SYDNEY BOYS' HIGH SCHOOL

MOORE PARK, SURRY HILLS

Year 11 YEARLY EXAMINATIONS – August 2000

MATHEMATICS

Time allowed — Two Hours
Examiners: E.Choy, A.M.Gainford

DIRECTIONS TO CANDIDATES

- *ALL* questions may be attempted.
- All necessary working should be shown in every question. Full marks may not be awarded for careless or badly arranged work.
- Approved calculators may be used.
- Use a new booklet for each question.
- If required, additional booklets may be obtained from the Examination Supervisor upon request.

Question 1. (18 Marks) **(Start a new booklet.)**

(a) Calculate $\sqrt{\dfrac{67}{4\cdot 7\times 2\cdot 3}}$ correct to two decimal places.

(b) Simplify $x-2(3-x)$.

(c) Solve the equation $\dfrac{x}{3}-\dfrac{x+1}{2}=4$.

(d) Simplify $\sqrt{32}-\sqrt{8}$.

(e) Find x if $\log_3 x=4$.

(f) Find θ to the nearest minute if $0^0 \le \theta \le 90^0$ and $\cos\theta = 0\cdot 613$.

(g) Solve the equation $3x^2 = 12$.

(h) Graph on a number line the solution of the inequality $|x-2|<3$.

(i) Simplify $\dfrac{(xy^2)^3}{x^3y^2}$.

(j) Find the exact value of $\sin 135^0 + \tan 480^0$.
Express your answer as a single fraction with rational denominator.

(k) Given that $f(x)=x-\dfrac{1}{x}$:

(i) Find $f(4)$.

(ii) Show that $f(x)$ is an odd function.

Question 2. (18 Marks) **(Start a new booklet.)**

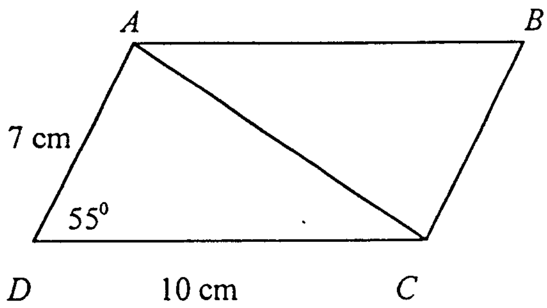
(a) Simplify $\frac{x^2 - y^2}{(x+y)^2}$.

(b) Sketch the graphs of the following:

(i) $y = (x-1)^2$ (ii) $y = \sqrt[3]{x}$

(c) Express the recurring decimal $0 \cdot 4232323\ldots$ as a common fraction.

(d) Given the parallelogram *ABCD*:

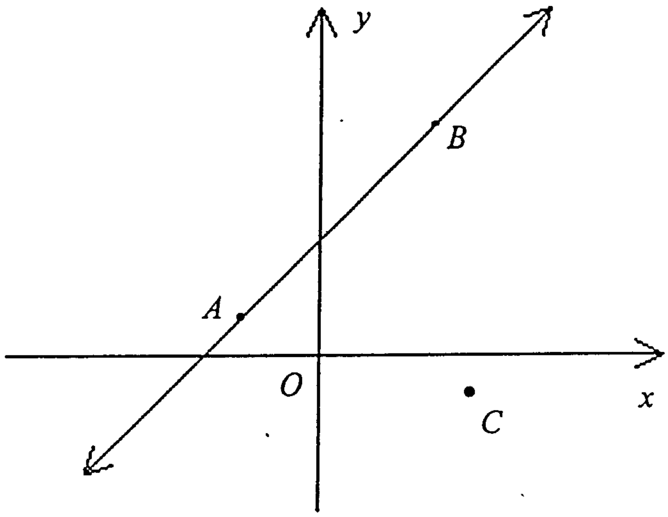


(i) Find the length of the diagonal *AC*, correct to 2 decimal places.

(ii) Calculate the area of the parallelogram, correct to 2 decimal places.

(e) State the natural domain and range of the function $f(x) = \sqrt{9 - x^2}$.

(f)



The diagram above shows the points $A(-2,1)$, $B(3,5)$, and $C(4,-1)$.

Copy the diagram to your answer booklet.

(i) Find the equation of the line through the points A and B.

(ii) Write the equation of the line through C perpendicular to AB.

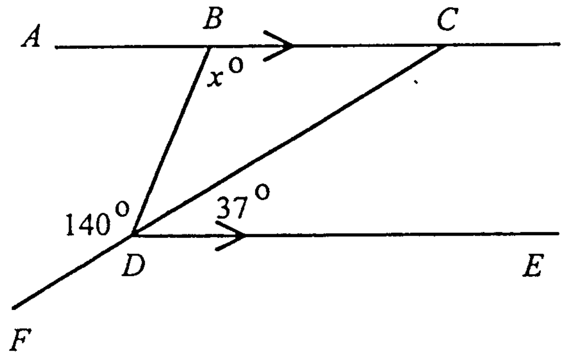
(iii) Hence or otherwise find the distance from C to AB.

Question 3. (18 Marks) **(Start a new booklet.)**

(a) (i) Find the points of intersection of the line $y = 4 - x$ and the circle $x^2 + y^2 = 16$.

(ii) Hence sketch the region where $y \geq 4 - x$ and $x^2 + y^2 < 16$ hold simultaneously.

(b)

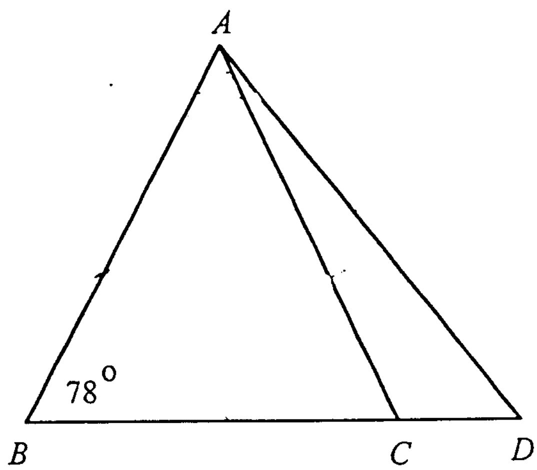


$AC \| DE$, CDF is a straight line.

Find the measure of x.

(c) $AB = AC = BD$.

Determine the size of $\angle ADB$ and $\angle DAC$ giving reasons.



(d) Solve the following equations:

(i) $x^4 - 13x^2 - 9 = 0$

(ii) $9^x - 8(3^x) - 9 = 0$

(e) For the parabola $y^2 - 6y - 2x + 7 = 0$ write down the

(i) equation of the axis of symmetry

(ii) coordinates of the vertex

(iii) equation of the directrix and coordinates of the focus.

(f) Show that the expression $x^2 - (k+2)x + (3k+6)$ is positive definite if $-2 < k < 10$.

Question 4. (18 Marks) **(Start a new booklet.)**

(a) Differentiate the following with respect to x:

(i) $-x^3+2x^2+\frac{1}{2}$

(ii) $\sqrt{7x}$

(iii) $\frac{ax^3-bx^2+cx}{x^2}$

(b) (i) Use the product rule to find $\frac{dy}{dx}$ if $y=2x(x+1)^8$.

(ii) Differentiate $y=\frac{5+t}{5-2t}$ by using the quotient rule.

(iii) If $f(x)=3x+\frac{1}{x^3}$, find

(α) $f'(2)$

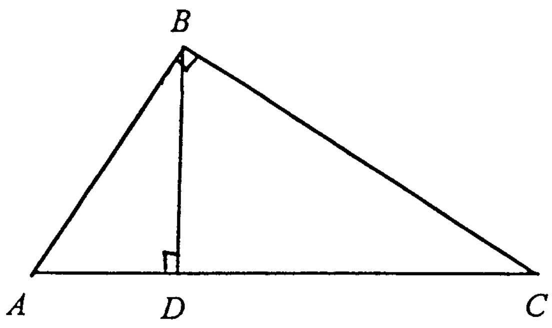
(β) $f''(2)$

(c) (i) For the curve $y=\frac{1}{x^2}$, find the gradient of the tangent to the curve at the point $\left(2,\frac{1}{4}\right)$. Also find the gradient of the normal to the curve at this point.

(ii) Given that $f(x)$ is defined as below, find the value of $f(-3)+f(4)+f(-1)$.

$$f(x)=\begin{cases}-5 & \text{for } x\le -3\\ 2x & \text{for } -3<x<0\\ x^2 & \text{for } x\ge 0\end{cases}$$

(d)



In the diagram ABC and ABD are right-angled triangles. $\angle ADB = \angle ABC = 90^0$.

Copy the diagram to your answer booklet.

(i) Prove that $\Delta ABD \parallel\!| \Delta ACB$.

(ii) Hence find AB if $AD = 4$ cm and $DC = 5$ cm.

(e) Find the value or values of k that will make the equation $x^2 + 16x - 4k = 0$ have:

(i) equal roots

(ii) two distinct real roots

(iii) roots which are reciprocals of one another

(iv) the sum of roots equal to their product.

Question 5. (18 Marks) **(Start a new booklet.)**

(a) The point $P(x,y)$ moves in the plane so that its distance from a point $A(-2,4)$ is always twice its distance from the point $B(4,1)$.

(i) Write down an expression connecting PA and PB.

(ii) Hence find the locus of P.

(b) Consider the function $y=\dfrac{1}{|x-1|}$.

(i) What is the natural domain of the function?

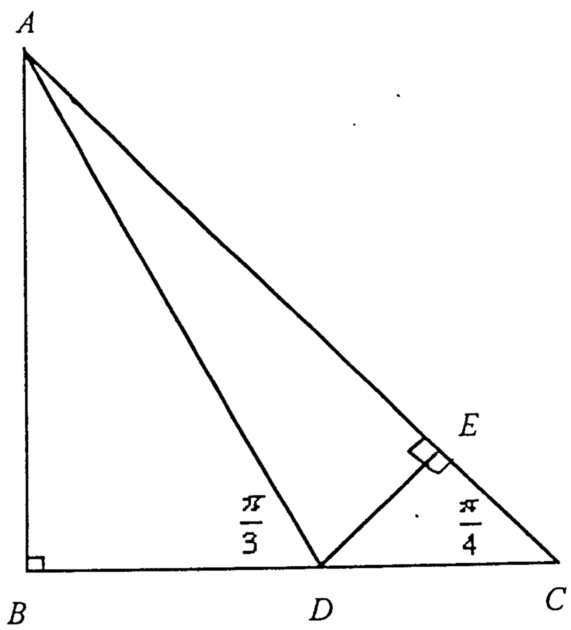
(ii) Write down the equations of the two branches of the function, and sketch its graph.

(c) Solve the equation $8\cos^2 x = 2\sin x + 7$ where $0^0 \le x \le 360^0$. Give your answer correct to the nearest minute.

(d) Prove the identity

$$\frac{1}{1+\sin\theta}+\frac{1}{1-\sin\theta}=2\sec^2\theta$$

(e) The diagram shows a right-angled triangle ABC, whose angle ACB is $\dfrac{\pi}{4}$. Line AD meets BC at D such that angle ADB is $\dfrac{\pi}{3}$, and length BD is one unit. Line DE meets line AC at right angles.



Find in **exact** form the:

(i) length AB

(ii) length DC

(iii) length DE

(iv) $\angle DAC$ in terms of π, and

hence show that $\sin\dfrac{\pi}{12}=\dfrac{\sqrt{3}-1}{2\sqrt{2}}$.

This is the end of the paper.

Sydney Boys' High School

Year 11

Yearly Examinations - August 2000

Mira Chow

Question 1

a. $\sqrt{\frac{67}{4.7\times 2.3}} = \sqrt{\frac{67}{10.81}} = \underline{\underline{2.49}}$ ✓

b. $x-2(3-x) = x-6+2x$

$= \underline{\underline{3x-6}}$ ✓

c. $\frac{x}{3} - \frac{x+1}{2} = 4$ $\times 6$

$2x - 3(x+1) = 24$

$2x - 3x - 3 = 24$ ✓

$-x = 27$

$\underline{\underline{x = -27}}$ ✓

d. $\sqrt{32} - \sqrt{8} = 4\sqrt{2} - 2\sqrt{2} = \underline{\underline{2\sqrt{2}}}$ ✓

e. $\log_3 x = 4$

$3^4 = x$

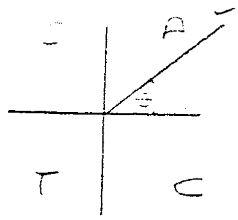
$\therefore \underline{\underline{x = 81}}$ ✓

IF $y = a^x$, then $x = \log_a y$

F. $\cos\theta = 0.613$

(Acute $\theta = 52°12'$)

$\underline{\underline{\theta = 52°12'}}$ ✓



g. $3x^2 = 12$

$3x^2 - 12 = 0$

$3(x^2-4) = 0$ ✓

$3(x-2)(x+2) = 0$

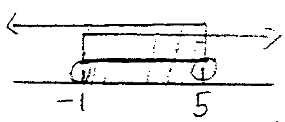
$\underline{\underline{x=2 \text{ OR } x=-2}}$ ✓

h. $|x-2| < 3$

$x-2 < 3$ $\quad$ $-x+2 < 3$

$x < 5$ $\quad$ $-x < 1$

$x > -1$



$\underline{\underline{-1 < x < 5}}$ ✓

i. $\frac{(xy^2)^3}{x^3y^2} = \frac{\cancel{x^3}y^6}{\cancel{x^3}y^2} = \underline{\underline{y^4}}$ ✓

j. $\sin 135 = \frac{1}{\sqrt{2}}$

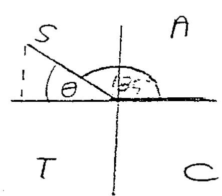
$\tan 480° = -\sqrt{3}$

$\sin 135° + \tan 480°$

$= \frac{1}{\sqrt{2}} - \sqrt{3}$ ✓

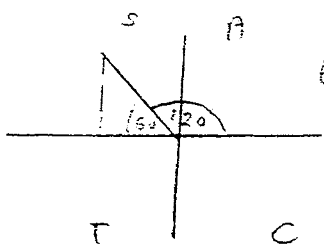
$= \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{1}$

$= \underline{\underline{\frac{\sqrt{2}-2\sqrt{3}}{2}}}$ ✓



$\sin 180° - 135° = 45°$

(Acute $\theta = 45°$)



$\tan 60° = -\sqrt{3}$

k. $f(x) = x - \frac{1}{x}$

(i). $f(4) = 4 - \frac{1}{4} = 3\frac{3}{4}$ ✓

(ii) $f(x) = x - \frac{1}{x}$

$f(-x) = (-x) - \frac{1}{(-x)}$

$= (-x) + \frac{1}{x}$

$= -x + \frac{1}{x}$ ✓ $= -\left(x - \frac{1}{x}\right) = -f(x)$

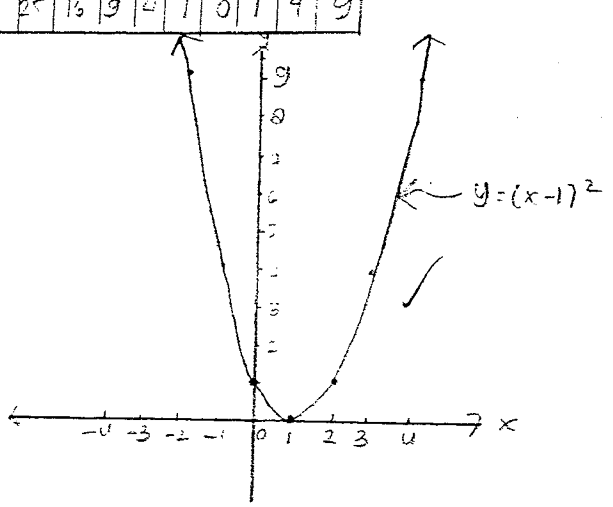
∴ It is an odd function.

Question 2

a. $\frac{x^2-y^2}{(x+y)^2} = \frac{x^2-y^2}{(x+y)(x+y)} = \frac{(x+y)(x-y)}{(x+y)(x+y)} = \frac{x-y}{x+y}$ ✓

b. (i). $y = (x-1)^2$

x	-4	-3	-2	-1	0	1	2	3	4
y	25	16	9	4	1	0	1	4	9



$y = (x-1)(x-1)$

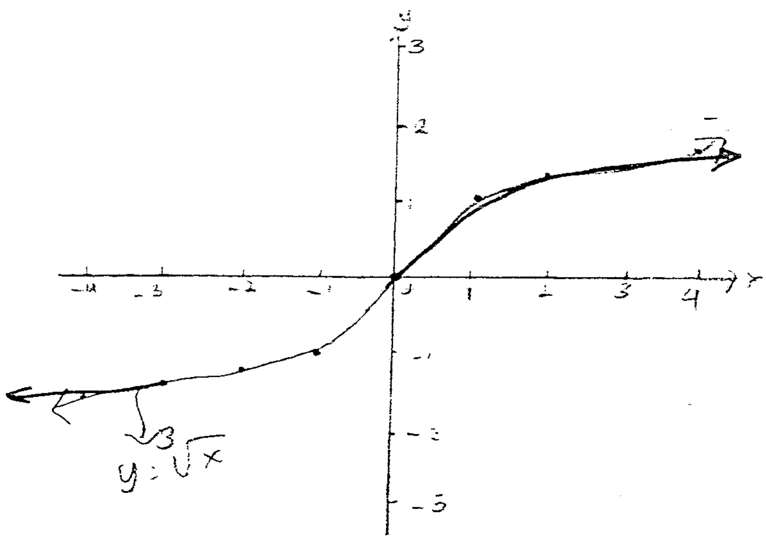
x-intercept, $y=0$ $0 = (x-1)(x-1)$

$x = 1$

y-intercept, $x=0$ $y = (-1)(-1)$

$= 1$

(ii). $y = \sqrt[3]{x}$



x	-4	-3	-2	-1	0	1	2	3	4
y	-1.6	-1.4	-1.3	-1	0	1	1.3	1.4	1.6

c. Let $x = 0.4\overline{23}2323\ldots\ldots$ (i)

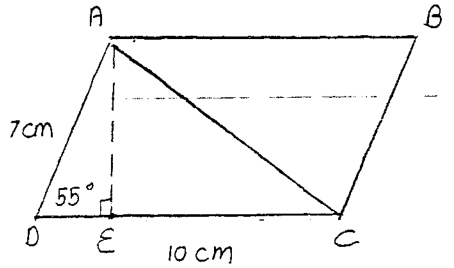
(i)×10 $10x = 4.\overline{23}2323\ldots\ldots$ (ii)

(ii)×100 $1000x = 423.\overline{23}2323\ldots\ldots$ (iii)

(iii)−(ii) $990x = 419$

$$x = \frac{419}{990}$$

d.



$$\frac{AE}{\sin 55^\circ} = \frac{7}{\sin 90^\circ}$$

$AE = 5.73$ cm

(i). $AC^2 = AD^2 + DC^2 - 2(AD)(DC)\cos 55^\circ$

$= 49 + 100 - 2(7)(10)\cos 55^\circ$

$= 149 - 140 \cos 55^\circ$

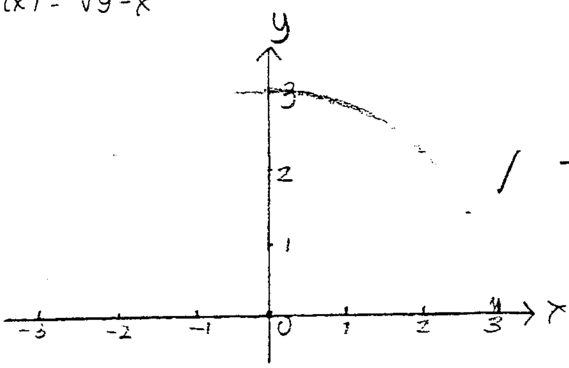
$AC = 8.29$ cm

(ii). Area = $DC \times AE$

$= 10\text{ cm} \times 5.73\text{ cm}$

$= 57.34\text{ cm}^2$

e. $f(x) = \sqrt{9 - x^2}$



Domain : $-3 \le x \le 3$

Range : $0 \le y \le 3$

F. (i). $A(-2,1)$, $B(3,5)$, $C(4,-1)$

$$M_{AB} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-1}{3-(-2)} = \frac{4}{5}$$

$y - y_1 = m(x - x_1)$

$y - 1 = \frac{4}{5}(x + 2)$

$y - 1 = \frac{4}{5}x + \frac{8}{5}$ ×5

$5y - 5 = 4x + 8$

$0 = 4x - 5y + 13$

(ii). $M_1 = \frac{4}{5}$

$M_2 = -\frac{5}{4}$ (lines are perpendicular)

$y - y_1 = m(x - x_1)$

$y + 1 = -\frac{5}{4}(x - 4)$

$y + 1 = -\frac{5}{4}x + 5$ ×4

$4y + 4 = -5x + 20$

$5x + 4y - 16 = 0$

(iii) $d = \frac{|ax_1 + by_1 + c_1|}{\sqrt{a^2 + b^2}}$

$= \frac{|4x - 5y + 13|}{\sqrt{16 + 25}}$

$= \frac{|(4)(4) + (-5)(-1) + 13|}{\sqrt{16+25}}$

$= \frac{|16 + 5 + 13|}{\sqrt{41}} = \frac{34}{\sqrt{41}} = \frac{34}{\sqrt{41}} \cdot \frac{\sqrt{41}}{\sqrt{41}} = \frac{34\sqrt{41}}{41}$