SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2004

YEAR 11

HALF YEARLY EXAMINATION

Mathematics Extension

General Instructions

- Working time – 90 minutes.
- Reading Time – 5 minutes.
- Write using black or blue pen.
- Only Board approved calculators may be used.
- All necessary working should be shown in every question if full marks are to be awarded.
- Start each section in a **SEPARATE** answer booklet.
- Marks may not be awarded for messy or badly arranged work

Total Marks - 82

- Attempt Sections A – D
- All sections are **NOT** of equal value.

Examiner: *C. Kourtesis*

SECTION A
[20 marks]

1. Convert (a) $\frac{5\pi}{2}$ radians to degrees — 1

   (b) 135° to radians — 1

2. The point $(9,k)$ lies on the line $x+3y=6$
   Find the value of k — 2

3. If $m(x)=3^x+3^{-x}$ find $m(-3)$ — 1

4. Solve the inequalities

   (a) $-4x+1\leq 12x$ — 2

   (b) $|x|<4$ — 1

5. Find the point of intersection of the straight lines

   $x+2y=6$ and $x-3y=10$ — 2

6. On separate diagrams sketch the graphs of:

   (a) $xy=4$ — 1

   (b) $y=2^{-x}$ — 1

   (c) $y=\log_{10}x$ — 1

7. Find the roots of the equation $\theta^2-\theta-2=0$ — 2

8. Sketch the region in the number plane indicated by $y\geq x^2$ — 2

9. The equation of a parabola is given by $x^2=8y$
   Find the (a) equation of the directrix — 1
   (b) coordinates of the focus — 2

SECTION B (Start a NEW Booklet)
[18 marks]

10. Explain why $f(x) = \sin x$ represents an odd function 2

11. For the function $f(x) = \dfrac{1}{(x+1)(1-3x)}$ write down the

(a) equations of the vertical asymptotes 2

(b) y intercept 1

12. The equation of a circle is given by

$$x^2 + y^2 + 4x - 8y = 0$$ 4

Find the (a) coordinates of the centre

(b) length of the radius

13. Find the acute angle between the lines, answer to the nearest minute.

$y = 2x - 1$ and $y = -\dfrac{1}{3}x$ 3

14. Given the quadratic equation $2x^2 - mx + 8 = 0$

Find the (a) discriminant 1

(b) value(s) of m for which the above equation has two distinct real roots 2

15. Find the coordinates of the point that divides the interval joining $A(1,4)$ and $B(5,10)$ externally in the ratio 3:2 3

SECTION C (Start a NEW Booklet)
[23 marks]

16. Solve the inequalities

(a) $(x+4)(x-2)(x-3)>0$ 2

(b) $\dfrac{x-3}{1-x}>2$ 4

17. Find the equation of the locus of a point $P(x,y)$ which moves so that its distance from $x-y+2=0$ is equal to its distance from the point $(1,-1)$ 3

18. Solve the inequality $\dfrac{1}{|4-3x|}<4$ 3

19. (a) Write down the algebraic definition of $|x|$ 1

(b) Sketch the graph of $y=x|x|$ 3

20. Find the value(s) of k if the equation

$$x^2-3kx+(k+3)=0$$

has:

(a) one root that is double the other 2

(b) one root that is the reciprocal of the other 2

21. Solve the equation

$$1-\cos x-2\sin^2 x=0 \quad \text{for} \quad 0\le x\le 2\pi$$ 3

SECTION D (Start a NEW Booklet)
[21 marks]

22. Solve $x^6 - 7x^3 - 8 = 0$ 3

23. Solve the inequality $(x+\frac{1}{x})^2 - (x+\frac{1}{x}) \leq 6$ 4

24. Prove that

$$\frac{\sin 2\theta + \sin\theta}{\cos 2\theta + \cos\theta + 1} = \tan\theta$$ 4

25. (a) Show that $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$ 2

(b) Hence show that

$$\tan 15° = 2 - \sqrt{3}$$ 3

26. A man travelling along a straight flat road passes three points at intervals of 200m. From those points he observes the angle of elevation of the top of a hill to the left of the road to be respectively 30°, 45° and again 45°.

(a) Draw a neat diagram to represent this information 1

(b) Find the height of the hill 4

THIS IS THE END OF THE EXAMINATION

SYDNEY BOYS HIGH SCHOOL
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2004

YEAR 11

HALF YEARLY EXAMINATION

Mathematics

Sample Solutions

Section	Marker
A	Mr Fuller
B	Mr Dowdell
C	Mr Boros
D	Ms Nesbitt
E	Ms Opferkuch

Section A

Question 1

(a) $x(x-y)-y(y-x)$

$= x^2 - xy - y^2 + xy$

$= x^2 - y^2$

(b) $\sqrt{\dfrac{3^2+12^2}{231-12^2}}$

$= \sqrt{\dfrac{9+144}{231-144}}$

$= \sqrt{\dfrac{153}{87}}$

$= 1.33$

(c) $\left(\dfrac{2}{5}\right)^{10} \times \left(\dfrac{15}{4}\right)^{10} \times \left(\dfrac{2}{3}\right)^{9}$

$= \left(\dfrac{3}{2}\right)^{10} \times \left(\dfrac{2}{3}\right)^{9}$

$= \left(\dfrac{3}{2}\right)^{9} \times \left(\dfrac{2}{3}\right)^{9} \times \left(\dfrac{3}{2}\right)$

$= \dfrac{3}{2}$

Question 2

(a) let $x = 0.727272\ldots$

$100x = 72.727272\ldots$

$99x = 72$

$x = \dfrac{72}{99}$

$x = \dfrac{8}{11}$

(b) $2\sqrt{\dfrac{9}{4}} + \sqrt[3]{\dfrac{8}{125}}$

$= 2 \times \dfrac{3}{2} + \dfrac{2}{5}$

$= 3 + \dfrac{2}{5}$

$= 3\dfrac{2}{5}$

(c) (i) $\tan 60^\circ = \sqrt{3}$

(ii) $\cos\dfrac{11\pi}{6} = \cos\dfrac{\pi}{6}$

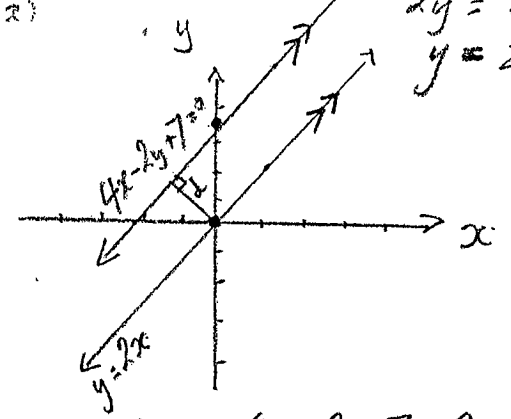
$= \dfrac{\sqrt{3}}{2}$

y) $y = 2x$ and $4x - 2y + 7 = 0$

$2y = 4x + 7$

$y = 2x + 3\frac{1}{2}$

x)



Use (0,0) on $4x - 2y + 7 = 0$

$d = \left| \frac{7}{\sqrt{16+4}} \right|$

$= \frac{7}{\sqrt{20}} = \frac{7}{2\sqrt{5}}$ or $(\doteqdot 1.56)$

$\frac{7}{2\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{7\sqrt{5}}{10}$ units (2)

b) (i) $5x^2 - 9xy - 2y^2$

$= (5x + y)(x - 2y)$ (1)

(ii) $x^2 + 4x + 4 - y^2$

$= (x+2)^2 - y$

$= (x + 2 - y)(x + 2 + y)$ (2)

(c) $|5 - 2x| = 3$

$5 - 2x = 3$ or $5 - 2x = -3$

$-2x = -2$ $-2x = -8$

$x = 1$ $x = 4$

(1)

2 unit

Q7(a)

$2x - 2y = 7$ (1)

$6x + 8y = 35$ (2)

$8x - 8y = -28$ (1)×4

$14x = 7$

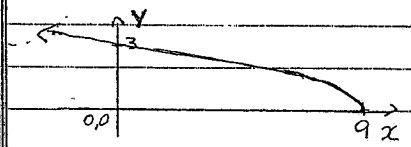
$x = \frac{1}{2}$

$2 \times \frac{1}{2} - 2y = -7$ (1)

$-2y = -8$

$y = 4$ $x = \frac{1}{2}$ (3)

(b)



Domain: $x \leq 9$

Range: $y \geq 0$ (2)

(c) $f(-x) = \frac{4^{-x} + 4^{x}}{2} = f(x)$

$\therefore f(x)$ is an even function (1)

Q8(a)

least value at centre axis

$x = -\frac{b}{2a}$

$x = 2$

least value $= 6$

or

$x^2 - 4x + 10 = (x-2)^2 + 6$

vertex $(2, 6)$

least value $= 6$ when $x = 2$ (1)

(b) No real root when $\Delta < 0$

$b^2 - 4ac < 0$

$2^2 - 4 \times 3 \times k < 0$

$\div 12 \quad k < -4$

$k > \frac{1}{3}$ (2)

(c) Third angle $= 180 - 80 - 32$

$= 68°$

$\frac{x}{\sin 68} = \frac{15}{\sin 80}$

$x = 15 \frac{\sin 68}{\sin 80}$

$x = 14.12$ (2dp) (3)

Q3 (a) $x(x-4) = 5$

$x^2 - 4x - 5 = 0$

$(x-5)(x+1) = 0$

$x = 5$ or $x = -1$ (2)

(b) $\frac{1}{\sqrt{3}-2} \times \frac{\sqrt{3}+2}{\sqrt{3}+2}$

$= \frac{\sqrt{3}+2}{3-4}$

$= -(\sqrt{3}+2)$ (2)

(c) $\sqrt{x} = \sqrt{50} - \sqrt{18}$

$= 5\sqrt{2} - 3\sqrt{2}$

$= 2\sqrt{2}$

$\therefore x = 8$ (2)

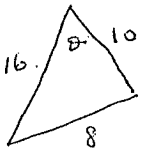
4 (a) $\sqrt{(a-4)(a+4)+16}$

$= \sqrt{a^2 - 16 + 16}$

$= \sqrt{a^2}$

$= a$ (as $a > 0$) (2)

(b)



$\cos\theta = \frac{16^2 + 10^2 - 8^2}{2 \times 16 \times 10}$

$= \frac{292}{320}$

$= \frac{73}{80}$

$\therefore \theta = 24.1468\ldots$

$\simeq 24° 9'$ (2)

(c) $4 - x \geq 3$

$-x \geq -1$

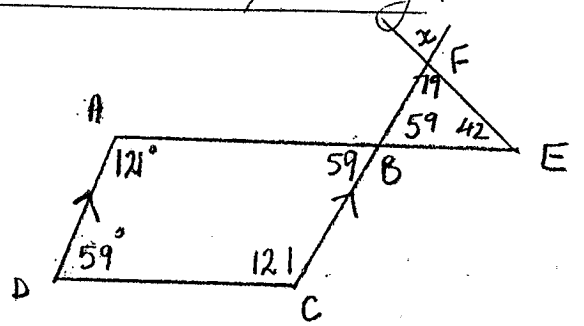
$x \leq 1$

(2)

04 Year 11 2 unit Half Yearly

ction C

) (a)



(1)

$A\hat{B}C = 180 - 121 = 59°$ co interior angles

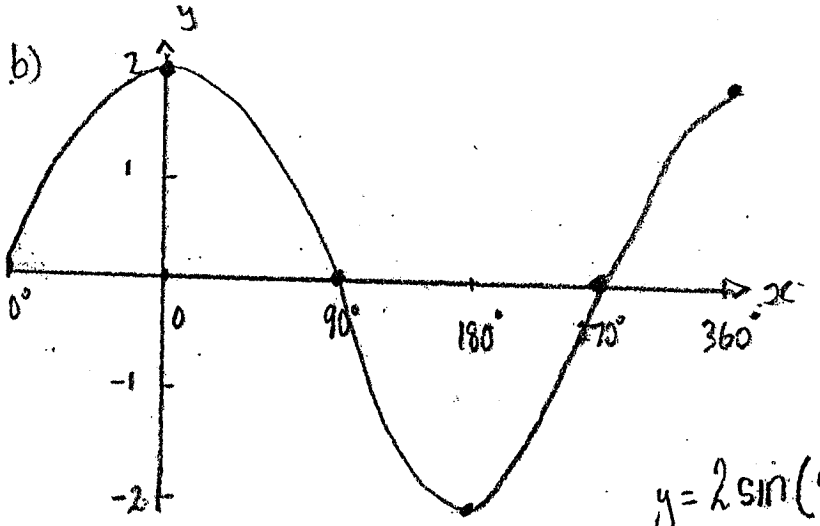
$A\hat{B}C = F\hat{B}E = 59°$ vert. opp.

$B\hat{F}E = 180 - (59 + 42)$

$= 79$

$\therefore x = B\hat{F}E = 79°$ vert. opp. (2)

b)



(3)

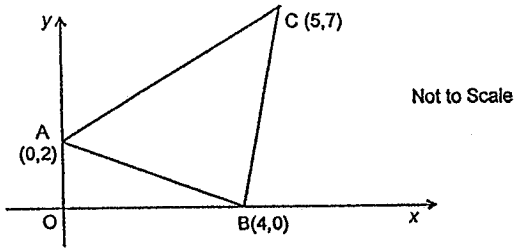
$y = 2\sin(90 - x)$

$= 2\cos x$

since $\sin(90 - x) = \cos x$

Year 11 Mathematics (Advanced)
Half Yearly Examination 2004
Solutions

Question 9



(a) $M: x=\frac{4+0}{2}, y=\frac{2+0}{2}$ (1)

$=2 \qquad =1$

$\therefore$ M (2,1)

(b) $m_{AB}=\frac{0-2}{4-0}$ (1)

$=-\frac{1}{2}$

$\therefore m_{AB}=-\frac{1}{2}$

(c) $m_1 \times m_2 = -1$ (1)

$-\frac{1}{2}\times m_2 = -1$

$m_2 = 2$

Using $y-y_1=m(x-x_1)$, where $x=2, y=1, m_2=2$

$y-1=2(x-2)$

$\therefore 2x-y-3=0$

(d) Using $2x-y-3=0$, where $x=5, y=7$ (1)

$2\times5-7-3=0$

$\therefore LHS = RHS$

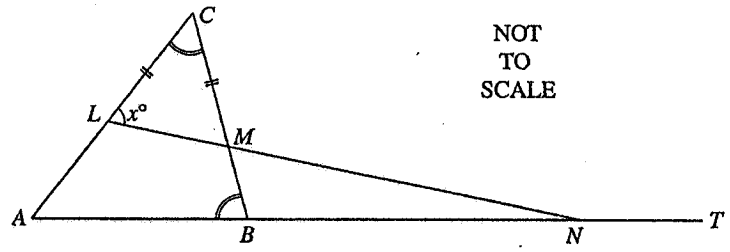
(e) In ΔABC (2)

$d_{AC}=d_{BC}=\sqrt{50}$

$d_{AB}=\sqrt{20}$

$\therefore \Delta ABC$ is isosceles

Question 10



(b) (i) In ΔCLM (1)

$\angle CLM = x^0$ (data)

$\angle CML = x^0$ (base $\angle$, isos. Δ)

$\therefore \angle ACB = 180-2x$ ($\angle$ sum of Δ)

In ΔABC

$\angle ACB = \angle ABC$ (data)

$\therefore \angle ABC = 180-2x$

(ii) In ΔMBN (2)

$\angle BMN = \angle CML$ (vert. opp. $\angle$s)

$= x^0$

$\angle MBN = 180-\angle ABC$ (supp. $\angle$s)

$= 180-(180-2x)$

$= 2x$

$\therefore \angle TNL = \angle BMN + \angle MBN$ (ext. $\angle$ = sum of two opp. int. $\angle$s)

$= x+2x$

$= 3x$

(a)