ$ (sides in the same ratio) ✓

$\therefore \angle RBC = \angle RPQ$ ✓ (corresp. ∠'s, similar triangles) ✓

$\therefore BC \parallel PQ$

c) PQ and SR are parallel, and are therefore straight lines)

If $AB \parallel SR$ and $BC \parallel PQ$, then AC must be a straight line.

Since points A, B and C lie on the straight line AC, ✓

they must be collinear. ✓

21 i. $x = BD$ (ΔBDC is an isosceles Δ) ✓

$BD = AD$ (ΔABD is an isosceles Δ)

$\therefore BD = AD = x$. ✓

ii. In Δ's ABC and BCD:

$\angle ABC = 72° = \angle ACB$ ✓ (given)

$\angle BCD = 72° = \angle BDC$ ✓

$\therefore \Delta ABC ||| \Delta BCD$ (equiangular)

iii. From (ii)

$\frac{BC}{CD} = \frac{AC}{BD}$

$\frac{x}{1-x} = \frac{1}{x}$

$\therefore x^2 = 1 - x$

$\therefore x^2 + x - 1 = 0$

$\therefore x = \frac{-1 \pm \sqrt{1-4(-1)}}{2} = \frac{-1 \pm \sqrt{1+4}}{2}$

$\therefore x = \frac{-1+\sqrt{5}}{2}$ since $x > 0$.