

SYDNEY BOYS' HIGH SCHOOL

MOORE PARK, SURRY HILLS



Year 11 YEARLY EXAMINATIONS – August 2000

MATHEMATICS

Time allowed — Two Hours
Examiners: E.Choy, A.M.Gainford

DIRECTIONS TO CANDIDATES

- *ALL* questions may be attempted.
- All necessary working should be shown in every question. Full marks may not be awarded for careless or badly arranged work.
- Approved calculators may be used.
- Use a new booklet for each question.
- If required, additional booklets may be obtained from the Examination Supervisor upon request.

Question 1. (18 Marks) (Start a new booklet.)

- (a) Calculate $\sqrt{\frac{67}{4 \cdot 7 \times 2 \cdot 3}}$ correct to two decimal places.
- (b) Simplify $x - 2(3 - x)$.
- (c) Solve the equation $\frac{x}{3} - \frac{x+1}{2} = 4$.
- (d) Simplify $\sqrt{32} - \sqrt{8}$.
- (e) Find x if $\log_3 x = 4$.
- (f) Find θ to the nearest minute if $0^\circ \leq \theta \leq 90^\circ$ and $\cos \theta = 0.613$.
- (g) Solve the equation $3x^2 = 12$.
- (h) Graph on a number line the solution of the inequality $|x - 2| < 3$.
- (i) Simplify $\frac{(xy^2)^3}{x^3y^2}$.
- (j) Find the exact value of $\sin 135^\circ + \tan 480^\circ$.
Express your answer as a single fraction with rational denominator.
- (k) Given that $f(x) = x - \frac{1}{x}$:
- Find $f(4)$.
 - Show that $f(x)$ is an odd function.

Question 2. (18 Marks) (Start a new booklet.)

(a) Simplify $\frac{x^2 - y^2}{(x + y)^2}$.

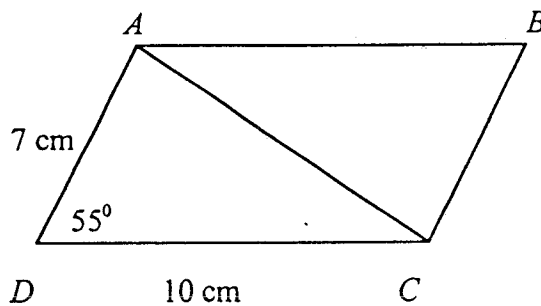
(b) Sketch the graphs of the following:

(i) $y = (x - 1)^2$

(ii) $y = \sqrt[3]{x}$

(c) Express the recurring decimal $0.4232323\dots$ as a common fraction.

(d) Given the parallelogram $ABCD$:

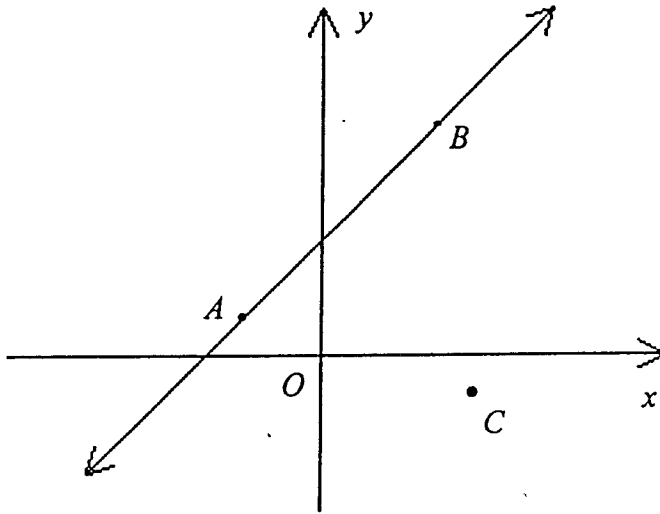


(i) Find the length of the diagonal AC , correct to 2 decimal places.

(ii) Calculate the area of the parallelogram, correct to 2 decimal places.

(e) State the natural domain and range of the function $f(x) = \sqrt{9 - x^2}$.

(f)



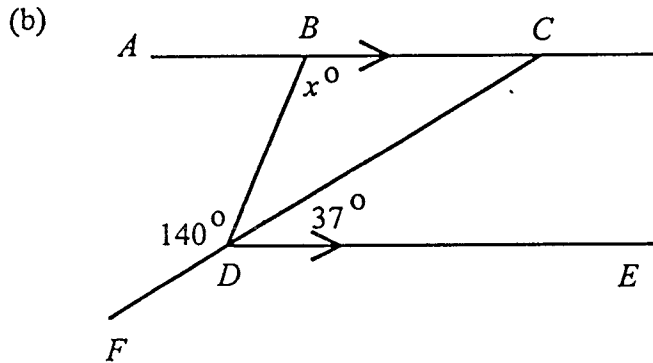
The diagram above shows the points $A(-2,1)$, $B(3,5)$, and $C(4, -1)$.

Copy the diagram to your answer booklet.

- (i) Find the equation of the line through the points A and B .
- (ii) Write the equation of the line through C perpendicular to AB .
- (iii) Hence or otherwise find the distance from C to AB .

Question 3. (18 Marks) (Start a new booklet.)

- (a) (i) Find the points of intersection of the line $y = 4 - x$ and the circle $x^2 + y^2 = 16$.
- (ii) Hence sketch the region where $y \geq 4 - x$ and $x^2 + y^2 < 16$ hold simultaneously.

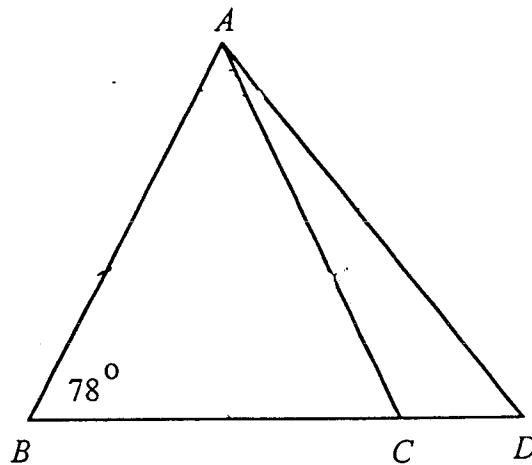


$AC \parallel DE$, CDF is a straight line.

Find the measure of x .

- (c) $AB = AC = BD$.

Determine the size of $\angle ADB$ and $\angle DAC$ giving reasons.



- (d) Solve the following equations:

(i) $x^4 - 13x^2 - 9 = 0$

(ii) $9^x - 8(3^x) - 9 = 0$

- (e) For the parabola $y^2 - 6y - 2x + 7 = 0$ write down the

- (i) equation of the axis of symmetry
- (ii) coordinates of the vertex
- (iii) equation of the directrix and coordinates of the focus.

- (f) Show that the expression $x^2 - (k + 2)x + (3k + 6)$ is positive definite if $-2 < k < 10$.

Question 4. (18 Marks) (Start a new booklet.)

(a) Differentiate the following with respect to x :

(i) $-x^3 + 2x^2 + \frac{1}{2}$

(ii) $\sqrt{7x}$

(iii) $\frac{ax^3 - bx^2 + cx}{x^2}$

(b) (i) Use the product rule to find $\frac{dy}{dx}$ if $y = 2x(x+1)^8$.

(ii) Differentiate $y = \frac{5+t}{5-2t}$ by using the quotient rule.

(iii) If $f(x) = 3x + \frac{1}{x^3}$, find

(α) $f'(2)$

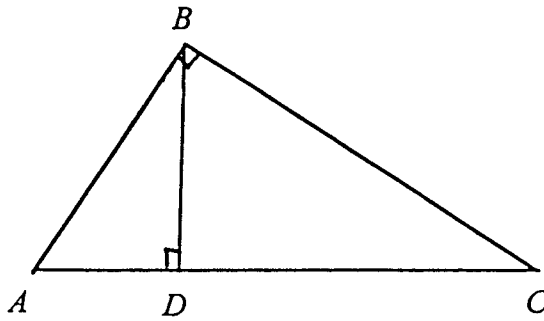
(β) $f''(2)$

(c) (i) For the curve $y = \frac{1}{x^2}$, find the gradient of the tangent to the curve at the point $\left(2, \frac{1}{4}\right)$. Also find the gradient of the normal to the curve at this point.

(ii) Given that $f(x)$ is defined as below, find the value of $f(-3) + f(4) + f(-1)$.

$$f(x) = \begin{cases} -5 & \text{for } x \leq -3 \\ 2x & \text{for } -3 < x < 0 \\ x^2 & \text{for } x \geq 0 \end{cases}$$

(d)



In the diagram ABC and ABD are right-angled triangles. $\angle ADB = \angle ABC = 90^\circ$.

Copy the diagram to your answer booklet.

- (i) Prove that $\triangle ABD \sim \triangle ACB$.
- (ii) Hence find AB if $AD = 4$ cm and $DC = 5$ cm.

(e) Find the value or values of k that will make the equation $x^2 + 16x - 4k = 0$ have:

- (i) equal roots
- (ii) two distinct real roots
- (iii) roots which are reciprocals of one another
- (iv) the sum of roots equal to their product.

Question 5. (18 Marks) (Start a new booklet.)

(a) The point $P(x, y)$ moves in the plane so that its distance from a point $A(-2, 4)$ is always twice its distance from the point $B(4, 1)$.

(i) Write down an expression connecting PA and PB .

(ii) Hence find the locus of P .

(b) Consider the function $y = \frac{1}{|x-1|}$.

(i) What is the natural domain of the function?

(ii) Write down the equations of the two branches of the function, and sketch its graph.

(c) Solve the equation $8\cos^2 x = 2\sin x + 7$ where $0^\circ \leq x \leq 360^\circ$. Give your answer correct to the nearest minute.

(d) Prove the identity

$$\frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta} = 2\sec^2\theta$$

(e) The diagram shows a right-angled triangle ABC , whose angle ACB is $\frac{\pi}{4}$. Line AD meets BC at D such that angle ADB is $\frac{\pi}{3}$, and length BD is one unit. Line DE meets line AC at right angles.

Find in exact form the:

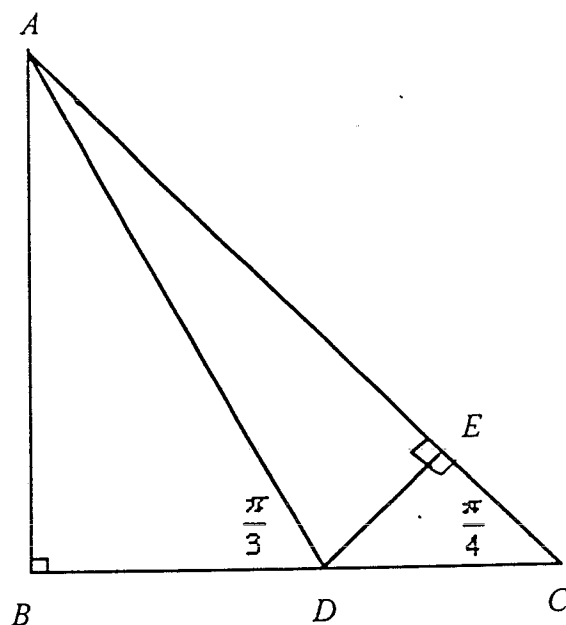
(i) length AB

(ii) length DC

(iii) length DE

(iv) $\angle DAC$ in terms of π , and

hence show that $\sin\frac{\pi}{12} = \frac{\sqrt{3}-1}{2\sqrt{2}}$.



This is the end of the paper.

Question 1

a. $\sqrt{\frac{63}{4.3 \times 2.3}} = \sqrt{\frac{63}{10.01}} = \underline{\underline{2.49}}$ ✓

b. $x - 2(3 - x) = x - 6 + 2x$
 $= \underline{\underline{3x - 6}}$ ✓

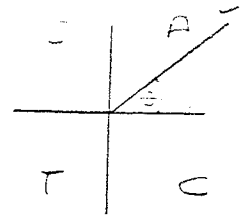
c. $\frac{x}{3} - \frac{x+1}{2} = 4$ $x \neq$
 $2x - 3(x+1) = 24$
 $2x - 3x - 3 = 24$ ✓
 $-x = 27$
 $x = \underline{\underline{-27}}$ ✓

d. $\sqrt{32} - \sqrt{8} = 4\sqrt{2} - 2\sqrt{2} = \underline{\underline{2\sqrt{2}}}$ ✓

e. $\log_3 x = 4$
 $3^4 = x$
 $\therefore x = \underline{\underline{81}}$ ✓

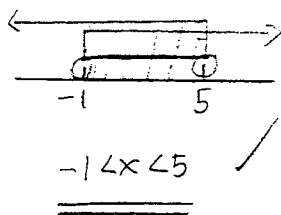
IF $y = a^x$,
 then $x = \log_a y$

f. $\cos \theta = 0.613$
 (Acute $\theta = 52^\circ 12'$)
 $\theta = \underline{\underline{52^\circ 12'}}$ ✓



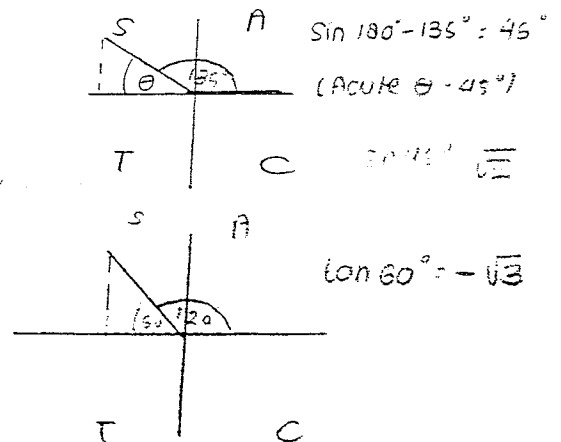
g. $3x^2 = 12$
 $3x^2 - 12 = 0$
 $3(x^2 - 4) = 0$ ✓
 $3(x-2)(x+2) = 0$
 $x = \underline{\underline{2}}$ OR $x = \underline{\underline{-2}}$ ✓

h. $|x-2| < 3$
 $x-2 < 3$ $-x+2 < 3$
 $x < 5$ $-x < 1$
 $x > -1$



i. $\frac{(xy^2)^3}{x^3y^2} = \frac{x^3y^6}{x^3y^2} = \underline{\underline{y^4}}$ ✓

j. $\sin 135 = \frac{1}{\sqrt{2}}$
 $\tan 400^\circ = -\sqrt{3}$
 $\sin 135^\circ + \tan 400^\circ$
 $= \frac{1}{\sqrt{2}} - \sqrt{3}$ ✓
 $= \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{1}$
 $= \underline{\underline{\frac{\sqrt{2} - 2\sqrt{3}}{2}}}$ ✓



k. $f(x) = x - \frac{1}{x}$

(i). $f(4) = 4 - \frac{1}{4} = 3\frac{3}{4} \checkmark$

(ii) $f(x) = x - \frac{1}{x}$

$$f(-x) = (-x) - \frac{1}{(-x)}$$

$$= (-x) + \frac{1}{x}$$

$$= -x + \frac{1}{x} \checkmark = -\left(x - \frac{1}{x}\right) = -f(x)$$

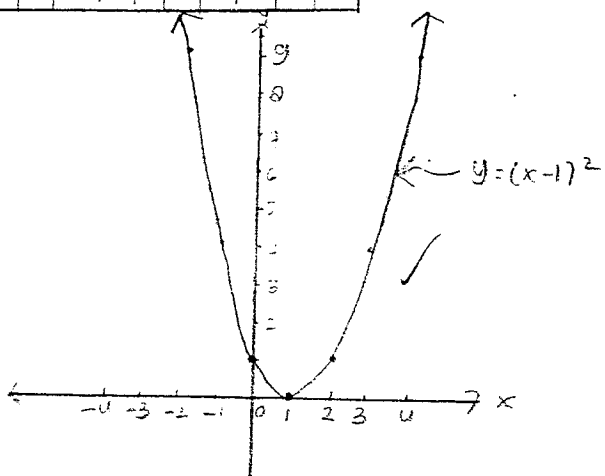
\therefore It is an odd function.

Question 2

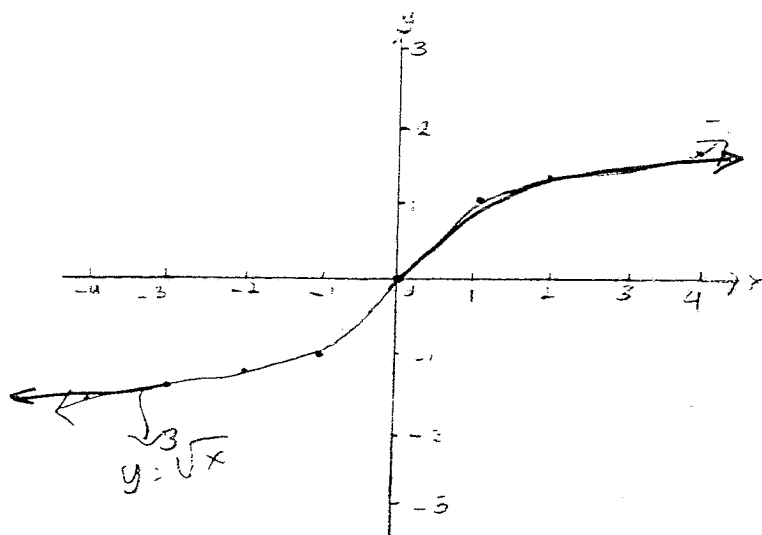
a. $\frac{x^2 - y^2}{(x+y)^2} = \frac{x^2 - y^2}{(x+y)(x+y)} = \frac{(x+y)(x-y)}{(x+y)(x+y)} = \frac{x-y}{x+y} \checkmark$

b. (i). $y = (x-1)^2$

x	-4	-3	-2	-1	0	1	2	3	4
y	25	16	9	4	1	0	1	4	9



(ii). $y = \sqrt[3]{x}$



$$y = (x-1)(x-1)$$

x-intercept, $y=0 \quad 0 = (x-1)(x-1)$

$$\underline{\underline{x=1}}$$

y-intercept, $x=0 \quad y = (-1)(-1)$

$$\underline{\underline{= 1}}$$

x	-4	-3	-2	-1	0	1	2	3	4
y	-16	-15	-13	-7	0	1	13	14	16

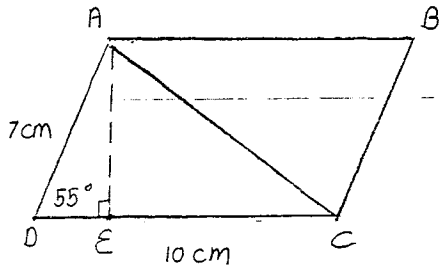
c. Let $x = 0.4232323 \dots$ (i)

(i) $\times 10$ $10x = 4.232323 \dots$ (ii)

(ii) $\times 100$ $100x = 423.232323 \dots$ (iii)

(iii) - (ii) $990x = 419$

$x = \frac{419}{990}$ ✓



$\frac{AE}{\sin 55^\circ} = \frac{7}{\sin 90^\circ}$

$AE = 5.73 \text{ cm}$

(i). $AC^2 = AD^2 + DC^2 - 2(AD)(DC) \cos 55^\circ$
 $= 49 + 100 - 2(7)(10) \cos 55^\circ$
 $= 149 - 140 \cos 55^\circ$

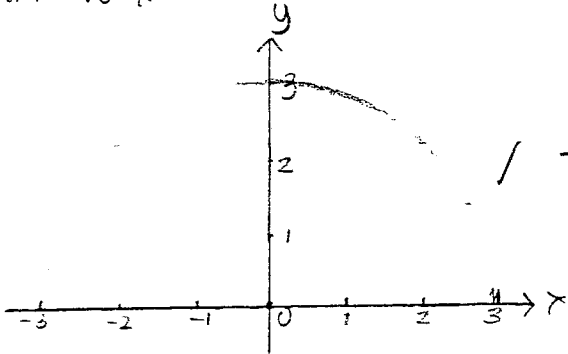
$AC = 8.29 \text{ cm}$ ✓

(ii). Area = $DC \times AE$

$= 10 \text{ cm} \times 5.73 \text{ cm}$

$= 57.34 \text{ cm}^2$ ✓

e. $f(x) = \sqrt{9-x^2}$



Domain: $-3 \leq x \leq 3$ ✓

Range: $0 \leq y \leq 3$ ✓

f. (i). $A(-2, 1)$, $B(3, 5)$, $C(4, -1)$

$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 1}{3 - (-2)} = \frac{4}{5}$

$y - y_1 = m(x - x_1)$

$y - 1 = \frac{4}{5}(x + 2)$ ✓

$y - 1 = \frac{4}{5}x + \frac{8}{5}$ $\times 5$

$5y - 5 = 4x + 8$

$0 = 4x - 5y + 13$ ✓

(ii). $m_1 = \frac{4}{5}$

$m_2 = -\frac{5}{4}$ (lines are perpendicular)

$y - y_1 = m(x - x_1)$

$y + 1 = -\frac{5}{4}(x - 4)$ ✓

$y + 1 = -\frac{5}{4}x + 5$ $\times 4$

$4y + 4 = -5x + 20$

$5x + 4y - 16 = 0$ ✓

(iii) $d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$

$= \frac{|4x - 5y + 13|}{\sqrt{16 + 25}}$ ✓

$= \frac{|(4)(4) + (-5)(-1) + 13|}{\sqrt{16 + 25}}$

$= \frac{116 + 5 + 13}{\sqrt{41}} = \frac{34}{\sqrt{41}} = \frac{34}{\sqrt{41}} \cdot \frac{\sqrt{41}}{\sqrt{41}} = \frac{34\sqrt{41}}{41}$ ✓

