S.B.H.S — HSC ASSESSMENT TASK 1 - 2004

MATHEMATICS

Question One (30 marks)

a. Solve the following:

(i) $\frac{4}{x}-\frac{3}{2x}=1$ (ii) $4^x-12(2)^x+32=0$

b. (i) Write in scientific notation $0\cdot00798$

(ii) How many significant zeros in $0\cdot003400708$?

(iii) With the aid of a diagram show how $\sqrt{3}$ may be constructed on a number line.

(iii) Find in general form the equation of the line perpendicular to the line $3x-5y+8=0$ and passing through the point $(2,-1)$.

c. Factorise the following;

(i) $2x^2-32$ (ii) $6x^2+23x-4$

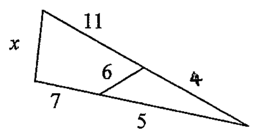
(iii) $2ax-a-8x+4$ (iv) $27a^3+8$

d. State whether the following are Odd, Even or Neither (show working)

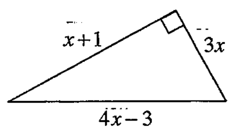
(i) $y=x^2+2x$ (ii) $y=-x$

e. Find x in the following;

(i)



(ii)



f. If α and β are the roots of $x^2-3x-5=0$, evaluate the following:

(i) $\alpha+\beta$ (ii) $\alpha\beta$

(iii) $\alpha^2+\beta^2$ (iv) $\frac{1}{\alpha}+\frac{1}{\beta}$

(v) $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$ (vi) $\alpha^3+\beta^3$

g. (i) Find the equation of the parabola with focus (3, 4) and directrix $y=-2$

(ii) Write down the focus and directrix of the parabola $x^2-4x+8y-36=0$

Question Two (25 marks)

a. (i) Solve and plot the solution on a number line: $x^2-3x-4\le 0$

(ii) State the domain of $y=\sqrt{4-x}$

(iii) If $f(x)=3x+4$ find $f(3x-2)$

(iv) Express with a rational denominator $\frac{\sqrt{5}-1}{\sqrt{5}+1}$

(v) Simplify $(1-\sec^2\theta)(\operatorname{cosec}^2\theta-1)$

(vi) If $0\le x\le\pi$, find the least value of $\frac{\sin^2 x}{1-\cos x}$

(vii) Find the centre and radius of the circle $x^2+y^2+4x-6y=0$

b. For what values of k does the equation have : $x^2+kx+(3-k)=0$

(i) equal roots (ii) real and distinct roots
(iii) roots equal in value but opposite in sign
(iv) roots which are reciprocals

c. One of the roots of $4kx^2+x-20k=0$ is $x=2$. Find the other root.

d. A person wishes to form a rectangular enclosure using an existing fence as one side. If he has 40 metres of fencing available to form the other three sides, find the area of the largest enclosure he can form and its dimensions.

Question Three (30 marks)

a. Sketch the following;

(i) $y=3\sin 2x$ for $0\le x\le 2\pi$

(ii) $xy=-4$

b. Find in simplest exact form the value of: $\cos 210^0$

c. Find the size of each interior angle in a regular 12 sided polygon.

d. Solve the following, where : $0\le x\le 2\pi$

(i) $\sin x=\frac{\sqrt{3}}{2}$ (ii) $\tan 2x=-1$

Question Two (25 marks)

a. (i) Solve and plot the solution on a number line:
$x^2 - 3x - 4 \leq 0$

(ii) State the domain of $y = \sqrt{4-x}$

(iii) If $f(x) = 3x + 4$ find $f(3x-2)$

(iv) Express with a rational denominator $\dfrac{\sqrt{5}-1}{\sqrt{5}+1}$

(v) Simplify $(1-\sec^2\theta)(\text{cosec}^2\theta - 1)$

(vi) If $0 \leq x \leq \pi$, find the least value of $\dfrac{\sin^2 x}{1-\cos x}$

(vii) Find the centre and radius of the circle $x^2 + y^2 + 4x - 6y = 0$

b. For what values of k does the equation have :
$x^2 + kx + (3-k) = 0$

(i) equal roots (ii) real and distinct roots
(iii) roots equal in value but opposite in sign
(iv) roots which are reciprocals

c. One of the roots of $4kx^2 + x - 20k = 0$ is $x = 2$. Find the other root.

d. A person wishes to form a rectangular enclosure using an existing fence as one side. If he has 40 metres of fencing available to form the other three sides, find the area of the largest enclosure he can form and its dimensions.

Question Three (30 marks)

a. Sketch the following;

(i) $y = 3\sin 2x$ for $0 \leq x \leq 2\pi$

(ii) $xy = -4$

b. Find in simplest exact form the value of: $\cos 210^0$

c. Find the size of each interior angle in a regular 12 sided polygon.

d. Solve the following, where : $0 \leq x \leq 2\pi$

(i) $\sin x = \dfrac{\sqrt{3}}{2}$ (ii) $\tan 2x = -1$

e. From a point P on level ground a person takes an elevation of the top of a building as $20°$. Walking 100 m towards the building the elevation becomes $45°$.
What is the height of the building correct to the nearest metre?

f. Find the locus of points equidistant from the points (2,3) and (-7,-5)

g. Find the distance between the parallel lines

$3x - 4y - 4 = 0$ and $3x - 4y - 7 = 0$

h. Use the k-method to find (in general form) the equation of the line through The intersection of the lines $x - y + 3 = 0$ and $2x + y - 3 = 0$ and passing through the point (-2,5)

i. Find to the nearest minute the largest angle in the triangle with sides 3 cm, 4 cm and 7 cm.

j. The diagonals of trapezium $ABCD$ in which AB is parallel to CD, meet at K.

(i) Prove that $AB.CK = CD.AK$

(ii) If K is the mid-point of AC, prove that the trapezium is a parallelogram.

END OF THE PAPER

QUESTION 1.

a) (i) $\frac{4}{x} - \frac{3}{2x} = 1$

$8 - 3 = 2x$

$2x = 5$

$\boxed{x = 2\tfrac{1}{2}}$ ✓

(ii) $4^x - 12 \cdot 2^x + 32 = 0$

$(2^x)^2 - 12 \cdot 2^x + 32 = 0$

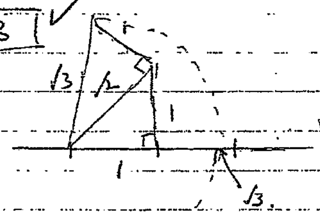
$(2^x - 8)(2^x - 4) = 0$

$2^x = 8, 4$

$\boxed{x = 2, 3}$ ✓

b) (i) $\boxed{7.98 \times 10^{-3}}$ ✓

(ii) $\boxed{3}$ ✓

(iii) 

✓✓

(iv) $3x - 5y + 8 = 0$ (A)

$5y = 3x + 8$

$y = \frac{3}{5}x + \frac{8}{5}$

∴ line ⊥ to (A) has slope $-\frac{5}{3}$

∴ line is $\frac{y+1}{x-2} = -\frac{5}{3}$

$3y + 3 = -5x + 10$

$\boxed{5x + 3y - 7 = 0}$ ✓✓

(c) (i) $2x^2 - 32 = \boxed{2(x-4)(x+4)}$ ✓

(ii) $\boxed{(6x-1)(x+4)}$ ✓

(iii) $a(2x-1) - 4(2x-1)$

$= \boxed{(a-4)(2x-1)}$ ✓

(iv) $\boxed{(3a+2)(9a^2 - 6a + 4)}$ ✓

d, (i) let $f(x) = x^2 + 2x$

$f(-x) = (-x)^2 + 2(-x)$

$= x^2 - 2x$

$\neq f(x)$

$\neq -f(x)$ $\therefore$ $\boxed{\text{NEITHER}}$

(ii) $f(x) = x^3 + x$

$f(-x) = (-x)^3 + (-x)$

$= -x^3 - x$

$= -f(x)$. $\therefore$ $\boxed{\text{ODD}}$

(e) (i) Δ's are similar as two sides proportional i.e. $\frac{4}{5} = \frac{12}{15}$ and included angle common.

$\therefore \frac{x}{6} = \frac{15}{5}$ $\therefore$ $\boxed{x = 18}$

(ii) $(4x-3)^2 = (3x)^2 + (x+1)^2$

$16x^2 - 24x + 9 = 9x^2 + x^2 + 2x + 1$

$6x^2 - 26x + 8 = 0$

$3x^2 - 13x + 4 = 0$

$(3x-1)(x-4) = 0$

$x = \frac{1}{3}, 4$

Clearly $x \neq \frac{1}{3}$ $\therefore$ $\boxed{x = 4}$

(f) (i) $\alpha + \beta = \boxed{3}$

(ii) $\alpha\beta = \boxed{-5}$

(iii) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= 9 + 10$

$= \boxed{19}$

(iv) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta}$

$= \boxed{-\frac{3}{5}}$

(v) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta} = \frac{19}{-5}$

$= \boxed{-\frac{19}{5}}$

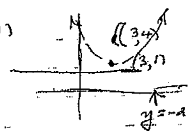
(vi) $\alpha^3 + \beta^3 = (\alpha+\beta)(\alpha^2 + \beta^2 - \alpha\beta)$

$= 3(19 + 5)$

$= 3 \times 24$

$= \boxed{72}$

(g) (i)



$(x-3)^2 = 4 \times 3(y-1)$

$\boxed{(x-3)^2 = 12(y-1)}$

(ii) $x^2 - 4x = -8y + 36$

$x^2 - 4x + 4 = -8y + 40$

$(x-2)^2 = -8(y-5)$

$\therefore$ $\boxed{S(2,3)}$ d: $\boxed{y = 7}$

QUESTION TWO:

a (i) $x^2 - 3x - 4 \leq 0$

$(x-4)(x+1) \leq 0 \Rightarrow \boxed{-1 \leq x \leq 4}$

-1 4

(ii) $4 - x \geq 0$

$\therefore$ $\boxed{x \leq 4}$

(iii) $f(3x-2) = 3(3x-2) + 4$

$= 9x - 6 + 4$

$= 9x - 2$.

(iv) $\frac{\sqrt{5}-1}{\sqrt{5}+1} = \frac{(\sqrt{5}-1)^2}{4}$

$= \frac{6-2\sqrt{5}}{4}$

$= \frac{3-\sqrt{5}}{2}$.

(v) $(1-\sec^2\theta)(\operatorname{cosec}^2\theta - 1)$

$= -\tan^2\theta \times \cot^2\theta$.

$= \boxed{-1}$

(vi) $\frac{\sin^2 x}{1-\cos x} = \frac{1-\cos^2 x}{1-\cos x}$

$= \frac{(1-\cos x)(1+\cos x)}{1-\cos x}$

$= 1 + \cos x$

now $-1 \leq \cos x < 1$ for $0 < x \leq \pi$.

$\therefore$ least value is $1 + -1$

$= \boxed{0}$

(vii) $x^2 + 4x + 4 + y^2 - 6y + 9 = 13$

$(x+2)^2 + (y-3)^2 = 13$

$\therefore$ centre $\boxed{(-2,3) \quad r = \sqrt{13}}$

b (i) $\Delta = 0 \Rightarrow k^2 - 4(3-k) = 0$

$k^2 + 4k - 12 = 0$

$(k+6)(k-2) = 0$

$\boxed{k = 2, -6}$

(ii) $\Delta > 0 \Rightarrow$ $\boxed{k > 2, \quad k < -6}$

(iii) $S_1 = 0$

$\therefore -k = 0$

$\boxed{k = 0}$

(iv) $S_2 = 1$

$\therefore 3 - k = 1$

$\boxed{k = 2.}$

4.

(c) $x=2$ is a root of $4kx^2+x-20k=0$ (A)

$\therefore$ $16k+2-20k=0$

$4k=2$

$\boxed{k=\frac{1}{2}}$

(A) becomes. $2x^2+x-10=0$.

now $S_1=2+\alpha=-\frac{1}{2}$

$\therefore \boxed{\alpha=-\frac{5}{2}}$

OR

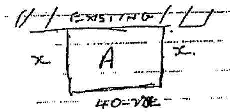
$S_2=\alpha\beta=\frac{-20k}{4k}$

$=-5$

$\therefore 2\beta=-5$

$\beta=-\frac{5}{2}$

(d)



$A=x(40-2x)$

$=40x-2x^2$

$=-2(x^2-20x+100)+200$

$=-2(x-10)^2+200$

$\therefore$ when $x=10$

$A=200$ which

is the maximum

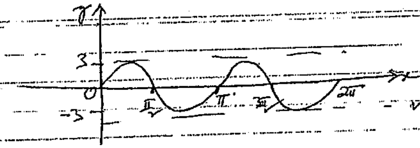
value.

$\therefore$ $\boxed{\text{MAX Area} = 200\,m^2 \text{ i.e. } 10\,m \times 20\,m}$

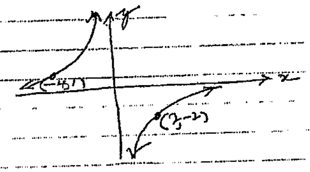
5.

QUESTION 3.

a (i)



(ii)



b. $\cos 210^\circ = -\cos 30^\circ$

$=\boxed{-\frac{\sqrt{3}}{2}}$

(c) Sum of exterior angles $=360^\circ$.

$\therefore$ each exterior angle $=\frac{360^\circ}{12}$

$=30^\circ$

$\therefore$ each interior angle $\boxed{150^\circ}$

d (i) $\sin x=\frac{\sqrt{3}}{2}$

$\boxed{x=\frac{\pi}{3},\frac{2\pi}{3}}$

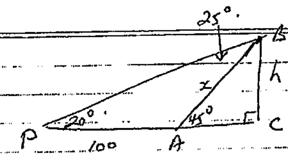
(ii) $\tan 2x=-1$ $\quad 0\le 2x\le 4\pi$.

$2x=\frac{3\pi}{4},\frac{7\pi}{4},\frac{11\pi}{4},\frac{15\pi}{4}$

$\boxed{x=\frac{3\pi}{8},\frac{7\pi}{8},\frac{11\pi}{8},\frac{15\pi}{8}}$

for $0\le x\le 2\pi$

e)



$$\frac{x}{\sin 20^\circ} = \frac{100}{\sin 25^\circ}$$

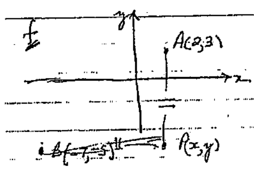
$$x = \frac{100 \sin 20^\circ}{\sin 25^\circ}$$

now $\frac{h}{x} = \frac{1}{\sqrt{2}}$

$$h = \frac{1}{\sqrt{2}} \times 100 \frac{\sin 20^\circ}{\sin 25^\circ}$$

$$\doteqdot 57\,m$$ ✓✓✓

f)



now $AP = PB$

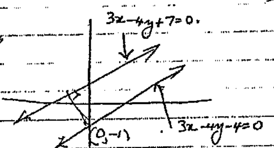
$$\sqrt{(x-2)^2+(y-3)^2} = \sqrt{(x+7)^2+(y+5)^2}$$

$$x^2-4x+4+y^2-6y+9 = x^2+14x+49+y^2+10y+25$$

$$-18x-16y-61 = 0$$

$$\boxed{18x+16y+61=0}$$ ✓✓ ✓

g)



Consider the distance of $(0,-1)$ from $3x-4y+7=0$,

$$d = \left| \frac{3\times 0 - 4\times -1 + 7}{\sqrt{3^2+(-4)^2}} \right|$$

$$= \boxed{\frac{11}{5}}$$ ✓✓

(h) The required line is of the form

$x-y+3+k(2x+y-3)=0$ which contains $(-2,5)$

$$\therefore -2-5+3+k(-4+5-3) = 0$$

$$-4+k\times -2 = 0$$

$$-2k = 4$$

$$k = -2.$$

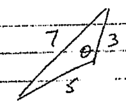
$$\therefore x-y+3-2(2x+y-3) = 0$$

$$x-y+3-4x-2y+6 = 0$$

$$-3x-3y+9=0$$

$$\boxed{x+y-3=0}$$ ✓✓.✓

i)



$$\cos\theta = \frac{3^2+5^2-7^2}{2\times 3\times 5}$$

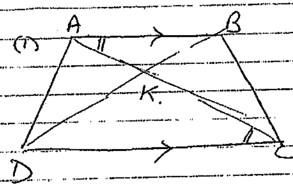
$$= \frac{9+25-49}{30}$$

$$= \frac{-15}{30}$$

$$= -\frac{1}{2}$$ ✓✓

$$\therefore \theta = 120^\circ.$$

j (i)



$\angle BAK = \angle KCD$ (alternate angles equal)

$\angle AKB = \angle CKD$ (vertically opposite)

$\therefore \triangle ABK \,|||\, \triangle CDK$ (equiangular)

$\therefore \frac{AB}{CD} = \frac{AK}{CK}$ (corresponding sides proportional)

$\therefore AB.CK = CD.AK.$ ✓✓

(ii) If K is the mid-point of AC then $\frac{AK}{CK} = 1$

$$\therefore \frac{AB}{CD} = 1 \Rightarrow AB = CD$$

now if a pair of opposite sides of a quadrilateral are equal and parallel then the quadrilateral is a parallelogram. ✓✓✓