SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

SEPTEMBER 2009

Yearly Examination

YEAR 11

Mathematics (2 unit) & Extension Continuers

Instructions:

- Each Question is to be returned in a separate booklet.
- **Question 1 & 2 are to be collected after 60 minutes** at which time the 2 unit Mathematics students will be dismissed.
- **Question 3 & 4 are to be collected after 105 minutes.**
- Write using black or blue pen. Pencil may be used for diagrams.
- Board approved calculators may be used.
- All necessary working should be shown in every question.
- Answer in simplest exact form unless otherwise stated.
- Full marks may not be awarded for careless or badly arranged work.

Examiner: *A Fuller*

(Use a SEPARATE writing booklet)

Question 1 (28 marks)

(a) Find the value of $\log_3 9$. [1]

(b) Solve the following for x: [6]

(i) $\log_6 x = 3$

(ii) $\log_x 3 = -1$

(iii) $2^{2x+1} = \frac{1}{16}$

(iv) $x^2 + 3x - 18 = 0$

(c) Differentiate the following with respect to x: [7]

(i) $2x + 5$

(ii) $\dfrac{1}{2x+5}$

(iii) $\dfrac{2x+5}{x}$

(iv) $\dfrac{x}{2x+5}$

(c). Two points $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ lie on the parabola $x^2 = 4ay$. The tangents at P and Q intersect at the point T. Let $S(0, a)$ be the focus of the parabola. [8]

(i) Show that the equation of the tangent to the parabola at P is given by

$y = px - ap^2$.

(ii) Find the co-ordinates of T.

(iii) Show that $SP = a(p^2 + 1)$.

(iv) Suppose P and Q move on the parabola so that $SP + SQ = 4a$.

Show that the locus of the point T is a parabola.

End of paper

(d) Sketch the following on separate axes showing any intercepts with the co-ordinate axes and any asymptotes: [4]

(i) $y = \frac{1}{x} + 1$

(ii) $y = \log_2(x+1)$

(e) Write $2x^2 - 7x - 4$ in the form $a(x+2)^2 + b(x+2) + c$. [3]

(f) Consider the arithmetic series: $-1 + 3 + 7 + 11 + 15 + \ldots$. [4]

(i) Which term of the series is 391?

(ii) Hence, find the sum up to the term which is 391.

(g) If $f(x) = x^3 - 3x^2 - 6x$. Evaluate: [3]

(i) $f(-2)$

(ii) $f'(2)$

(Use a SEPARATE writing booklet)

Question 2 (28 marks)

(a) Consider the geometric series: $27 + 18 + 12 + 8 + \ldots$. [2]

(i) Explain why the series has a limiting sum.

(ii) Find the limiting sum of the series.

(b) Find the co-ordinates of the focus and the equation of the directrix of the parabola $y = \frac{x^2}{4} - 1$. [3]

(c) State whether the following functions are odd, even, or neither: [3]

(i) $f(x) = x^6 + 10$

(ii) $f(x) = \frac{x^2}{2-x}$

(iii) $f(x) = \log_{10} 2^x$

(d) Find using first principles the derivative $f'(x)$ given that $f(x) = x^3$. [2]

(e) Let $\log_5 3 = a$ and $\log_5 2 = b$. [4]

(i) Find the following in terms of a and b:

(α) $\log_5 6$

(β) $\log_5 \left(\frac{1}{4}\right)$

(ii) Evaluate 5^{2a}.

(f) Find the value(s) of k for which $x^2 - kx + 4 = 0$ has: [5]

(i) one root equal to -1

(ii) real roots

(iii) one root double the other.

(g) Find the domain and the range of the following: [4]

(i) $y = \sqrt{1-x}$

(ii) $y = \sqrt{1-x^2}$

(h) Caleb plans to deposit an amount of money into an account which will pay him 1% interest each month on the balance of his account at the time. [5]

Immediately after each interest payment is made, Caleb plans to withdraw \$1000.

Let his deposit be \$$D$.

(i) Show that when he has made his second withdrawal, the balance of his account will be $\$[D(1.01)^2 - 1000(1 + 1.01)]$.

(ii) Caleb wants his deposit to be sufficient to be able to make withdrawals for 10 years. Find, to the nearest \$100, what his deposit must be.

(Use a SEPARATE writing booklet)

Question 3 (18 marks)

(a) Find the point dividing the interval from $(-3, 4)$ to $(5, -2)$ in the ratio $1 : 3$. [2]

(b) Find the acute angle between the lines $y = 3x - 2$ and $x + 2y - 3 = 0$. [3]

Give the answer to the nearest degree.

(c) (i) Show that $(x + 2)$ is a factor of $6x^3 + 7x^2 - 9x + 2$. [3]

(ii) Hence, or otherwise find all of the factors of $6x^3 + 7x^2 - 9x + 2$.

(d) Find the general solution for $\tan\theta + 1 = 0$ (in radians). [3]

(e) (i) Write $\cos\theta - \sqrt{3}\sin\theta$ in the form $R\cos(\theta + \alpha)$. [4]

(ii) Hence, or otherwise, solve $\cos\theta - \sqrt{3}\sin\theta = 1$ for $0 \leq \theta \leq 2\pi$.

(f) Given that $\sin\theta = \frac{1}{\sqrt{3}}$ and $\frac{\pi}{2} < \theta < \pi$. Find the exact value of the following: [3]

(i) $\tan\theta$

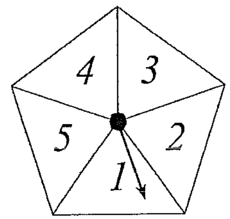
(ii) $\cos 2\theta$

(Use a SEPARATE writing booklet)

Question 4 (17 marks)

(a) Solve $\dfrac{x+4}{x-2} \geq 3$. [3]

(b) [6]



(i) The arrow on the regular pentagon is spun twice and the sum of the two numbers is recorded. Find the probability of getting:

(α) an odd result

(β) a result of at least 7

(ii) How many times must the arrow on the regular pentagon be spun to be 99.9% sure of getting at least one 5?

1 (a) $\log_3 9 = 2$ [1]

(b) (i) $\log_6 x = 3$

$x = 6^3$

$x = 216$ [1]

(ii) $\log_x 3 = -1$

$3 = x^{-1}$

$x = \frac{1}{3}$ [1]

(iii) $2^{2x+1} = \frac{1}{16}$

$2^{2x+1} = 2^{-4}$

$2x+1 = -4$

$x = -2\frac{1}{2}$ [2]

(iv) $x^2 + 3x - 18 = 0$

$(x+6)(x-3) = 0$

$x = -6$ or 3 [2]

(c) (i) $\frac{d}{dx}(2x+5) = 2$ [1]

(ii) $\frac{d}{dx}\left(\frac{1}{2x+5}\right) = \frac{d}{dx}\left((2x+5)^{-1}\right)$

$= -1 \times (2x+5)^{-2} \times 2$

$= \frac{-2}{(2x+5)^2}$ [2]

(iii) $\frac{d}{dx}\left(\frac{2x+5}{x}\right) = \frac{d}{dx}\left(2 + \frac{5}{x}\right)$

$= 5 \times -1x^{-2}$

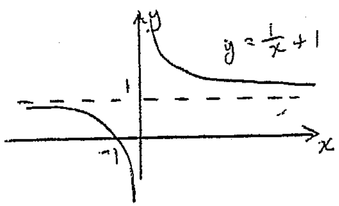
$= \frac{-5}{x^2}$ [2]

(iv) $\frac{d}{dx}\left(\frac{x}{2x+5}\right) = \frac{(2x+5).1 - x.2}{(2x+5)^2}$

$= \frac{2x+5-2x}{(2x+5)^2}$

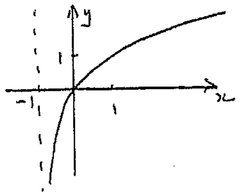
$= \frac{5}{(2x+5)^2}$ [2]

(d) (i)



[2]

(ii)



[2]

(e) $2x^2 - 7x - 4 \equiv a(x+2)^2 + b(x+2) + c$

Equating leading coefficients $a = 2$

Let $x = -2$: $8 + 14 - 4 = c$

$\therefore c = 18$

Let $x = 0$: $-4 = 2\times 4 + b \times 2 + 18$

$-4 = 2b + 26$

$2b = -30$

$b = -15$ [3]

$2x^2 - 7x - 4 = 2(x+2)^2 - 15(x+2) + 18$

(f) $a = -1$

$d = 4$

(i) $T_n = -1 + (n-1).4 = 391$

$4(n-1) = 392$

$n - 1 = 98$

$n = 99$ [2]

(ii) $S_{99} = \frac{99}{2}(-1 + 391)$

$= \frac{99}{2} \times 390$

$= 19305$ [2]

(g) (i) $f(-2) = -8 - 12 + 12$

$= -8$ [1]

(ii) $f'(x) = 3x^2 - 6x - 6$

$f'(2) = 12 - 12 - 6$

$= -6$ [2]

28

e) $\log_5 3 = a$

$\log_5 2 = b$

i) α) $\log_5 6 = \log_5 3 + \log_5 2$

$= a + b$ (1)

β) $\log_5(1/4) = \log_5 1 - \log_5 4$

$= 0 - \log_5 2^2$

$= 0 - 2\log_5 2$

$= -2b$ (1)

ii) $5^{2a} = (5^a)^2$

$= 3^2$

$= 9$ (2)

f) $x^2 - kx + 4 = 0$

i) $(-1)^2 + k + 4 = 0$

$k = -5$ (1)

ii) $\Delta > 0$

$\therefore \sqrt{b^2 - 4ac} \geq 0$

$b^2 - 4ac \geq 0$

$(-k)^2 - 4 \times 1 \times 4 \geq 0$

$k^2 - 16 \geq 0$

$k^2 \geq 16$

$k \geq 4,\ k \leq -4$ (2)

iii) $\alpha + \beta = -b/a$

$\alpha + 2\alpha = k$

$3\alpha = k$

$\alpha \times 2\alpha = 4$

$2\alpha^2 = 4$

$\alpha^2 = 2$

$\alpha = \pm\sqrt{2}$

$\therefore k = 3\sqrt{2},\ -3\sqrt{2}$ (2)

g) i) D: $x \leq 1$ (1)

R: $y \geq 0$ (1)

ii) D: $-1 \leq x \leq 1$ (1)

R: $0 \leq y \leq 1$ (1)

h) i) After 1 month

$A_1 = (\$D \times 1.01) - 1000$

After 2nd month

$A_2 = (A_1 \times 1.01) - 1000$

$= [(\$D \times 1.01) - 1000] \times 1.01 - 1000$

$= \$[D \times (1.01)^2 - 1000 \times 1.01 - 1000]$

$= \$[D(1.01)^2 - 1000(1.01 + 1)]$

$= \$[D(1.01)^2 - 1000(1 + 1.01)]$ (2)

$D(1.01)^{120} - 1000[1.01^{119} + 1.01^{118} + 1.01^{117} + \ldots + 1.01 + 1] \geq 0$

this part is a series

$a = 1,\ r = 1.01 \quad n = 120$

$S_n = \frac{1(1.01^{120} - 1)}{1.01 - 1}$

$D(1.01)^{120} - 1000 \times \frac{1(1.01^{120} - 1)}{1.01 - 1} \geq 0$

$D(1.01)^{120} \geq 1000 \times \frac{1(1.01^{120} - 1)}{1.01 - 1}$

$D \geq \dfrac{1000 \times \frac{(1.01^{120} - 1)}{1.01 - 1}}{(1.01)^{120}}$

$D \geq \$69,700.$ (nearest \$100)

ii) $\cos\theta - \sqrt{3}\sin\theta = 1$.

$2\cos(\theta + \pi/3) = 1$.

$\cos(\theta + \pi/3) = 1/2$

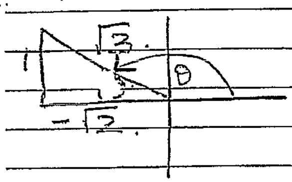
as $0 \le \theta \le 2\pi$.

$\pi/3 \le \theta + \pi/3 \le 2\pi + \pi/3$.

$\theta + \pi/3 = \pi/3, \frac{5\pi}{3}, \frac{7\pi}{3}$.

$\theta = 0, \frac{4\pi}{3}, 2\pi$ #

f) $\sin\theta = \frac{1}{\sqrt{3}}, \frac{\pi}{2} < \theta \le \pi$



i) $\tan\theta = -\frac{1}{\sqrt{2}}$.

ii) $\cos 2\theta = 1 - 2\sin^2\theta$

$= 1 - 2\left(1/\sqrt{3}\right)^2$

$= +1/3$ #

THEN

$y = px - ap^2$

$= pa(p+q) - ap^2$

$= ap^2 + apq - ap^2$

$= apq$

So $x = a(p+q)$

$y = apq$

iii) NOW

$SP = \sqrt{(2ap - 0)^2 + (ap^2 - a)^2}$

$= \sqrt{4a^2p^2 + a^2p^4 - 2a^2p^2 + a^2}$

$= \sqrt{a^2p^4 + 2a^2p^2 + a^2}$

$= \sqrt{(ap^2 + a)^2}$

$= ap^2 + a$

$= a(p^2 + 1)$

iv) $SQ = a(q^2 + 1)$

AND

$SP + SQ = a(p^2+1) + a(q^2+1)$

$a(p^2+1) + a(q^2+1) = 4a$

$\Rightarrow p^2 + q^2 = 2$

NOW

$\frac{x}{a} = p + q$, $\frac{y}{a} = pq$

$\frac{x^2}{a^2} = p^2 + 2pq + q^2$

$= (p^2 + q^2) + 2pq$

$\frac{x^2}{a^2} = 2 + \frac{2y}{a}$

$\frac{x^2}{a^2} = 2 + \frac{2y}{a}$

$x^2 = 2ay + 2a^2$

$x^2 = 2a(y + a)$

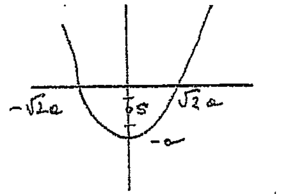
$x^2 = 4 \cdot \frac{a}{2}(y + a)$

$X^2 = 4AY$

Parabola

Vertex $(0, -a)$

Focal length $\frac{a}{2}$



Focus is point S