SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2004

YEAR 11

HIGHER SCHOOL CERTIFICATE
ASSESSMENT TASK # 2

Mathematics

General Instructions
- Working time – 2 periods.
- Write using black or blue pen.
- Board approved calculators may be used.
- All necessary working should be shown in every question if full marks are to be awarded.
- Marks may not be awarded for messy or badly arranged work.
- Hand in your work in 3 **SEPARATE** bundles

Total Marks -70
- Attempt questions 1 - 3
- All sections are **NOT** of equal value.

Examiner: *P. Bigelow*

This is an assessment task only and does not necessarily reflect the content or format of the Higher School Certificate.

Question One (20 marks)

a. Solve for x:
(i) $4^x = 0 \cdot 125$ (ii) $\log_2 x = 5$

b. Write an equivalent expression for $y = a^4$ using logarithmic notation.

c. In the arithmetic series $3 + 8 + 13 + 18 + \ldots$, find:
(i) the 20^{th} term
(ii) the sum of the first 20 terms
(iii) the sum of the 21^{st} to 30^{th} terms.

d. By expressing $0 \cdot \dot{4}\dot{5}$ as an infinite geometric series, find its value in simplest fractional form.

e. Find the values of x for which the function given by
$$y = 2x^3 - 3x^2 - 12x + 5$$
is decreasing

f. Evaluate $\log_7 4$ correct to two decimal places.

g. Find the second derivative of $f(x) = \dfrac{x+1}{x-1}$

h. Find the primitive of the following:
(i) $2x - 3$ (ii) $\dfrac{2}{x^3} - \dfrac{4}{\sqrt{x}} + \dfrac{4x}{3}$

Question Two (20 marks)

a. Given that $\log_a b = 2 \cdot 75$ and $\log_a c = 0 \cdot 25$ find the value of :
(i) $\log_a \left(\dfrac{b}{c}\right)$ (ii) $\log_a (bc)^2$

b. The number of unemployed people u at time t was studied over a period of time. At the start of this period, the number of unemployed was 700 000.

(i) Throughout the period, $\dfrac{du}{dt} < 0$. What does this say about the number of unemployed during the period?

(ii) It is also observed that, throughout the period, $\dfrac{d^2u}{dt^2} > 0$.
Sketch the graph of u against t

c. Factorise $6^y + 15^y$

d. A junior employee was offered a starting wage of \$480 per week and a pay increase of \$2.50 per week for each additional week worked.

(i) What would be her pay in the twentieth week of employment?

(ii) How much would she earn by the end of the first 20 weeks of employment?

(iii) Determine her average weekly pay if she was employed for 52 weeks

e. Solve the equation $2\log_5 x = \log_5 x - \log_5 6$.

f. Solve $3^x = 15$ (giving your answer correct to two decimal places)

Question Three (30 marks)

a. A series is defined by $S_n = 5n^2 + 6n$ where S_n is the sum of n terms.

(i) Find S_7 (ii) Find T_6

(iii) Find an expression for T_n and show that the series is arithmetic.

b. If $\frac{d^2y}{dx^2} = 2x + 1$ and there is a stationary point at $(3,2)$, find the equation of the curve.

c. Evaluate $\sum_{n=1}^{\infty} \frac{3}{2^n}$

d. A can of baked beans is in the shape of a closed cylinder with height h cm and radius r cm.

(i) The volume of the can is 500 cm^3.
Find an expression for h in terms of r.

(ii) Show that the surface area S cm^3 of the can is given by
$$S = 2\pi r^2 + \frac{1000}{r}.$$

(iii) If the area of metal used to make the can is to be minimised, find the radius of the can.

e. Consider the curve given by $y = 1 + 3x - x^3$, for $-2 \le x \le 3$

(i) Find the stationary points and determine their nature.

(ii) Find the point of inflexion.

(iii) Sketch the curve for $-2 \le x \le 3$

(iv) What is the minimum value of y for $-2 \le x \le 3$?

f. A farmer borrows \$180 000 to purchase new equipment.
The interest is calculated monthly at the rate of 2% per month, and is compounded each month.
The farmer intends to repay the loan with interest in two equal annual instalments of $\$X$ at the end of the first and second years.

(i) How much does the farmer owe at the end of the first month?

(ii) Write an expression involving X for the total amount owed by the farmer after 12 months, just after the first instalment of $\$X$ has been paid.

(iii) Find an expression for the amount owed at the end of the second year and calculate X.

(iv) What is the total interest over the two year period?

End of the paper

Q1

a) (i) $4^x = \frac{1}{8}$

$2^{2x} = 2^{-3}$

$2x = -3$

$\boxed{x = -\frac{3}{2}}$ ✓

(ii) $\log_2 x = 5$

$x = 2^5$

$\boxed{x = 32}$ ✓

b) $y = e^4 \Rightarrow \boxed{\log_e y = 4}$ ✓

c) $3 + 8 + 13 + 18 + \ldots$

(i) $T_{20} = 3 + 19 \times 5$

$= \boxed{98}$ ✓

(ii) $S_{20} = \frac{20}{2}[3 + 98]$

$= 10 \times 101$

$= \boxed{1010}$ ✓

(iii) $T_{21} + \ldots + T_{30}$

$= S_{30} - S_{20}$

$= \frac{30}{2}[6 + 29 \times 5] - S_{20}$

$= 15[6 + 145] - S_{20}$

$= 15 \times 151 - S_{20}$

$= 2265 - 1010$

$= \boxed{1255}$ ✓✓

d) $0.45 + 0.0045 + \ldots = \frac{0.45}{1 - 0.01} = \boxed{\frac{5}{11}}$ ✓✓

e) $y = 2x^3 - 3x^2 - 12x + 5$

$y' = 6x^2 - 6x - 12$

$= 6(x^2 - x - 2)$

$= 6(x - 2)(x + 1) < 0$

$\boxed{-1 < x < 2}$ ✓✓

f) $\log_7 4 = \frac{\log 4}{\log 7} = \underline{0.71}$ (2DP) ✓✓

g) $f(x) = \frac{x+1}{x-1}$

$f'(x) = \frac{(x-1).1 - (x+1).1}{(x-1)^2} = \frac{-2}{(x-1)^2} = -2(x-1)^{-2}$

$f''(x) = 4(x-1)^{-3}$

$= \boxed{\frac{4}{(x-1)^3}}$ ✓✓

h) (i) $y' = 2x - 3$

$y = x^2 - 3x + c$ ✓✓

(ii) $y' = 2x^{-3} - 4x^{-\frac{1}{2}} + \frac{4}{3}x$

$y = \frac{2x^{-2}}{-2} - \frac{4x^{\frac{1}{2}}}{\frac{1}{2}} + \frac{4}{3}\frac{x^2}{2} + c$

$= \boxed{\frac{-1}{x^2} - 8\sqrt{x} + \frac{2}{3}x^2 + c}$ ✓✓✓

Q2.

a) (i) $\log_a \frac{b}{c} = \log_a b - \log_a c$

$= 2.75 - 0.25$ ✓✓

$= \boxed{2.5}$

(ii) $\log_a (bc)^2 = 2 \log_a bc$

$= 2\left[\log_a b + \log_a c\right]$

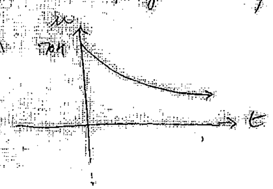
$= 2\left[2.75 + 0.25\right]$

$= 2 \times 3$ ✓✓

$= \underline{6}$

b) (i) ~~unemployment is falling~~ ✓✓

(ii)



✓✓

c) $6^y + 15^y = 3^y 2^y + 3^y 5^y$

$= \boxed{3^y\left[2^y + 5^y\right]}$ ✓✓

d) 480 + 482.50 + . . . 20 terms

(i) $T_{20} = 480 + 19 \times 2.50$

$= 480 + 47.50$ ✓✓

$= \boxed{\$527.50}$

(ii) $S_{20} = \frac{20}{2}\left[480 + 527.50\right]$

$= 10 \times 1007.50$ ✓✓

$= \boxed{\$10{,}075.00}$

(iii) $\frac{S_{52}}{52} = \frac{\frac{52}{2}\left[960 + 51 \times 2.50\right]}{52}$

✓✓ $= \frac{960 + 51 \times 2.50}{2}$

$= \underline{\$543.75}$

e) $2\log_5 x = \log_5 x - \log_5 6$

$\log_5 x^2 = \log_5 \frac{x}{6}$

$x^2 = \frac{x}{6}$

$6x^2 - x = 0$

$x(6x - 1) = 0$

$x = 0, \frac{1}{6}$ clearly $x \neq 0$

$\therefore \underline{x = \frac{1}{6}}$ ✓✓

f) $3^x = 15$

$x = \frac{\log 15}{\log 3} = \underline{2.46}$ (2DP) ✓✓

Q3 a) (i) $S_7 = 5\times7^2+6\times7$
$= 5\times49+42$
$= 245+42$
$= \boxed{287}$ ✓

(ii) $T_n = S_n - S_{n-1}$
$= 5n^2+6n-[5(n-1)^2+6(n-1)]$
$= 5n^2+6n-[5(n^2-2n+1)+6n-6]$
$= 5n^2+6n-[5n^2-4n-1]$
$= 10n+1$

$S_6 - S_5 = 5\times36+6\times6 - (5\times5^2+6\times5)$
$= 55+6$
$= 61$

$\therefore T_6 = 10\times6+1$
$= \boxed{61}$ ✓✓

(iii) $T_n = 10n+1$ ✓ ✓

$T_n - T_{n-1} = 10n+1-(10(n-1)+1)$
$= \boxed{10 \therefore \text{common diff.}}$ ✓

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b) $\frac{dy}{dx} = x^2+x+c$ when $0 = 9+3+c$, $c=-12$

$y' = x^2+x-12$
$y = \frac{x^3}{3}+\frac{x^2}{2}-12x+k$
$2 = 9+\frac{9}{2}-36+k$
~~$24\frac{1}{2} = k$~~
~~$y = \frac{1}{3}x^3+\frac{x^2}{2}-12x+24\frac{1}{2}$~~

$\boxed{y = \frac{x^3}{3}+\frac{x^2}{2}-12x+\frac{49}{2}}$ ✓✓

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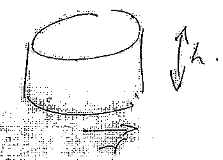
c) $\sum_{n=1}^{\infty}\frac{3}{2^n} = \frac{3}{2}+\frac{3}{4}+\frac{3}{8}+\dots$
$= \frac{3/2}{1-1/2}$ ✓
$= \boxed{3}$

1

d) 

(i) $V = \pi r^2 h = 500$
$\boxed{h = \frac{500}{\pi r^2}}$ ✓

(ii) $S = 2\pi r^2 + 2\pi r h$
$= 2\pi r^2 + 2\pi r \times \frac{500}{\pi r^2}$
$\boxed{S = 2\pi r^2 + \frac{1000}{r}}$ ✓

$\frac{dS}{dr} = 4\pi r - \frac{1000}{r^2}$

$\frac{d^2S}{dr^2} = 4\pi + \frac{2000}{r^3} > 0$

$\therefore$ st pt is MIN. ✓

$4\pi r = \frac{1000}{r^2}$

$r^3 = \frac{1000}{4\pi}$

$\boxed{r = \frac{10}{(4\pi)^{\frac{1}{3}}}}$ ✓✓

Q. $y = 1 + 3x - x^3$

$y' = 3 - 3x^2$

$y'' = -6x$

(i) For st. pts. $3 - 3x^2 = 0$

$x^2 = 1$

$x = \pm 1$ ∴ $(1, 3)$ ✓

or $(-1, -1)$ ✓✓

at $(1,3)$ $y'' = -6$ ∴ max stn.

at $(-1,-1)$ $y'' = 6$ ∴ min stn.

(ii) Possible inflexion at $y'' = 0$

$-6x = 0$

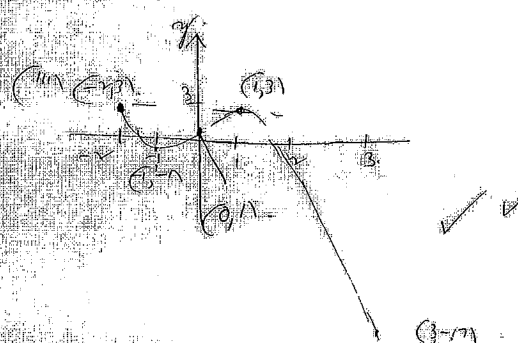
$x = 0$ ∴ $(0, 1)$ ✓✓

Testing

x	$-\frac{1}{2}$	0	$\frac{1}{2}$
y''	3	0	-3

∴ change in concavity ∴ pt. of inflexion

(iii)



✓✓

✓

(iv) -17

f. (i) $180\,000(1.02)^1 = \$183\,600$ ✓

(ii) $180\,000(1.02)^{12} - x = \$228\,283.52 - x$ ✓

(iii) $[180\,000(1.02)^{12} - x](1.02)^{12} - x$

$= 180\,000(1.02)^{24} - x(1 + 1.02^{12}) = 0$ ✓✓

$x = \dfrac{180\,000(1.02)^{24}}{1 + 1.02^{12}}$

$x = 177,640.14$ ✓✓

(iv) $2 \times \$177,640.14 - \$180\,000$

$= \$175,280.28$ ✓✓