SYDNEY BOYS HIGH SCHOOL

YEAR 11 HALF YEARLY EXAMINATION – MAY 2001

MATHEMATICS

EXTENSION 1

Time allowed – 60 minutes

Examiner: C Kourtesis

DIRECTIONS TO CANDIDATES

- ***ALL*** **questions may be attempted.**
- **All necessary working should be shown in every question. Full marks may not be awarded for careless or badly arranged work.**
- **Approved calculators may be used.**
- **Use a new booklet for each question.**
- **If required, additional booklets may be obtained from the Examination Supervisor upon request.**

Question 1 **[15 marks]**

(a) Convert $\dfrac{3\pi}{4}$ radians to degrees

(b) Factorize $x^3 - 27$

(c) Sketch the graphs (on separate diagrams) of:

(i) $x^2 + y^2 = 4$ (ii) $y = 4^{x+1}$ (iii) $y = |x| + 2$

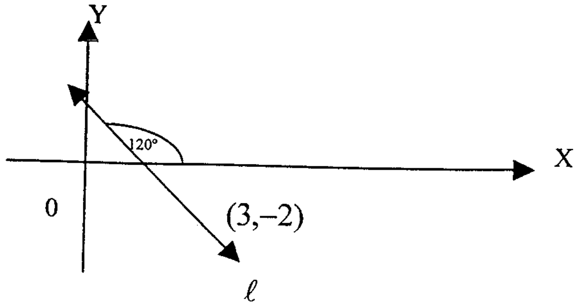
(d) If $\dfrac{1}{F} = \dfrac{1}{u} + \dfrac{1}{v}$ evaluate F when $u = 3\cdot 6$, $v = 6\cdot 4$

(e) Solve the inequality $x^2 > 4$

(f) If $F(x) = 3x^2 + k$, find the value of k if $F(-3) = 10$

(g) Find the perpendicular distance from the point $G(4,-1)$ to the straight line $5x - 12y + 36 = 0$

(h)



Find the equation of line ℓ

Question 2 [15 marks]

(a) If $g(x) = x^2 - x$ simplify $g(x) - g(x-1)$

(b) Find the centre and radius of the circle

$$x^2 + y^2 + 4x - 6y = 3$$

(c) For the function $f(x) = \frac{1}{x^2} - 3$

(i) Find the domain and range

(ii) Prove that the function is even

(d) Sketch the graphs of:

(i) $y = \log_2(-x)$ for $x < 0$

(ii) $y = \frac{x}{|x|}$

(e) Solve the inequality

$$\frac{a+4}{3-a} < 1$$

Question 3 [15 marks)

(a) Solve simultaneously the system of equations

$$\begin{aligned} 5a - 2b + 6c &= 3 \\ 6a + 4b - 4c &= 0 \\ 3a - 4b + 8c &= 3 \end{aligned}$$

(b) (i) Show that $\cos 2A = 1 - 2\sin^2 A$

(ii) Prove that

$$\frac{\sin 2A}{1 - \cos 2A} = \cot A$$

(iii) Hence find the values of a and b if

$$\cot(67\frac{1^\circ}{2}) = \sqrt{a} + b$$

where a and b are integers.

(c) (i) Solve the inequality $|2x+1| < |1-x|$

(ii) Hence graph the solution on a number line.

Question 4 **[15 marks]**

(a) The elevation of a lighthouse at a point D due east of it is 33^0 and at a point F due south of it, the elevation is 25^0. The distance from D to F is 650 metres.

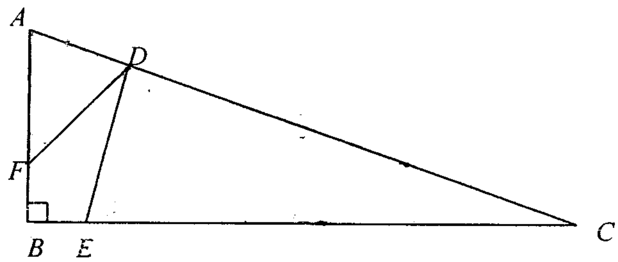
(i) Draw a diagram of the above

(ii) Show that the height H of the lighthouse is given by

$$H = \frac{650}{\sqrt{\tan^2 57^0 + \tan^2 65^0}}$$

(iii) Find the height of the lighthouse to 4 significant figures.

(b)



In the right triangle ABC, $AF=AD$ and $DC=EC$

Find the size of $\angle FDE$ (give reasons).

(c) In any triangle ABC prove that:

(i) $\tan(A+B) = -\tan C$

(ii) $\tan A + \tan B + \tan C = \tan A \tan B \tan C$

1

$\frac{59}{60}$ V. Good!

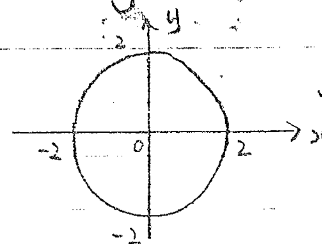
a) $\frac{3\pi}{4} \times \frac{180^\circ}{\pi}$

$= \underline{\underline{135^\circ}}$ ✓

b) $x^3 - 27$

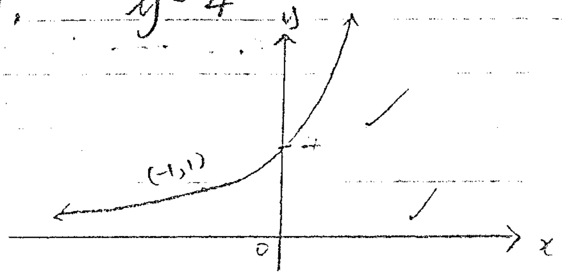
$= \underline{\underline{(x-3)(x^2+3x+9)}}$ ✓

c) i, $x^2 + y^2 = 4$



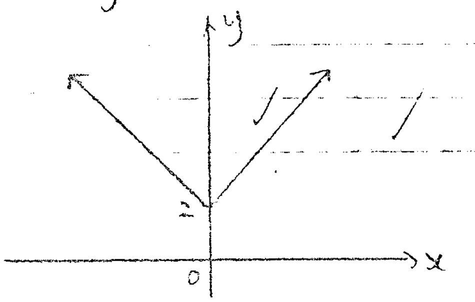
✓✓

ii, $y = 4^{x+1}$



✓ ✓

iii, $y = |x| + 2$



✓ ✓

d) $\frac{1}{F} = \frac{1}{3.6} + \frac{1}{6.4}$ ✓

$\underline{\underline{F = 2.3}}$ ✓ (2 sig. fig.)

15

e) $x^2 > 4$

$\therefore \underline{\underline{x < -2 \text{ OR } x > 2}}$ ✓ ✓

f) Put $x = -3$.

$\Rightarrow (-3)^2 + k = 10$

$\underline{\underline{k = -17}}$ ✓

g) $d = \left| \frac{5(4) - 12(-1) + 36}{\sqrt{5^2 + (-12)^2}} \right|$

$= \frac{68}{13}$ ✓

$= \underline{\underline{5\frac{3}{13} \text{ units}}}$ ✓

h) $m = \tan 120^\circ$

$= -\tan 60^\circ$

$= -\sqrt{3}$ ✓

$\therefore \frac{y+2}{x-3} = -\sqrt{3}$

$y + 2 = -\sqrt{3}x + 3\sqrt{3}$ ✓

$\underline{\underline{-\sqrt{3}x - y + 3\sqrt{3} - 2 = 0}}$

(2)

a) $g(x-1) = (x-1)^2 - (x-1)$

$= x^2 - 2x + 1 - x + 1$

$= x^2 - 3x + 2$ ✓

$\therefore \quad g(x) - g(x-1) = x^2 - x - x^2 + 3x - 2$

$= 2x - 2$

$= 2(x-1)$ ✓

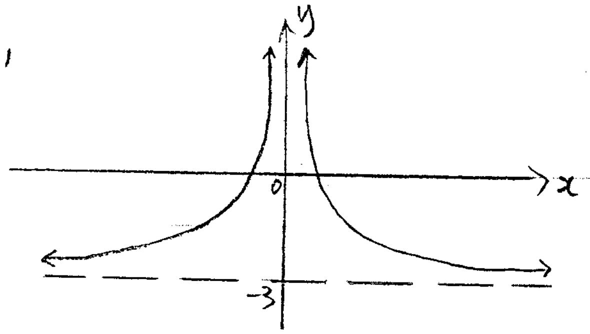
b) $x^2 + y^2 + 4x - 6y = 3$

$x^2 + 4x + 4 + y^2 - 6y + 9 = 3 + 4 + 9$ ✓

$(x+2)^2 + (y-3)^2 = 16$

$\therefore$ centre : $(-2, 3)$ ✓ radius : 4 units ✓

c) i,



D : $x \neq 0$ ✓

R : $y > -3$ ✓

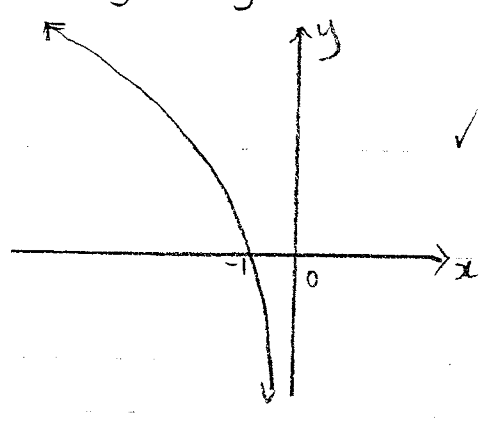
ii, $f(-x) = \frac{1}{(-x)^2} - 3$ ✓

$= \frac{1}{x^2} - 3 = f(x)$ ✓

$\therefore$ even function

14

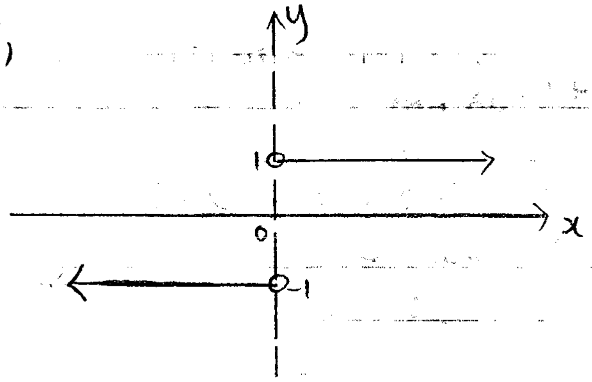
d) i, $y = \log_{10}(-x)$ for $x < 0$



✓

✓

ii)



$y = \frac{x}{|x|}$

$x \neq 0$

e)

$$\frac{a+4}{3-a} < 1$$

$$(a+4)(3-a) < (3-a)^2$$

$$3a - a^2 + 12 - 4a < 9 - 6a + a^2$$

$$2a^2 - 5a - 3 > 0$$

$$(2a+1)(a-3) > 0$$

$$\therefore \underline{\underline{a < -\tfrac{1}{2} \quad \text{OR} \quad a > 3}}$$

③ $(12+1)$

a) $5a-2b+6c=3 \quad -\ (i)$

$6a+4b-4c=0 \quad -\ (ii)$

$3a-4b+8c=3 \quad -\ (iii)$

(ii) + (iii) : $9a+4c=3$

$$a=\frac{3-4c}{9} \quad ✓$$

from (iii) : $3\left(\frac{3-4c}{9}\right)-4b+8c=3$

$$3-4c-12b+24c=9$$

$$12b=20c-6$$

$$b=\frac{10c-3}{6} \quad ✓$$

from (i) : $5\left(\frac{3-4c}{9}\right)-2\left(\frac{10c-3}{6}\right)+6c=3$

$$15-20c-30c+9+54c=27$$

$$4c=3$$

$$c=\frac{3}{4} \quad ✓$$

$$\therefore a=\frac{3-4\left(\frac{3}{4}\right)}{9}$$

$$=0$$

$$\therefore b=\frac{10\left(\frac{3}{4}\right)-3}{6}$$

$$=\frac{3}{4} \quad ✓$$

$$\therefore a=0,\quad b=\frac{3}{4},\quad c=\frac{3}{4}$$

b) i, $\cos 2A = \cos^2 A - \sin^2 A$ ✓

$= (1-\sin^2 A) - \sin^2 A$

$= 1-2\sin^2 A$ ✓

ii, L.H.S. $= \dfrac{\sin 2A}{1-\cos 2A}$ ✓

$= \dfrac{2\sin A\cos A}{1-(1-2\sin^2 A)}$ ✓

$= \dfrac{\cos A}{\sin A} = \cot A = $ R.H.S.

iii, $\cot 67.5^\circ = \dfrac{\sin 135^\circ}{1-\cos 135^\circ}$

$= \dfrac{\sin 45^\circ}{1+\cos 45^\circ}$

$= \dfrac{\sqrt{2}}{2} \times \dfrac{2}{2+\sqrt{2}}$

$= \dfrac{2\sqrt{2}-2}{4-2}$ ✓

$= \sqrt{2}-1$ ✓

$\therefore \ \underline{\underline{a = 2}}$ ✓, $\underline{\underline{b=-1}}$ ✓

c) i, $2x+1 < 1-x$

$3x < 0$

$x < 0$

OR

$-(2x+1) < 1-x$ ✓

$-2x-1 < 1-x$

$x > -2$

OR

$2x+1 < -(1-x)$

$2x+1 < -1+x$

$x < -2$

OR

$-(2x+1) < -(1-x)$

$-2x-1 < -1+x$

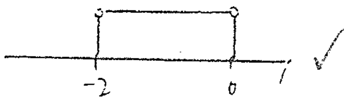
$3x > 0$ ✓

$x > 0$

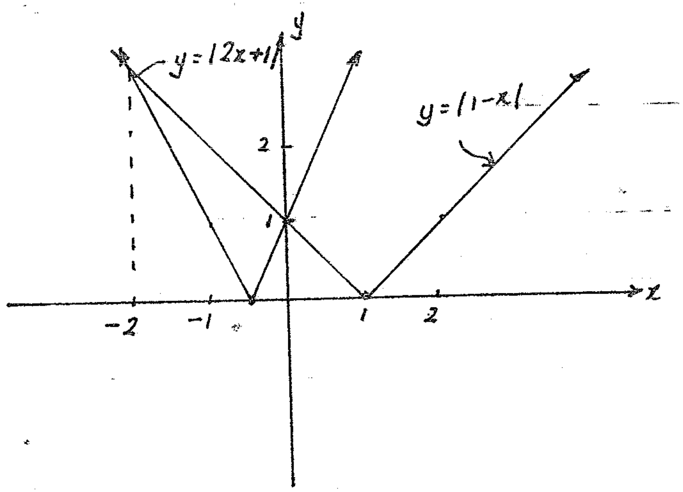
Use the graphs of $y = |2x+1|$ and $y = |1-x|$ try again.

$\therefore \ \underline{\underline{-2 < x < 0}}$ ✓

ii,



Alternatively, solve $|2x+1| < |1-x|$ graphically.

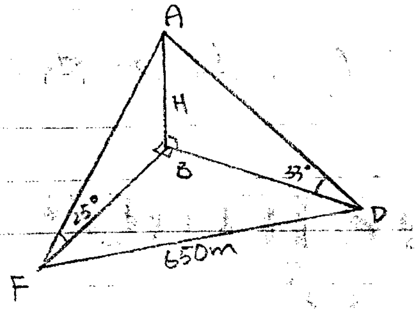


For $|2x+1| < |1-x|$

$\therefore \ \boxed{-2 < x < 0}$

(4)

a) i)



15

ii) $\angle BAD = 57°$ (remaining ∠)

$\angle BAF = 65°$ (remaining ∠)

$\therefore \tan 57° = \frac{BD}{H}$

$BD = H \tan 57°$

$\therefore \tan 65° = \frac{BF}{H}$

$\therefore BF = H \tan 65°$

$BF^2 + BD^2 = 650^2$ (Pyth. th.)

$H^2 \tan^2 65° + H^2 \tan^2 57° = 650^2$

$H^2 = \frac{650^2}{\tan^2 65° + \tan^2 57°}$

$H = \frac{650}{\sqrt{\tan^2 65° + \tan^2 57°}}$

iii) $H = 246.2$ m (4 sig. fig.)

b) In $\triangle ADF$,

$\angle ADF = \angle AFD = \alpha$ (base ∠s, isos. △)

$\alpha = \frac{180° - \angle BAC}{2}$ (∠ sum of △)

In $\triangle CDE$,

$\angle CDE = \angle CED = \beta$ (base ∠s, isos. △)

$\beta = \frac{180° - \angle BCA}{2}$ (∠ sum of △)

$\because \angle BAC + \angle BCA = 90°$ (remaining ∠)

$\therefore \alpha + \beta = \frac{180° - \angle BAC}{2} + \frac{180° - \angle BCA}{2}$

$= \frac{360° - (\angle BAC + \angle BCA)}{2}$

$= \frac{360° - 90°}{2}$

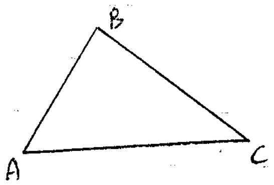
$= 135°$

$\because \angle FDE = 180^\circ - (\alpha+\beta)$ ($\angle$s on st. line)

$= 180^\circ - 135^\circ$

$= 45^\circ$

c) i,



$\because A + B + C = 180^\circ$ ($\angle$ sum of $\triangle$)

$C = 180^\circ - (A+B)$

If $A+B$ is acute $\angle$,

then $\tan(A+B) = (+ve)$ (1^{st} quadrant)

$\therefore \tan 180^\circ - (A+B) = -\tan(A+B)$ (2^{nd} quadrant)

$\tan C = -\tan(A+B)$

$\tan(A+B) = -\tan C$

If $A+B$ is obtuse $\angle$,

then $\tan(A+B) = (-ve)$ (2^{nd} quadrant)

$\therefore \tan 180^\circ - (A+B) = -\tan(A+B)$ (1^{st} quadrant)

$\tan C = -\tan(A+B)$

$\tan(A+B) = -\tan C$

ii,

$$\because \frac{\tan A + \tan B}{1 - \tan A \tan B} + \tan C = -\tan C + \tan C$$

$$\frac{\tan A + \tan B}{1 - \tan A \tan B} + \tan C = 0$$

$$\tan A + \tan B + \tan C - \tan A \tan B \tan C = 0$$

$$\tan A + \tan B + \tan C = \tan A \tan B \tan C$$