SYDNEY BOYS' H.S 2003 Y-11 Y-ly.

Total marks – 130
Attempt Questions 1–10

Answer each Section in a SEPARATE writing booklet. Extra writing booklets are available upon request.

Section A Use a SEPARATE writing booklet. **Marks**

Question 1 (16 marks)

(a) Express each of the following as a rational number.

(i) $49^{-\frac{1}{2}} \times 27^{\frac{2}{3}}$ **1**

(ii) The quotient of $\sqrt{7}$ and $\sqrt{63}$ **1**

(iii) $\log_2 8$ **1**

(iv) $\dfrac{\sqrt{32}-\sqrt{8}}{3\sqrt{2}}$ **2**

(b) Find, correct to 2 decimal places, $\dfrac{(3{\cdot}24)^2}{5{\cdot}73-2{\cdot}84}$. **1**

(c) Solve for x, $\dfrac{2x}{x-5} = \dfrac{3}{5}$. **2**

(d) Factorise fully the expression x^3-x^2-x+1. **2**

(e) Find the centre and radius of the circle $x^2+y^2-6x+4y-12=0$. **2**

(f) Solve the inequality $x^2-4x<0$. **2**

(g) Given $v^2=u^2-2aS$, $v=2{\cdot}5$, $u=2{\cdot}3$, and $a=7$, find S correct to 3 significant figures. **2**

Section A continued **Marks**

Question 2 (14 marks)

(a) Given the points $A(-5, 3)$, $B(1, -5)$, and $C(2, 2)$,

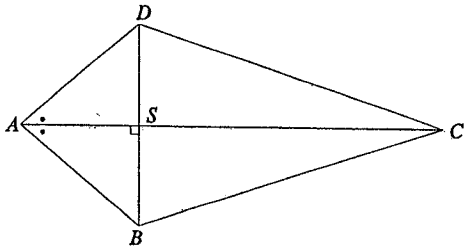
(i) Find the length of AB. **1**

(ii) Find the equation of the line AB written in general form. **2**

(iii) Find the perpendicular distance of C from the line AB. **2**

(iv) Hence or otherwise, calculate the area of the triangle ABC. **1**

(b)



In the above diagram, not to scale, $ABCD$ is a quadrilateral. The diagonals AC and DB intersect at right angles at point S. $D\hat{A}S = B\hat{A}S$.

(i) Prove that ΔASB is congruent to ΔASD. **2**

(ii) Hence prove that $DA = BA$. **1**

(c) Given $f(x) = x^2+3x+2$,

(i) Evaluate $f(-3)$. **1**

(ii) Find a simple expression for $f(a+2)$. **2**

(d) Solve for x, $|2x-1| \leq 5$. **2**

Section B Use a SEPARATE writing booklet. **Marks**

Question 3 (13 marks)

(a) Doug observes a clifftop A from his yacht at position P. He then sails 500 m towards the cliff to position Q. The angle of elevation to the clifftop from P is 5° and from Q is 8°.

(i) Draw a diagram to illustrate the above information and use the Sine Rule to calculate AQ correct to the nearest metre. **2**

(ii) Hence or otherwise find the distance QB correct to the nearest 10 metres. **2**

(b) Simplify the expression $\frac{\tan\theta}{\cot\theta} - \frac{\sec^2\theta}{1}$. **2**

(c) (i) Find θ given that $\sin\theta = \frac{1}{2}$ where $0° \leq \theta \leq 180°$. **1**

(ii) Hence find the exact values of $\tan\theta$ and $\sec\theta$. **2**

(d) Simplify $\sin\theta\cos(90° - \theta) + \cos\theta\sin(90° - \theta)$. **2**

(e) If $\sin\theta = \frac{8}{17}$ and θ is an acute angle, find the exact values of $\cos\theta$ and $\tan\theta$. **2**

Section B continued **Marks**

Question 4 (15 marks)

(a) Sketch the area defined by the inequality $y \leq x^2$. **2**

(b) Let α and β be the roots of the equation $x^2 - 5x + 2 = 0$. Find the values of:

(i) $\alpha + \beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $(\alpha + 1)(\beta + 1)$ **1**

(c) State the domain and range of each of these functions.

(i) $x^2 + 3 = y$ **2**

(ii) $y = 3^x$ **2**

(d) Solve for x the equation $9^x - 9(3)^x = 0$. **3**

(e) The roots of the quadratic equation $px^2 - x + q = 0$ are -1 and 3. Find p and q. **3**

Section C Use a SEPARATE writing booklet. **Marks**

Question 5 (15 marks)

(a) A parabola has the equation $y = x^2 - 12x + 20$. Find

(i) where it cuts the x and y axes, **2**

(ii) its axis of symmetry and vertex, **2**

(iii) the focus, by first expressing it in the form $(x-h)^2 = 4a(y-k)$, **2**

(iv) the equation of the directrix. **1**

(b) For each of the quadratics below, evaluate the discriminant and state the relevance of this with regard to the roots of the equation.

(i) $x^2 + 3x + \frac{9}{4} = y$ **2**

(ii) $3x^2 - 2x = y + 5$ **2**

(c) Find the values of M for which the equation $4x^2 - Mx + 9 = 0$ has

(i) exactly one real root, **2**

(ii) real roots. **2**

Section C continued **Marks**

Question 6 (14 marks)

(a) Solve the following equations simultaneously, **2**
$4x - y = 3$ and $10x + 3y = 2$.

(b) Find x given $2\log_9\sqrt{3} + \log_9 81 = x$. **3**

(c) Find x correct to 3 decimal places given that $7^x = 15$. **2**

(d) If Ron invests \$500 at 12·5 % p.a. compound interest, how long would it take the investment to grow to a sum of \$1000. (Answer in years, correct to 2 decimal places.) **3**

(e) The r^{th} term of a series is $3 \times 2^{(r-4)}$. Determine which of the numbers 96, ¾, 256 belong to the series. **2**

(f) Evaluate $\sum_{n=-1}^{7} (2n + 3)$. **2**

Section D Use a SEPARATE writing booklet. **Marks**

Question 7 (10 marks)

(a) Let A (0, -2) and B (1, 0) be 2 fixed points and let P (x, y) be a variable point. **3**
Find the locus of P such that the length $(PA)^2$ equals the length $(PB)^2$.

(b) Find the locus of points 2 units away from the line $y = 3$. **2**

(c) Sketch the graph of $y = \sin\theta$ given $-180° < \theta < 90°$. **3**

(d) Find the values of a and b given that **2**
$$(ax-3)^2 + b = 4x^2 - 12x + 15.$$

Section D continued **Marks**

Question 8 (12 marks)

(a) Differentiate each of the following:

(i) $y = 2x^3 - 8$ **1**

(ii) $y = (2x-1)^3$ **1**

(iii) $y = \dfrac{2x}{1-3x}$ **2**

(iv) $y = x^2\sqrt{x}$ **2**

(v) $y = \dfrac{7}{2x^3}$ **1**

(b) (i) Find the gradient of the normal to the curve $y = 1 - \frac{1}{2}x^2$ at the point (1, 3). **3**

(ii) Find the point on $y = 1 - \frac{1}{2}x^2$ where the tangent to the curve is parallel to this normal. **2**

Section E Use a SEPARATE writing booklet. **Marks**

Question 9 (11 marks)

(a) Evaluate the following limits:

(i) $\lim_{x \to 5} \left(\frac{x-5}{x^2-25} \right)$ **2**

(ii) $\lim_{x \to \infty} \left(\frac{7-2x-3x^2}{5x^2+3} \right)$ **3**

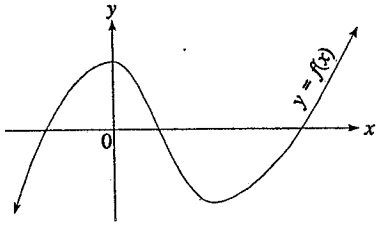
(b) Two cars depart town A at the same time. Car X travels at 60 km/h on a bearing of 345°T, whilst car Y travels at 100 km/h on a bearing of 085°T. How far apart would the cars be after 3 hours? Answer correct to one decimal place. **3**

(c) Differentiate $y = x^2 + x$ from first principles. **3**

Section E continued **Marks**

Question 10 (10 marks)

(a) The diagram below shows $y = f(x)$. **3**



Copy or trace this diagram into your answer booklet and sketch a possible graph for $y = f'(x)$ on the same set of axes.

(b) A farmer wishes to make a rectangular enclosure using a river as one boundary and 400 m of fencing on the other three sides.

(i) Find the maximum possible area of the enclosure. **3**

(ii) What are the dimensions of this enclosure? **1**

(c) Find x correct to 3 decimal places, given that $\log_7 6 - 2\log_7 3 = x$. **3**

END OF THE PAPER

$\frac{100.5}{130} \approx 77\%$

2003 Y-11 Trial.

1. a.(i) $49^{-1/2} \times 27^{2/3}$
$= \frac{1}{49^{1/2}} \times \sqrt[3]{27}$
$= \frac{1}{\sqrt{49}} \times 9$
$= \frac{9}{7}$ ✓

(ii) $\sqrt{7} \div \sqrt{63}$
$= \sqrt{7} \div 3\sqrt{7}$
$= \frac{1}{3}$ ✓

(iii) $\log_2 8$. $= 3$. ✓

(iv) $\frac{\sqrt{8} \cdot \sqrt{4}}{3\sqrt{8}}$
$= \frac{4\sqrt{8} + 2\sqrt{8}}{3\sqrt{8}}$ ✓
$= \frac{2}{3}\frac{\sqrt{8}}{\sqrt{8}}$
$= 2/3$. ✓

b. (a). $\frac{(3.04)^2}{5.073 - 2.04}$

$= \frac{10.4976}{2.033}$.

$= 3.63(2dp)$. ✓

(b). $\frac{2x}{x-5} = \frac{3}{5}$.

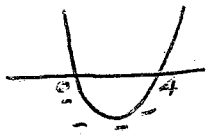
$10x = 3x - 15$. ✓
$7x = -15$ ✓
$x = -2\frac{1}{7}$.

13

(d). $x^3 - x^2 - x + 1$.
$x^2(x-1) - (x-1)$ ✓
$= (x^2-1)(x-1) = (x+1)(x-1)(x-1)$
$= (x+1)(x-1)^2$

(e). $x^2 + y^2 - 6x + 4y = 12$.
$(x^2 - 6x + 9) + (y^2 + 4y + 4) = 12 + 9 + 4$.
$(x-3)^2 + (y+2)^2 = 25$. ✓
$\therefore$ Centre at $(3, -2)$
Radius $= 5$.

(f). $x^2 - 4x < 0$.
$x(x-4) < 0$



$\therefore 0 < x < 4$. ✓

(g). $v^2 = u^2 - 2as$ $v = 2.5, u = 2.03, a = 7$.
$6.25 = 5.29 - 14s$. ✓
$14s = -0.96$
$s = -0.0686(3s.f)$ ✓

2003 Yr 11 Maths.

2. (a). (i) A (−5, 3), B (1, −5), C (2, 3)

Length AB $= \sqrt{(1-(-5))^2 + (-5-3)^2}$

$= \sqrt{36+64}$

$= \sqrt{100}$

$= 10.$ ✓

(ii). $m_{AB} = \frac{-5-3}{1+5}$

$= -1\frac{1}{3}.$

$y - 3 = -1\frac{1}{3}(x+5)$

$3y - 9 = -4x - 20.$ ✓

$4x + 3y + 11 = 0.$ ✓

(iii). $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$

$= \frac{|4 \times 2 + 3 \times 2 + 11|}{\sqrt{4^2+3^2}}$ ✓

$= \frac{25}{5}$

$= 5$ units ✓

(iv) $\frac{1}{2}ab$

$= 25$ units^2 ✓

(b) (i) In Δ's ASB & ASD

$D\hat{A}S = B\hat{A}S$ (given) ✓

AS is common ✓

$A\hat{S}B = 90 = A\hat{S}D$ (supplementary).

~~(ii)~~ So $\therefore$ $\Delta ASB \equiv \Delta ASD$ (A.A.S)

(ii) Since $\Delta ASB \equiv \Delta ASD$

$\therefore$ $DA = DA$ (corresponding sides in $\equiv \Delta$'s).

(c). (i) $f(x) = x^2 + 3x - 2.$

$f(-3) = 9 - 9 - 2$

$= -2.$ ✓

(ii). $f(a+2) = (a+2)^2 + 3(a+2) - 2.$ (careless)

$= a^2 + 4a + 4 + 3a + 6 - 2.$ ✓

$= a^2 + 7a + 8$

(d). $|2x - 1| \leq 5.$

$2x - 1 \leq 5$ or $-2x + 1 \leq 5.$

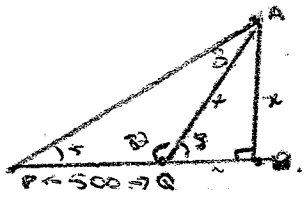
$2x \leq 6.$ ✓ $-2x \leq 4$

$x \leq 3$ $x \geq -2$ ✓

$-2 \leq x \leq 3.$

12

8.



(a) (i). $\frac{AQ}{\sin 5} = \frac{500}{\sin 3}$ Write clearly. ✓

$AQ = \frac{500 \sin 5}{\sin 3}$

$AQ = 1444.833\text{m}$

(ii). $AQ = 1444.833$

$\therefore QB = \cos 5 = \frac{AB}{AQ}$ $\frac{QB}{QA}$

$1444.833 \sin 3 = AB$

$AB = 208.63$

$= 210$ (nearest 10m).

$QB = 832.656\ldots \times \cos 5°$

$= 820\text{m}$ (to nearest 10m).

(b) $\frac{\tan\theta}{\cot\theta} - \sec^2\theta$

$= \left(\frac{\tan\theta}{1} \times \frac{\tan\theta}{1}\right) - \frac{1}{\cos^2\theta}$

$= \frac{\tan^2\theta}{1} - \frac{1}{\cos^2\theta}$ ✓

$= \frac{\sin^2\theta}{\cos^2\theta} - 1$ ✓

$= \frac{-\cos^2\theta}{\cos^2\theta}$

$= -1$.

(c) (i). $\sin\theta = \frac{1}{2}$ $0° \le \theta \le 180°$

$\theta = 30, 150$. ✓

(ii). $\tan\theta = \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}$.

$\sec\theta = \frac{1}{\cos\theta} = \frac{2}{\sqrt{3}}, -\frac{2}{\sqrt{3}}$. ✓

(d). $\sin\theta \cos(90-\theta) + \cos\theta \sin(90-\theta)$

$= \sin\theta \sin\theta + \cos\theta \cos\theta$ ✓

$= 1$ ✓

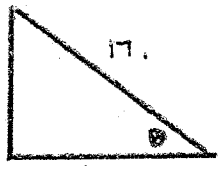
(e). $\sin\theta = \frac{8}{17}$. $0° \le \theta \le 90°$.

$c^2 = 17^2 - 8^2$

$17^2 = 8^2 + c^2$

$c^2 = 225$

$c = 15$ ✓

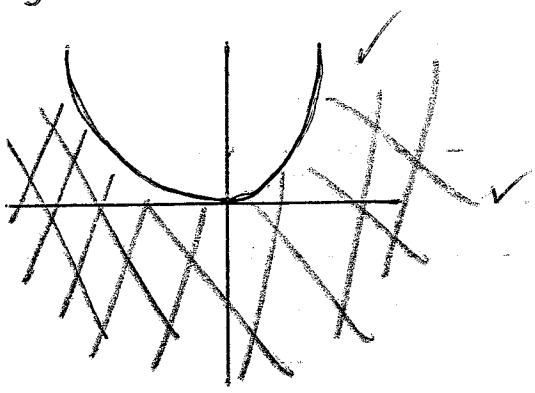


careless!

$\cos\theta = \frac{15}{17}$

$\tan\theta = \frac{8}{15}$. ✓

4. a). $y \leq x^2$.



(b) $x^2 - 5x + 2 = 0$.

(i) $\alpha + \beta = \frac{-b}{a} = -\frac{-5}{1}$

$= 1 \cdot 5$

(ii). $\alpha\beta = \frac{c}{a}$.

$= 2$. ✓

(iii) $(\alpha + 1)(\beta + 1)$

$= \alpha\beta + \alpha + \beta + 1$ ✓

$= 2 + 5 + 1$

$= 8$

(c). (i). $x^2 + 3 = y$.

Domain = All real x ✓

Range = $y \geq 3$. ✓

(ii). $y = 3^x$

Domain = All real x ✓

Range = $y > 0$. ✓

(a). $9^x - 9(3)^x = 0$.

$3^{2x} - 3^2 \times 3^x = 0$.

$3^{2x} - 3^{2+x} = 0$. $\Rightarrow$ $(3^x)^2 - 3^x \cdot 3^2 = 0$

$\therefore 2x - 2 - x = 0$. ✗ $3^x(3^x - 9) = 0$

$x - 2 = 0$ $\therefore 3^x = 0$ or $3^x = 3^2$

$x = 2$. No soln. or $x = 2$

(b). $px^2 - x + 2 = 0$. zeros: $-1, 3$.

$p + \frac{1}{2} + q = 0$. $9p - 3 + q = 0$.

$p + q = -1$ (1). $9p - 3 = -q$

$q = -9p + 3$ ✓

$p - 9p + 3 = -1$

$-8p = -4$

$p = \frac{1}{2}$. ✓ $\therefore \frac{9}{2} - 3 = -q$

$q = -1\frac{1}{2}$. ✓

12

13

5. (a). (i) $y = x^2 - 12x + 20$.

x int: $x^2 - 12x + 20 = 0$

$(x-2)(x-10)$

$\therefore x = 2, 10.$

$\therefore$ (2,0) & (10,0) ✓

y int. $y = (0)^2 - 12(0) + 20$

$= 20.$

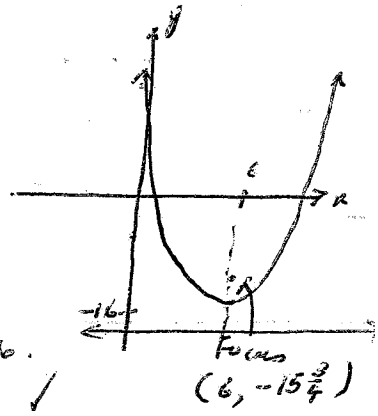
$\therefore$ (0,20) ✓

(ii). axis symetry at $x = 6$. ✓

Vertex. $f(6) = 36 - 72 + 20$

$= -16.$

$\therefore$ (6,-16). ✓



(iii). $(x-h)^2 = 4a(y-k)$

$(x-6)^2 = 4a(y+16)$

$x^2 - 12x = -20 + y$

$x^2 - 12x + 36 = 16 + y$

$(x-6)^2 = y + 16.$

$\therefore a = \frac{1}{4}$ ✓

(iv) $\therefore$ Directer: $y = -\frac{1}{a}$

$y = -16\frac{1}{4}$

(b). (i) $x^2 + 3x + \frac{9}{4} = y.$

$\Delta = b^2 - 4ac$

$= 9 - 9$ ✓

$= 0.$

$\therefore$ One Root ✓

(ii). $3x^2 - 2x = y + 5$

$3x^2 - 2x - 5 = y.$

$\Delta = b^2 - 4ac$

$= 4 + 60$

$= 64$

Two real Roots ✓, which are rational. ✓

(c) (i). $4x^2 - Mx + 9 = 0.$

One Root

$\Delta = 0.$ ✓

$\therefore M^2 - 144 = 0.$

$M^2 = 144$

$M = \pm 12.$ ✓

(ii) Real Roots.

$\Delta > 0.$

$M^2 - 144 > 0.$ ✓

$M^2 > 144$

$M > 12$ or $M < -12$ ✓

13 1/2

6. (a) $4x - y = 3$ (i)
$10x + 3y = 2$ (ii)

$4x - 3 = y$.

$\therefore 10x + 12x - 9 = 2$. $2 - y = 3$

$22x = 11$ ✓ $y = -1$ ✓

$x = 1/2$.

(b). $2\log_9\sqrt{3} + \log_9 81 = x$

$\log_9 3 + \log_9 81 = x$ ✓

$\log_9 243 = x$ ✓

$9^x = 243$.

$9^x = 9^{2.5}$

$x = 2.5$ ✓

$3 \times 3^x = 9^x$

$3^5 = (3^2)^x$

$5 = 2x$

$\frac{5}{2} = x$

(c). $7^x = 15$.

$\frac{\log 15}{\log 7} = x$ ✓

$x = 1.392 (3dp)$ ✓

(d). $500 \times 1.125^n = 1000$. ✓

$1.125^n = 2$.

$\frac{\log 2}{\log 1.125} = n$ ✓

$n = 5.88 \text{ yrs} (2\text{ dp})$. ✓

(e). $96 = 3 \times 2^{(r-4)}$

$32 = 2^{(r-4)}$

$2^5 = 2^{(r-4)}$

$5 = (r-4)$

$9 = r$ ✓

9th Term

$\frac{3}{x} = 3 \times 2^{(r-9)}$ $256 = 3 \times 2^{(r-4)}$

$\frac{1}{4} = 2^{(r-9)}$ $85\frac{1}{3} = 2^{(r-4)}$

$2^{-2} = 2^{(r-4)}$ Doesn't Belong to series

$-2 = (r-4)$

$r = 2$. ✓

2nd Term

(f). $\sum_{n=-1}^{17} (2n+3)$ $a = 1, L = 17$

$= S_9 = 9/2 [1 + 17]$ ✓

$= 81$ ✓

7. (a). $A(0,-2)$ $B(4,0)$ $P(x,y)$

$(PA)^2 = (PB)^2$ Try this $(x-0)^2 + (y+2)^2$

$PA^2 = (0-x)^2 + (-2-y)^2$ $PB^2 = (1-x)^2 + y^2$ ✓

$= x^2 + 4 + 4y + y^2$ $= 1 - 2x + x^2 + y^2$

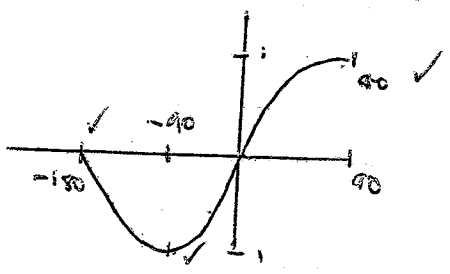
$4 + 4y = 1 - 2x$ ✓

$2x + 4y + 3 = 0$.

(b). $y = 3$. Locus dunts away

$\therefore y = 1$ or $y = 5$. ✓

(c). $y = \sin\theta \quad (-180 < \theta < 90)$



9

(d). $(ax-3)^2 + b = 4x^2 - 12x + 15$

$a^2x^2 - 6ax + 9 + b = 4x^2 - 12x + 15.$

$\therefore \quad b + 9 = 15. \qquad a^2x^2 - 6ax = 4x^2 - 12x$

$b = 6$ ✗ $\qquad a = 2.$ ✓

$b = 15 - 2 = 13.$

8. (a) (i) $y = 2x^3 - 8.$

$y' = 6x^2$ ✓

(ii) $y = (2x-1)^3$

$y' = 3(2)(2x-1)^2$

$= 6(2x-1)^2$ ✓

9

(iii). $y = \frac{2x}{1-3x}$ Quotient rule, try again.

$y' = \frac{2}{-3}$ ✗

$= -\frac{2}{3}.$

(iv) $y = x^2\sqrt{x}$ $\quad u = x^2 \quad v = x^{\frac{1}{2}}$ $\quad u' = 2x \quad v' = \frac{1}{2}x^{-1/2}$

$uv' + vu'$

$y' = x^2(\frac{1}{2}x^{-1/2}) + x^{1/2}(2x)$ ✓

$= \frac{1}{2}x^{1\frac{1}{2}} + 2x^{1\frac{1}{2}}$

$= 2\frac{1}{2}x^{3/2}.$

$= 2\frac{1}{2}\sqrt{x^3}$ ✓

Quicker to simplify, using index laws first.

$y = x^2 \cdot x^{1/2}$

$= x^{5/2}$

$\therefore y' = \frac{5}{2}x^{3/2}$

(v) $y = \frac{7}{2x^3}$ $\qquad y = \frac{7}{2}x^{-3}$ Need more practice practise with derivatives.

$y' = 0.$ ✗

(b)(i). $y = 1 - \frac{1}{2}x^2$ $\qquad$ Tangent $y = x$

$y' = -1x^1$ $\qquad \therefore \quad a+(1,3) = 1.$ ✓

$= -x$

a+ tangent.

(ii) $y' = -x$

$\therefore -x = 1$ ✓ $\qquad f(-1) = 1 - \frac{1}{2}$

$x = -1$ $\qquad = \frac{1}{2}$

$\therefore \quad a+(-1, \frac{1}{2})$ ✓

5

9.(a)(i). $\lim_{x \to 5} \frac{(x-5)}{x^2-25}$.

$= \lim_{x \to 5} \frac{x-5}{(x-5)(x+5)}$ ✓

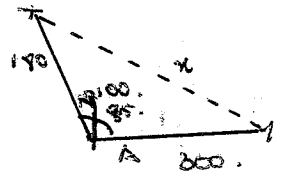
$= \frac{1}{10}$ ✓

(ii) $\lim_{x \to \infty} \frac{7-2x-3x^2}{5x^2+3}$.

$= \lim_{x \to \infty} \frac{x^2\left(\frac{7}{x^2} - \frac{2}{x} - 3\right)}{x^2\left(5 + \frac{3}{x^2}\right)}$ Note: $\lim_{x \to \infty} \frac{1}{x} = 0$

$= -\frac{3}{5}$

(b).



$\therefore a^2 = b^2 + c^2 - 2bc\cos A$ ✓

$a^2 = 180^2 + 800^2 - 108000\cos 100$

$= 153554$ ✓

$a = 391.9$ (1 d.p.)

(c). $y = x^2 + x$

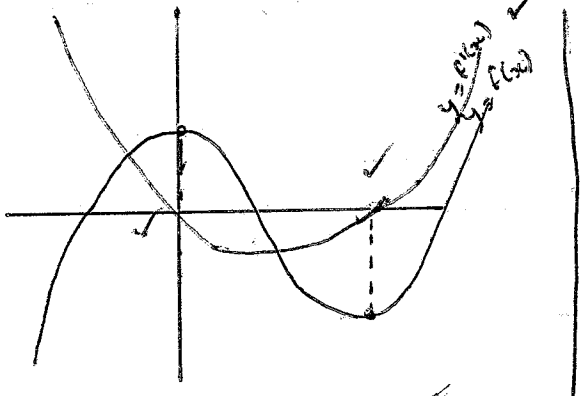
$y' = 2x + 1$ ✗

Use $\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ which $f(x) = x^2 + x$

Try again.

6

10.(a).



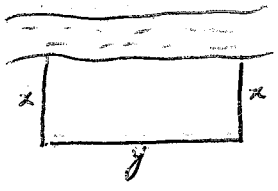
(b) (i). 400 ✱

100×100

$= 10000\ m^2$

(ii) 100m by 100m ✗

✱ Using a diagram



$2x + y = 400$ $\quad A = xy$

$y = 400 - 2x$ $\quad A = x(400 - 2x)$

$= 400x - 2x^2$

$A' = 400 - 4x = 0$

$x = 100, y = 200$

(c). $\log_7 6 - 2\log_7 3 = x \Rightarrow \log_7 6 - \log_7 3^2 = x$

~~$\log_7 6 = 6 - 7^3 = 9 = x$~~ ?

$\log_7\left(\frac{6}{3^2}\right) = x$

$\frac{\log 6}{\log 7} = 0.92$ $\quad \frac{\log 9}{\log 7} = 1.129$ ✓

$\therefore \left(\frac{\log_{10} 3^2}{\log_{10} 7}\right) = x$

$\frac{\log 6}{\log 7} - \frac{\log 9}{\log 7} = x$ ✓

$-0.208 = x$

$0.92 - 1.129 = x$

$\therefore x = -0.208$ (3 s.f.).