SYDNEY BOYS HIGH SCHOOL

MOORE PARK SURRY HILLS

1) 12 (70)
2) 16
3) 13
4) 17
5) 12

YEAR 11

HALF YEARLY 2000

MATHEMATICS

2 UNIT COMMON

Time allowed: *1-1/2 hours*
(plus five minutes reading tme)
Examiner : *E Choy*

DIRECTIONS TO CANDIDATES

- *ALL* questions may be attempted.
- All necessary working should be shown in every question. Full marks may not be awarded for careless or badly arranged work.
- Approved calculators may be used.
- Start each question in a new answer booklet. Indicate your name, class and teacher on each new booklet
- Additional answer bookets may be obtained from the supervisor upon request.

NOTE: This paper does not necessarily reflect the content or format of the final Higher School Certificate Examination Paper for this subject.

Question One

(a) Find the exact value of $64^{\frac{1}{2}} \times 64^{-\frac{1}{3}}$

(b) The point (k, 3) lies on the line $x + 5y = 10$. Find the value of k.

(c) Graph the solution to $5 - 2x < 8$ on a number line.

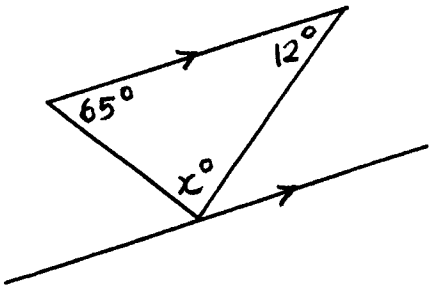
(d) Simplify $(2a - 1)^2 - 5(1 - a)$

(e) Find the value of $\frac{4}{\sqrt{2} - 5}$ correct to 3 significant figures.

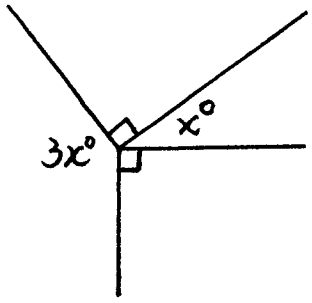
(f) Rationalise the denominator of $\frac{1}{1 + \sqrt{1 + a}}$

(g) Find x in the following

(i)



(ii)

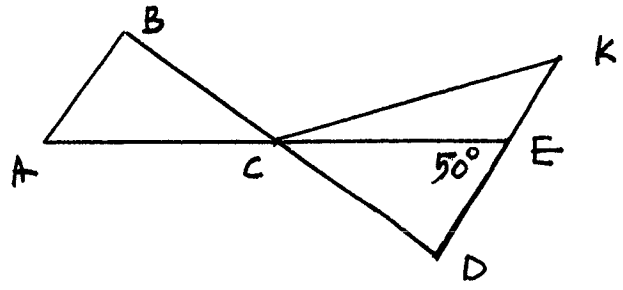


(h) Shade in the region represented by the following inequality.

$\{(x, y) \quad : \quad y \leq x + 1, y \geq 2x \}$

Question two

(a) In the accompanying figure.



$AB \,//\, DK$ and $< ABD = 90^0$
AE and BD intersect at C
$EC = EK$ and $\angle DEC = 50^0$
Find the size of $\angle KCD$ (give reasons)

(b) Find the value of x *if*

$$\sqrt{x} = \sqrt{50} - \sqrt{18}$$

(c) Factorise.

(i) $4x^2 + x - 3$

(ii) $1 + \sin^3 \theta$

(iii) $xy - 8x + y - 8$

(d) Simplify

$$\frac{x}{4} + \frac{3x-1}{3}$$

(e) Solve the equations

(i) $8^n = \frac{1}{16}$

(ii) $|2 - 3n| = 14$

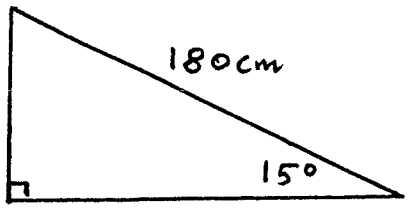
(f) Solve simlultaneously:

$$3x - 2y = 7$$
$$xy = 3$$

Question Three

(a) If $a:b = 3:1$, find $\dfrac{a+b}{a-b}$

(b)



Mr **Jones** insisted on having his bed raised at an angle of elevation of 15^0 in order to read in bed. If the slanted length was 180cm, find the height above the floor, correct to the nearest centimetre.

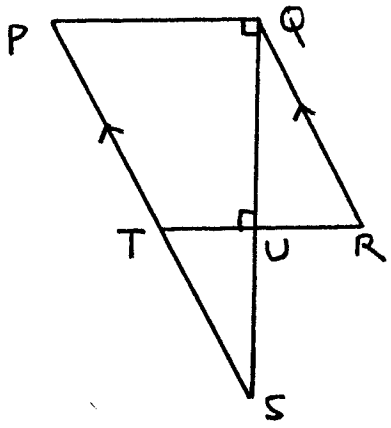
(c) Find the gradient of the line that passes through *(-3,2)* and *(5,8)*
Hence, find the angle the line makes with the positive *x-axis*.

(d) Solve the equations

$3a + 4b = -5$
$-9a + 8b + 25 = 0$

(e) If $\sin x = \dfrac{5}{12}$, and x is obtuse, find the value of $\sec x$ in exact form.

(f) In the diagram, $SQ \perp PQ$ and
$RU \perp SQ$ and PS is parallel to QR



(i) Copy the diagram

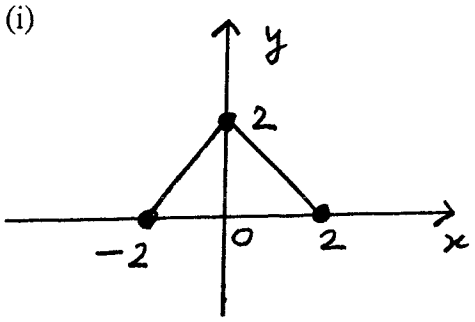
(ii) Prove $\Delta\ RQU$ is similar to $\Delta\ PQS$

(iii) If $RU = x$ units
$QR = y$ units
and PS is four times the length of RU,
find the length of PQ in terms of x and y.

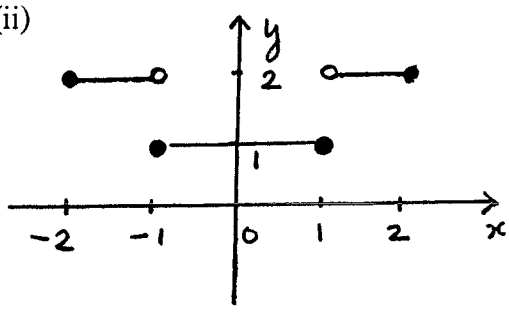
Question Four

(a) State the function rule for the following curves

(i)



(ii)



(b) Sketch the function

(i) $$f(x) = \begin{cases} x, if\ -1 \le x \le 1 \\ 1, if\ x > 1 \end{cases}$$

(ii) State the domain and the range

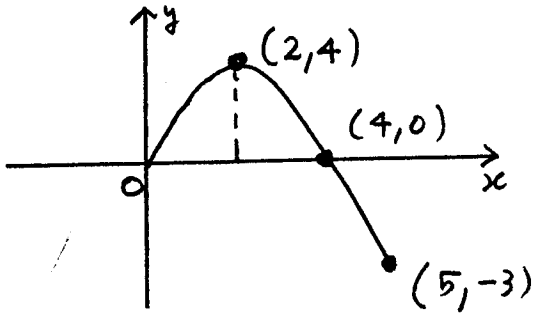
(iii) find $2f(\frac{1}{2}) - f(0) + f(2)$

(c) Starting from a point A boat sails due west for 6 km to a point B, then turns to the bearing of 200^0 T and sails for 12km to C. Find the distance AC.

(d) The graph of a function is shown below for $x \ge 0$. Copy the graph and complete it for $x < 0$ if it is :

(i) even

(ii) odd



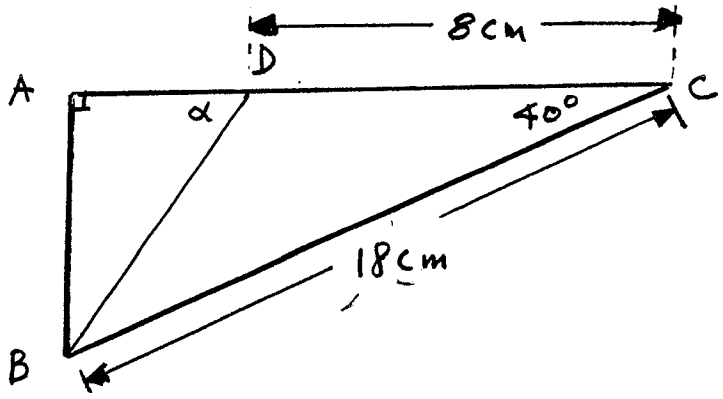
(e) Find the equation of the line that makes an angle of 45^0 with the positive $x-axis$ and also passes through the intersection of $3x + 2y - 5 = 0$ and $x + y + 2 = 0$

Question Five

(a) (i) ~~Explain why $x = 0$ is a boundary for $y > \frac{1}{x}$~~

(ii) ~~Hence~~ sketch the region $y > \frac{1}{x}$

(b)

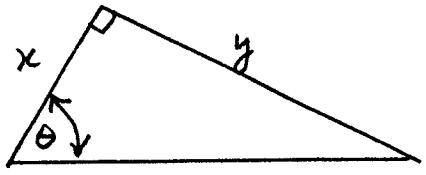


(i) Show that AC = 18 cos 40^0

(ii) find an expression for AB

(iii) Find angle α to the nearest minute.

(c)



Show that

$$\sin^2\theta = \frac{y^2}{x^2 + y^2}$$

(d) Find the exact value of

$3\tan 210^0 \sec 210^o - Sin330^0 \cot 135^0$

(e) Find the perpendicular distance between $(-2,1)$ and the line $12x - 5y + 3 = 0$

(f) Draw separate sketches of the following curves showing their main features.

(i) $y = -\sqrt{4 - x^2}$

(ii) $y = 3^x$

(iii) $y = |x - 1|$

Sydney Boys High - Yr 11 Half-Yearly 2000

Omar Salem

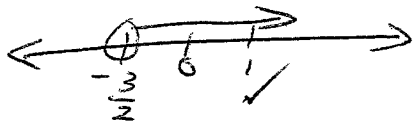
Check corrections!

Question One

a) $\sqrt{64} \times \frac{1}{\sqrt[3]{64}} = 8 \times \frac{1}{4} = 2$ ✓

b) when $y = 3$, $x + 15 = 10$
$x = -5$ ✓
$\therefore k = -5$ ✓

c) $5 - 2x < 8$
$-2x < 3$
$2x > -3$
$x > -\frac{3}{2}$ ✓



✓

d) $(2a-1)^2 - 5(1-a)$ careless!
$= 4a^2 - 4a + 1 - 5 + 5a$
$= 4a^2 + a - 4$

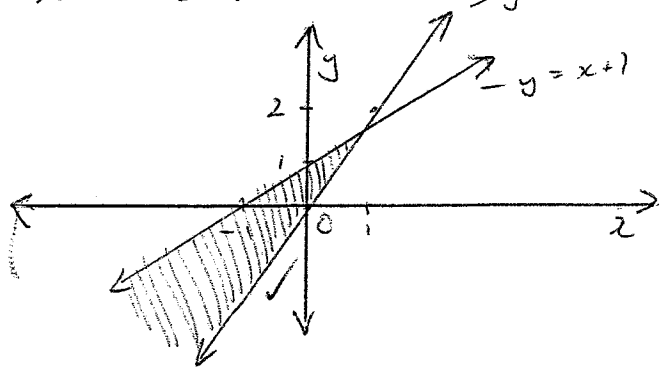
e) -1.12 ✓ (3 sig fig) ✓

f) $\frac{1}{1+\sqrt{1+a}} \times \frac{1-\sqrt{1+a}}{1-\sqrt{1+a}}$ ✓ $= \frac{1-\sqrt{1+a}}{1-(1+a)} = \frac{1-\sqrt{1+a}}{1-1-a} = \frac{1-\sqrt{1+a}}{-a}$ ✓

g) i) $x = 180 - 65 - 12$ ($\angle$ sum Δ)
$x = 103°$ ✓

ii) $3x + 90 + x + 90 = 360°$
($\angle$ sum revolution)
$4x + 180 = 360$
$4x = 180$ ✓
$x = 45°$ ✓

h)



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Question Two:

a) $\angle KEC = 180 - 50^\circ$ ($\angle$ sum line)
$= 130^\circ$ ✓

$\angle EKC = \angle ECK$ (base $\angle$'s of isosceles $\triangle$)

$\angle EKC = \frac{180-130}{2}$ (isosceles $\triangle$)

$\angle EKC = \angle ECK = 25^\circ$ ✓

$\angle CDE = 90^\circ$ (alternate $\angle$'s and // lines)

$\angle DCE = 180 - 90 - 50$ ($\angle$ sum $\triangle$)
$= 40^\circ$ ✓

$\angle KCD = \angle DCE + \angle KCE$ (addition of $\angle$'s)
$= 40^\circ$ ✓ $+ 25^\circ$
$= 65^\circ$ ✓

b) $\sqrt{50} = \sqrt{25} \times \sqrt{2} = 5\sqrt{2}$

$\sqrt{18} = \sqrt{9} \times \sqrt{2} = 3\sqrt{2}$

$\sqrt{x} = 5\sqrt{2} - 3\sqrt{2}$ ✓
$= 2\sqrt{2}$

$\sqrt{x} = \sqrt{8}$

$x = 8$ ✓

(12)

c) i) $\frac{(4x+4)(4x-3)}{4} = (x+1)(4x-3)$ ✓

ii) $(1+\sin\theta)(1-\sin\theta+\sin^2\theta)$ ✓

iii) $xy+y-8x-8$ ✓
$y(x+1)-8(x+1)$ $= (y-8)(x+1)$ ✓

d) $\frac{3x+4(3x-1)}{12} = \frac{3x+12x-4}{12} = \frac{15x-4}{12}$ ✓

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e) i) $\ln 8^n = \ln 16^{-1}$

$n \ln 8 = \ln 16^{-1}$

$n = \dfrac{\ln 16^{-1}}{\ln 8}$ ✓

$n = -\dfrac{4}{3}$ ✓

At this stage, should only use INDEX LAWS

$8^n = 16^{-1}$

$(2^3)^n = (2^4)^{-1}$

$\therefore 3n = -4 \Rightarrow n = -\frac{4}{3}$

ii) $2 - 3n = 14$ or $2 - 3n = -14$

$-3n = 12$ $-3n = -16$

$n = -4$ ✓ or $3n = 16$ ✓

$n = 5\frac{1}{3}$

$n = -4, 5\frac{1}{3}$ ✓

f) $3x - 2y = 7$ — ①

$xy = 3$ — ②

$x = \dfrac{3}{y}$ ✓

$3\left(\frac{3}{y}\right) - 2y = 7$

$\frac{9}{y} - 2y = 7$

$9 - 2y^2 = 7y$

$-2y^2 - 7y + 9 = 0$ ✓

$2y^2 + 7y - 9 = 0$

$(2y + 9)(y - 1) = 0$ ✓

$y = -\frac{9}{2}, \; y = 1$ ✓

when $y = 1, x = 3$ ✓

when $y = -\frac{9}{2}, x = -\frac{2}{3}$ ✓

Question Three

a) $a = 3x, \; b = x$ ✓ $\quad \dfrac{3x + x}{3x - x} = \dfrac{4x}{2x} = \dfrac{4}{2} = 2$ ✓

b) $\sin 15 = \dfrac{x}{180}$ ✓

$x = 180 \sin 15$

$x = 47\text{cm}$ (greatest cm)

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c) $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8-2}{5+3} = \frac{6}{8} = \frac{3}{4}$ ✓

$\tan\theta = m$

$\tan\theta = \frac{3}{4}$ $\theta = 36°52'$ ✓ (nearest minute)

d) $3a + 4b = -5$ — (1)

$-9a + 8b + 25 = 0$ — (2)

(2) $-9a + 8b = -25$ — (2)

(1) $\times 2$

$6a + 8b = -10$ ✓ — (1)

(1) − (2)

$15a = 15$

$a = 1$ ✓

$\boxed{\begin{array}{l} a = 1 \\ b = -2 \end{array}}$

sub into (1)

$3(1) + 4b = -5$

$3 + 4b = -5$ ✓

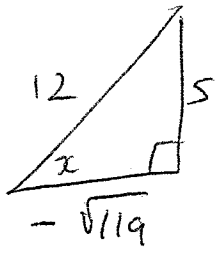
$4b = -8$

$b = -2$ ✓

e) $12^2 - 5^2 = 119$ (pythag theorem)

$\cos x = \frac{-\sqrt{119}}{12}$ ✓

$\sec x = \frac{12}{-\sqrt{119}} = \frac{-12}{\sqrt{119}}$ ✓



f) ii) In $\triangle RQV$ and $\triangle PQS$ there is,

$\angle QVR = \angle SQP$ (right $\angle$'s)

$\angle QRT = \angle TPQ$ (opposite $\angle$'s of // gram are equal)

$\therefore \triangle RQV \,|||\, \triangle PQS$ (equiangular)

Need to show that PQ // TR,

$\angle PQS = \angle RUQ = 90°$ (Alternate $\angle$s)

$\therefore$ PQ // TR.

iii) $PS = 4x$, $VR = x$, $QR = y$

$\frac{4x}{y} = \frac{PQ}{x}$ ✓ (corresponding sides of similar $\triangle$'s)

$4x^2 = PQ\,y \Rightarrow PQ = \frac{4x^2}{y}$

$PQ = 4x^2$ ✓

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Question 4

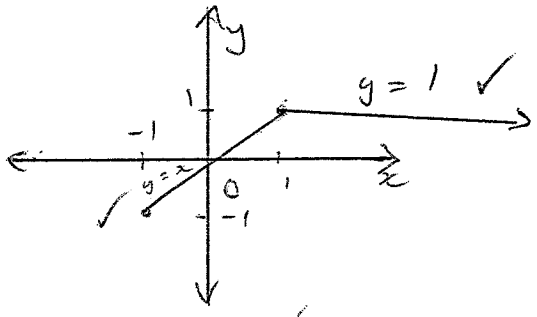
a) i) continuous ✓ $D: -2 \leq x \leq 2$ ✓

$R: 0 \leq y \leq 2$ ✓

ii) discontinuous $D: -2 \leq x \leq 2$ ✓

$R: \ldots, y=1, y=2$ ✓

b) i)



ii) $D: x \geq -1$ ✓

$R: -1 \leq y \leq 1$ ✓

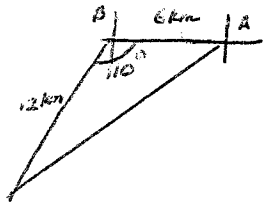
iii) $2\left(\frac{1}{2}\right) - 0 + 1 = 1 + 1 = 2$ ✓

c) $a^2 = b^2 + c^2 - 2bc\cos A$

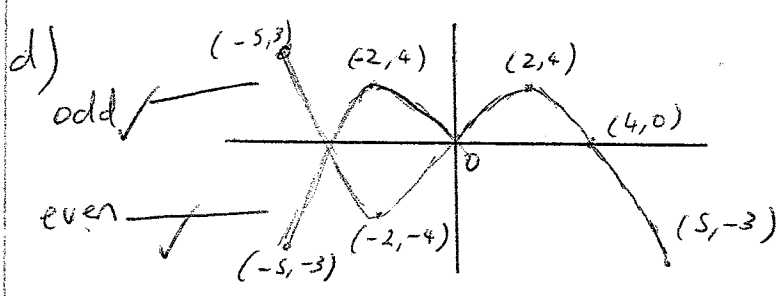
$AC = \cancel{6.68}$ km (2dp)

15.14 km (to 2dp)

Should draw a diagram.



d)



e) $\tan\theta = m$

$\tan 45 = m$

$m = 1$ ✓

$y + 11 = 1(x - 9)$

$y + 11 = x - 9$

$y = x - 20$ ✓

$x - y - 20 = 0$

or alternatively use this method for the equation of the line

$(x + y + 2) + k(3x + 2y - 5) = 0$

$(1 + 3k)x + (1 + 2k)y + (2 - 5k) = 0$

$\therefore m = -\frac{(1+3k)}{1+2k} = 1$

$\therefore -1 - 3k = 1 + 2k$

$\Rightarrow 5k = -2 \Rightarrow k = -\frac{2}{5}$

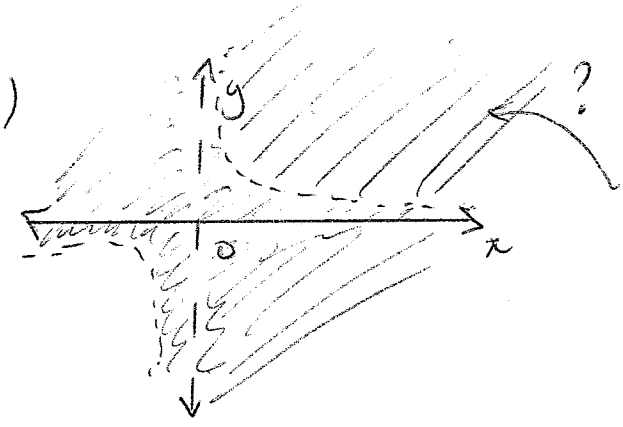
sub into $(x + y + 2) - \frac{2}{5}(3x + 2y - 5) = 0$

$5x + 5y + 10 - 6x - 4y + 10 = 0$

$-x + y + 20 = 0$

Question Five

a) ii)



b) i) $\cos 40 = \frac{AC}{18}$

$AC = 18 \cos 40$ ✓

ii) $\sin 40 = \frac{AB}{18}$ $\quad AB = 18 \sin 40$ ✓

iii) $\tan a = \frac{11.57}{5.789}$ ✓

$a = 63°25'$ ✓

c) $\sin\theta = \frac{y}{\sqrt{x^2+y^2}}$ ✓ (pythag theorem)

$\sin^2\theta = \frac{y^2}{x^2+y^2}$ ✓

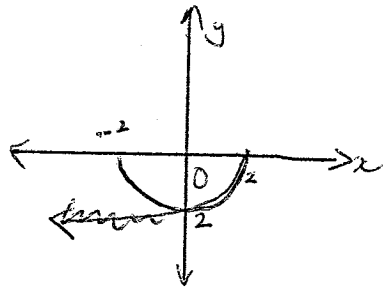
d) $\left(3 \times \frac{1}{\sqrt{3}}\right) + \left(\frac{2}{-\sqrt{3}}\right) - \left(-\frac{1}{2} \times -1\right) = -2.5$ ✓

e) $\frac{|-24-5+3|}{\sqrt{169}}$ ✓

$= \frac{|-26|}{13}$

$= 2$ ✓

f) i)



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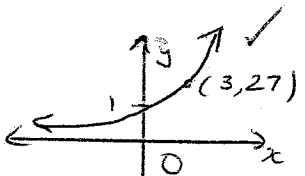

iii)