SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2008

HIGHER SCHOOL CERTIFICATE
ASSESSMENT TASK #1

Mathematics Extension 2

General Instructions

- Reading time – 5 minutes.
- Working time – 90 minutes.
- Write using black or blue pen. Pencil may be used for diagrams.
- Board approved calculators may be used.
- All necessary working should be shown in every question if full marks are to be awarded.
- Marks may **NOT** be awarded for untidy or badly arranged work.
- Start each **NEW** section in a separate answer booklet.

Total Marks - 90 Marks

- Attempt questions 1 - 3
- All questions are **NOT** of equal value.

Examiner: *E. Choy*

This is an assessment task only and does not necessarily reflect the content or format of the Higher School Certificate.

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n \neq -1;\ \ x \neq 0,\ \text{if } n<0$$

$$\int \frac{1}{x}\,dx = \ln x,\ \ x>0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax},\ \ a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax,\ \ a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\ \ a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,\ \ a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax,\ \ a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\ \ a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a},\ \ a>0,\ -a<x<a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right),\ \ x>a>0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x,\ \ x>0$

Total marks – 90
Attempt Questions 1 - 3
All questions are not of equal value

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Question 1 (30 marks) **Use a SEPARATE writing booklet** **Marks**

(a) (i) For the complex number $z=\sqrt{3}-i$ find

(α) $|z|$; **1**

(β) $\arg z$. **1**

(ii) For $z=\sqrt{3}-i$, show on an Argand diagram (clearly labelled) $z, \bar{z}, z^2$ and $\frac{1}{z}$. **4**

(b) Sketch on an Argand diagram the region defined by

(i) $|\arg z| \le \frac{\pi}{4}$ and $z+\bar{z}<6$ and $|z|>3$ **3**

(ii) $\arg\left(\frac{z+2i}{z}\right)=-\frac{\pi}{3}$ **2**

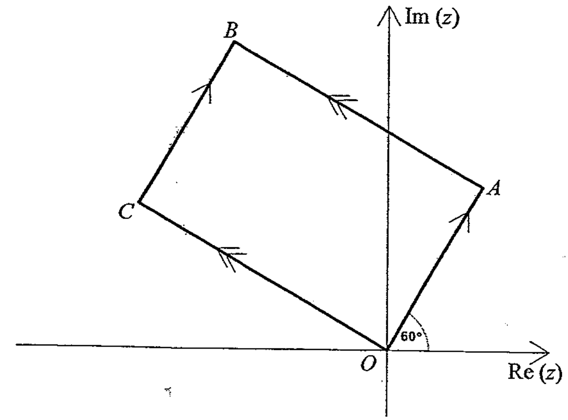
(c) (i) Express $\frac{\sqrt{3}}{2}+\frac{1}{2}i$ in modulus - argument form. **1**

(ii) Using (i) above, find the value(s) of n such that $\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)^n=1$, where n is a positive integer. **3**

Question 1 is continued on page 3

Question 1 continued **Marks**

(d)



The above figure shows a parallelogram $OABC$ in an Argand diagram.

$|OA|=2$ and OA makes an angle of $60°$ with the positive real axis.

Let z_1, z_2 and z_3 be the complex numbers represented by vertices A, B and C respectively.

It is given that $z_3=\left(\sqrt{3}i\right)z_1$.

(i) Find z_1 and z_3 in the form $x+iy$ **4**

(ii) Show that $\frac{z_2}{z_1}=1+\sqrt{3}i$. **4**

(iii) Let $\omega=\cos\theta+i\sin\theta$, where $0° \le \theta < 360°$. Point E is a point on the Argand diagram representing the complex number ωz_3.

Find the value(s) of θ in each of the following cases:

(α) E represents the complex number z_3; **3**

(β) Points E, O and A lie on the same straight line. **4**

Question 2 starts on page 4

Question 2 (20 marks) **Use a SEPARATE writing booklet** **Marks**

(a) Find the constants p and q such that $x-2$ is a common factor of $x^3-x^2-2px+3q$ and qx^3-px^2+x+2. **3**

(b) $P(x)$ is a cubic polynomial with real coefficients. **5**

One zero of $P(x)$ is $1+2i$ and the constant term is -15.

Also, $P(2)=5$.

Write $P(x)$ in the form ax^3+bx^2+cx+d.

(c) Factorise $4x^4+1$ as a product of real quadratic polynomials. **3**

(d) If α and $-\alpha$ are both roots of $x^3+mx^2+nx+p=0$, show that $mn=p$. **4**

(e) (i) Explain briefly why any rational root of $x^n+a_{n-1}x^{n-1}+...+a_2x^2+a_1x+a_0=0$, must be an integer, where $a_i\ (i=0,1,2...,n-1)$ are integers. **2**

(ii) Find the integral roots of $x^3-6x^2+6x+8=0$ and hence find all the roots **3**

Question 3 starts on page 5

Question 3 (40 marks) **Use a SEPARATE writing booklet** **Marks**

(a) (i) Evaluate ~~Evaluate~~ $\int_1^5 \frac{dx}{x^2+5x+6}$ **3**

(ii) Find ~~Evaluate~~ $\int \frac{1+\sin x}{1+\cos x}\,dx$ **3**

(iii) Evaluate ~~Evaluate~~ $\int_{\frac{1}{\sqrt{2}}}^{\frac{\sqrt{3}}{2}} \frac{dx}{x^2\sqrt{1-x^2}}$ **3**

(iv) Find $\int x^2\cos x\,dx$ **3**

(b) (i) By means of the substitution $u=a-x$ prove that **2**

$$\int_0^a f(x)\,dx=\int_0^a f(a-x)\,dx$$

(ii) Hence evaluate $\int_0^{\frac{\pi}{4}} \sqrt{1-\sin 2x}\,dx$. **3**

(c) Let $y=x^n\sin x$, where n is a positive integer.

(i) Find $\frac{dy}{dx}$ **2**

(ii) Hence, show that $\int x^n\cos x\,dx=x^n\sin x-n\int x^{n-1}\sin x\,dx$, where $n\geq 1$. **2**

(iii) Use this result, in (ii) ~~(i)~~ above, to show that $\int_0^\pi x^n\cos x\,dx=-n\int_0^\pi x^{n-1}\sin x\,dx$. **2**

(iv) Hence evaluate $\int_0^\pi x\cos x\,dx$. **2**

Question 3 continued on page 6

Question 3 continued **Marks**

(d) How many ways are there to split 4 red, 5 blue and 7 black balls among:

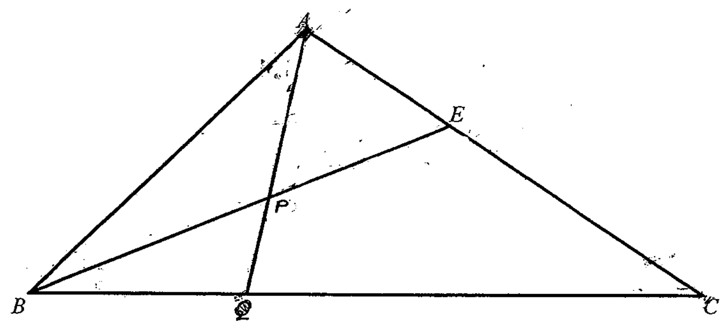
(i) Two boxes, without any restriction? **2**

(ii) Two boxes, with no box empty? **2**

(e) In the ΔABC below, BE bisects $\angle ABC$. ~~5~~ 3

APQ is a straight line such that $AP = AE$.

Prove that AB is a tangent to the circle that passes through the points A, Q and C.



(f) A straight line is drawn to the curve $y = x^4 - 4x^3 - 18x^2$ so that it is a common tangent at two distinct points on the curve.

(i) If the equation of the tangent is $y = mx + b$ and its points of contact are $x = p$ and $x = q$, show that

(α) $p + q = 2$; **2**

(β) $p^2q^2 = -b$. **1**

(ii) Hence, or otherwise, find the equation of the common tangent. ~~3~~ 5

End of paper

Boys' High School

Student No.: ____________

Paper: ____________

Section: 1

Sheet No.: 1 of 1 for this Section.

Q.No	Tick	Mark
1	✓	$25\frac{1}{2}$
2		
3		
4		
5		
6		
7		
8		
9		
10		

Q1

a)(i) (α) $z = \sqrt{3} - i$

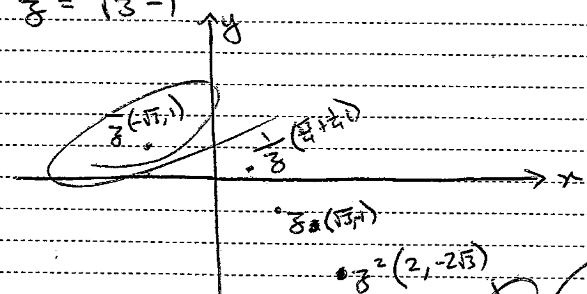
$|z| = \sqrt{(\sqrt{3})^2 + (-1)^2}$

$= 2$ ✓

(β) $\arg z = \tan^{-1}\left(\frac{-1}{\sqrt{3}}\right)$

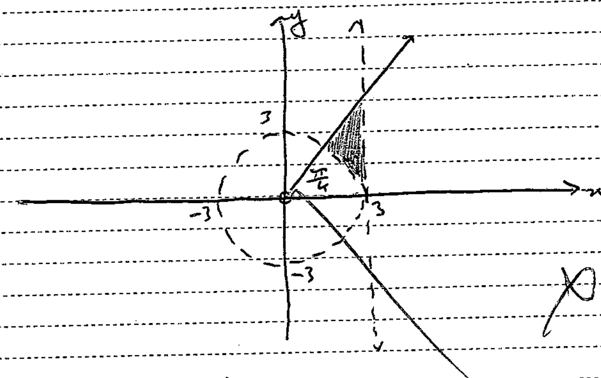
$= \frac{-\pi}{6}$ ✓

(ii) $z = \sqrt{3} - 1$



3

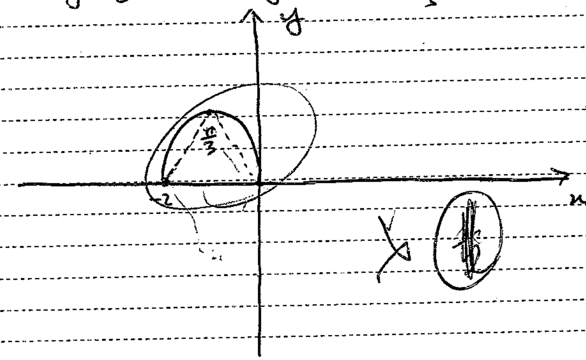
b)(i) $|\arg z| \leq \frac{\pi}{4}$ $\quad z + \bar{z} < 6 \quad |z| > 3$



$2\frac{1}{2}$

(ii) $\arg\left(\frac{z+2i}{z}\right) = \frac{-\pi}{3}$

$= \arg(z+2i) - \arg(z) = \frac{-\pi}{3}$



c)(i) $\frac{\sqrt{3}}{2} + \frac{1}{2}i$

$= 1\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$ ✓

(ii) $\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)^n = 1$

$\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)^n = 1$

$\therefore \cos\frac{n\pi}{6} + i\sin\frac{n\pi}{6} = 1$

$i\sin\frac{n\pi}{6} = 0$

$\cos\frac{n\pi}{6} = 1$

$\frac{n\pi}{6} = 2k\pi$

$n\pi = 12k\pi$

$n = \frac{12k\pi}{\pi}$

$\therefore n = 12k$ where $k = 0, 1, 2, \ldots, (n-1)$

3

d) (i) $z_1 = 2\text{cis}\frac{\pi}{3}$

$= 1 + i\sqrt{3}$

$z_3 = (\sqrt{3}i) z_1$

$= (\sqrt{3}i)(1 + i\sqrt{3})$

$= \sqrt{3}i + 3i^2$

$= -3 + i\sqrt{3}$

4

(ii) $z_2 = z_1 + z_3$

$= (1 + i\sqrt{3}) + (-3 + i\sqrt{3})$

$= -2 + i2\sqrt{3}$

$\therefore \frac{z_2}{z_1} = \frac{-2 + i2\sqrt{3}}{1 + i\sqrt{3}} \times \frac{1 - i\sqrt{3}}{1 - i\sqrt{3}}$

$= \frac{-2 + 2i\sqrt{3} + 2i\sqrt{3} - i^2 6}{1 - 3i^2}$

$= \frac{4 + 4i\sqrt{3}}{4}$

$\therefore \frac{z_2}{z_1} = 1 + i\sqrt{3}$

4

(iii) $\omega = \cos\theta + i\sin\theta \quad 0° \le \theta < 360°$

(α) $\omega = 1$

$\cos\theta + i\sin\theta = 1$

$\therefore \theta = 0°$

3

(β) $\angle AOC = \frac{\pi}{2}$

If E, O & A lie on the same line

$\angle EOC = \frac{\pi}{2}$

$\therefore E = \omega z_3 = iz_3$

Sydney Boys' High School

Student No.:	
Paper:	
Section:	2
Sheet No.:	1 of 1 for this Section.

Q.No	Tick	Mark
1		
2	✓	$14\frac{1}{2}$
3		
4		
5		
6		
7		
8		
9		
10		

Q2

a) let $P(x) = x^3 - x^2 - 2px + 3q$

$Q(x) = qx^3 - px^2 + x + 2$

$(x-2)$ is a common factor

$P(2) = 2^3 - 2^2 - 2p(2) + 3q = 0$

$\therefore 4 - 4p + 3q = 0 \quad -(1)$

$Q(2) = q(2)^3 - p(2)^2 + 2 + 2 = 0$

$\therefore 8q - 4p + 4 = 0 \quad -(2)$

$(1) - (2)$

$(2) - (1)$

$5q = 0$

$\therefore q = 0$

sub in (1)

$4 - 4p + 0 = 0$

$4(1-p) = 0$

$\therefore p = 1$

3

$\therefore p = 1 \quad q = 0$

(b) Let the complex roots be $1+2i$, $1-2i$ as the cubic poly^{n}. has real coeffs.

$\therefore\ P(x) = (x^2-2x+5)(ax-3)$ since the constant term is -15

Now $P(2)=5$

$\therefore\ P(2) = (4-4+5)(2a-3) = 5$

$\therefore\ 2a-3 = 1$

$2a = 4$

$a = 2$

$\therefore\ P(x) = (x^2-2x+5)(2x-3)$

$= 2x^3-3x^2-4x^2+6x+10x-15$

$= \underline{2x^3-7x^2+16x-15}$

(c) $4x^4+1=0$

$= (2x^2)^2 - (i^2)$

$= (2x^2+i)(2x^2-i)$

$= (2x^2-1)(2x^2+1)$

Let $x^4 = -\frac{1}{4}$

$= \frac{1}{4}\ \text{cis}\ \pi$

$\therefore\ x_k = \left[\frac{1}{4}\ \text{cis}(\pi+2k\pi)\right]^{\frac{1}{4}}$

$= \left(\frac{1}{4}\right)^{\frac{1}{4}} \text{cis}\left(\frac{\pi+2k\pi}{4}\right)$

$x_0 = \left(\frac{1}{4}\right)^{\frac{1}{4}} \text{cis}\ \frac{\pi}{4}$

$x_1 = \left(\frac{1}{4}\right)^{\frac{1}{4}} \text{cis}\left(\frac{3\pi}{4}\right)$

$x_2 = \left(\frac{1}{4}\right)^{\frac{1}{4}} \text{cis}\left(\frac{5\pi}{4}\right)$

$x_3 = \left(\frac{1}{4}\right)^{\frac{1}{4}} \text{cis}\left(\frac{7\pi}{4}\right)$

$\therefore\ x_0 = \frac{1}{\sqrt2}\cdot\left(\frac{1}{\sqrt2}+i.\frac{1}{\sqrt2}\right)$

$x_1 = \frac{1}{\sqrt2}\left(-\frac{1}{\sqrt2}+i.\frac{1}{\sqrt2}\right)$

$x_2 = \frac{1}{\sqrt2}\left(-\frac{1}{\sqrt2}-i.\frac{1}{\sqrt2}\right)$

$x_3 = \frac{1}{\sqrt2}\left(+\frac{1}{\sqrt2}-i\frac{1}{\sqrt2}\right)$

$x_0 = \frac{1}{2}(1+i)$ $\quad x_3 = \bar{x}_0 = \frac{1}{2}(1-i)$, $\quad x_2 = \frac{1}{2}(-1-i)$, $x_1 = \frac{1}{2}(-1+i)$

$\therefore\ P(x) = 4x^4+1$

$= (x^2-x+1)(x^2+x+1)$

d) $x^3+mx^2+nx+p=0$

let roots be $\alpha, -\alpha, \beta$

$\alpha + -\alpha + \beta = -m$

$\therefore\ \beta = -m$ — ①

$\alpha\beta + (-\alpha\beta) + (\alpha\cdot-\alpha) = n$

$\cancel{\alpha\beta} - \cancel{\alpha\beta} - \alpha^2 = n$

$\therefore -\alpha^2 = n$ — ②

$\alpha\cdot-\alpha\cdot\beta = -p$

$-\alpha^2\beta = -p$ — ③

sub ① & ② into ③

$\therefore\ n\cdot-m = -p$

$-mn = -p$

$\therefore\ mn = p$

4

e) (i) $x^n + a_{n-1}x^{n-1} + \dots + a_2x^2 + a_1x + a_0 = 0$

let p be a root of this equation

$P(p) = p^n + a_{n-1}p^{n-1} + \dots + a_2p^2 + a_1p + a_0 = 0$

$p(p^{n-1} + a_{n-1}p^{n-2} + \dots + a_2p + a_1) = -a_0$

$\therefore\ p$ is a factor of a_0

Since a_0 is an integer

~~p, a root of~~

p, which is a factor of a_0 must then be an integer.

2

(ii) $P(x) = x^3-6x^2+6x+8=0$

$P(1) = 1-6+6+8 \neq 0$

$P(-1) = -1-6-6+8 \neq 0$

$P(4) = 64-96+24+8$

$= 0$

$\therefore\ 4$ is a root of $P(x)$

$$\begin{array}{r} x^2-2x-2 \\ x-4\overline{)x^3-6x^2+6x+8} \\ \underline{x^3-4x^2} \\ -2x^2+6x \\ \underline{-2x^2+8x} \\ -2x+8 \\ \underline{-2x+8} \\ 0 \end{array}$$

$\therefore\ P(x) = (x-4)(x^2-2x-2)$

$= (x-4)(x-1-\sqrt3)(x-1+\sqrt3)$

$x = \frac{2\pm\sqrt{4-4(1)(-2)}}{2}$

$= \frac{2\pm\sqrt{12}}{2}$

$= \frac{2\pm2\sqrt3}{2}$

$= 1\pm\sqrt3$

$\therefore$ roots are $4, (1+\sqrt3), (1-\sqrt3)$

3

Q3 (a) (i) $\displaystyle\int_1^5 \frac{dx}{(x+2)(x+3)}$

$\Rightarrow \dfrac{1}{(x+2)(x+3)} = \dfrac{A}{x+2} + \dfrac{B}{x+3}$

$1 = A(x+3) + B(x+2)$

Let $x=-3$: $1 = -B$, $\underline{B=-1}$

Let $x=-2$: $\underline{A=1}$

$= \displaystyle\int_1^5 \frac{1}{x+2} - \frac{1}{x+3}\,dx$

$= \left[\ln\left|\dfrac{x+2}{x+3}\right|\right]_1^5$

$= \ln\left(\frac{5}{8}\right) - \left(\ln\left(\frac{3}{4}\right)\right)$

$= \ln\left(\dfrac{5}{\cancel{8}_2}\times\dfrac{\cancel{4}}{3}\right)$

$= \underline{\ln\frac{5}{6}}$

(ii) $\displaystyle\int \frac{1+\frac{2t}{1+t^2}}{1+\frac{1-t^2}{1+t^2}}\cdot\frac{2}{1+t^2}\,dt$

Using $t = \tan\frac{x}{2}$

$\dfrac{dt}{dx} = \frac{1}{2}\sec^2\frac{x}{2}$

$\therefore dt = \frac{1}{2}(1+t^2)\,dx$

$\dfrac{2}{1+t^2}\,dt = dx$

$= \displaystyle\int \frac{1+t^2+2t}{1+t^2+1-t^2}\cdot\frac{2}{1+t^2}\,dt$

$= \displaystyle\int \frac{(t+1)^2}{\cancel{2}}\cdot\frac{\cancel{2}}{1+t^2}\,dt$

$= \displaystyle\int \frac{t^2+2t+1}{t^2+1}\,dt$

$$\begin{array}{r|l} & 1 \\ t^2+1 & t^2+2t+1 \\ & t^2 \quad\;\; +1 \\ \hline & 2t \end{array}$$

$= \displaystyle\int 1 + \frac{2t}{t^2+1}\,dt$

$= t + \ln(t^2+1) + c$

$= \tan\frac{x}{2} + \ln\left[\left(\tan^2\frac{x}{2}+1\right)\right] + c$

$= \underline{\tan\frac{x}{2} + \ln\left[\sec^2\frac{x}{2}\right] + c}$

(3) (a) (iii) $\displaystyle\int_{\frac{1}{\sqrt2}}^{\frac{\sqrt3}{2}} \frac{dx}{x^2\sqrt{1-x^2}}$

Let $x = \sin\theta$ When $x = \frac{\sqrt3}{2}$, $\theta = \frac{\pi}{3}$

$\dfrac{dx}{d\theta} = \cos\theta$ $x = \frac{1}{\sqrt2}$, $\theta = \frac{\pi}{4}$

$= \displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\cancel{\cos\theta}\,d\theta}{\sin^2\theta\cdot\cancel{\cos\theta}}$

$= \displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \operatorname{cosec}^2\theta\,d\theta$

Note: $\dfrac{d}{dx}(\cot\theta) = -\operatorname{cosec}^2\theta$

$= \left[-\cot\theta\right]_{\frac{\pi}{4}}^{\pi/3}$ $= -\left[\dfrac{1}{\sqrt3} - \dfrac{1}{1}\right]$

$= \underline{\left[1 - \dfrac{1}{\sqrt3}\right]}$

3

(iv) $\int x^2\cos x\,dx$

let $u = x^2$ $u' = 2x$

$v' = \cos x$ $v = \sin x$

$\therefore \int x^2\cos x\,dx = x^2\sin x - \int 2x\sin x\,dx$

let $u = 2x$ $u' = 2$

$v' = \sin x$ $v = -\cos x$

$\therefore \int 2x\sin x\,dx = -2x\cancel{\sin x}\cos x - \int -2\cos x\,dx$

$= -2x\cancel{\sin x}\cos x + \int 2\cos x\,dx$

$= -2x\cancel{\sin x}\cos x + 2\sin x$

$\therefore \int x^2\cos x\,dx = \cancel{x^2\sin x - [2x\sin x + 2\sin x]}$

$\cancel{= x^2\sin x - 2x\sin x - 2\sin x}$

$\cancel{= \sin x\,(x^2 - 2x - 2)}$

$\int x^2\cos x\,dx = x^2\sin x - \left[-2x\cos x + 2\sin x\right]$

$= x^2\sin x + 2x\cos x - 2\sin x$

(b) (i) let $u=a-x \iff x=a-u$ $\quad \frac{du}{dx}=-1$

when $x=a$ $\quad u=0$

$x=0$ $\quad u=a$ $\quad dx=-du$

$\therefore \int_a^0 f(a-u)\,-du$

$= -\int_a^0 (a-u)\,du$

$= \int_0^a (a-u)\,du$

let variable u be substituted by variable x

$\therefore \int_0^a (a-x)\,dx$

$\therefore \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

(ii) $\int_0^{\frac{\pi}{4}} \sqrt{1-\sin 2x}\,dx$

$= \int_0^{\frac{\pi}{4}} \sqrt{1-\sin(2(a-x))}\,dx$

$= \int_0^{\frac{\pi}{4}} \sqrt{1-\sin(2(\frac{\pi}{4}-x))}\,dx$

$= \int_0^{\frac{\pi}{4}} \sqrt{1-\sin\left[\frac{\pi}{2}-2x\right]}\,dx$

$= \int_0^{\frac{\pi}{4}} \sqrt{1-\cos 2x}\,dx$

$= \int_0^{\frac{\pi}{4}} \sqrt{1-(2\cos^2 x-1)}\,dx$

$= \int_0^{\frac{\pi}{4}} \sqrt{-2\cos^2 x}\,dx$

$= \sqrt{2}\int_0^{\frac{\pi}{4}} \cos x\,dx$

$= \sqrt{2}\left[\sin x\right]_0^{\frac{\pi}{4}}$

$= \sqrt{2}\left[\frac{1}{\sqrt{2}}-0\right]$

$= 1-0$

$= 1$

$\int_0^{\frac{\pi}{4}} \sqrt{2\sin^2 x}\,dx$

$= \int_0^{\frac{\pi}{4}} \sqrt{2}\sin x\,dx$

$= \sqrt{2}\left[-\cos x\right]_0^{\frac{\pi}{4}}$

$= \sqrt{2}\left[-\frac{1}{\sqrt{2}}+1\right]$

$= -1+\sqrt{2}$

$= \sqrt{2}-1$

Sydney Boys' High School

Student No.:

Paper:

Section: 3

Sheet No.: 2 of 2 for this Section.

Q.No	Tick	Mark
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

(c) (i) $y = x^n \sin x$

~~let $u=x^n$ $u'=n(x)^{n-1}$ $v'=\sin x$ $v=-\cos x$~~

$\frac{dy}{dx} = x^n.\cos x + nx^{n-1}.\sin x$

$= x^n\cos x + nx^{n-1}.\sin x$

(ii) $\int x^n \cos x\,dx$

let $u=x^n$ $\quad u'=nx^{n-1}$

$v'=\cos x$ $\quad v=\sin x$

$\int x^n\cos x = x^n\sin x - \int nx^{n-1}\sin x\,dx$

$= x^n\sin x - n\int x^{n-1}\sin x$ (QED)

(iii) $\int_0^{\pi} x^n \cos x \, dx = \left[x^n \sin x\right]_0^{\pi} - n\int_0^{\pi} x^{n-1} \sin x \, dx$

$= (0-0) - n\int_0^{\pi} x^{n-1} \sin x \, dx$

$= -n\int_0^{\pi} x^{n-1} \sin x \, dx$ ✓

(iv) $\int_0^{\pi} x^n \cos x \, dx$; where $n=1$

$= -1\int_0^{\pi} x^{1-1} \sin x \, dx$

$= -1\int_0^{\pi} \sin x \, dx$

$= -1 \left[-\cos x\right]_0^{\pi}$

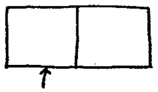
$= -(1-1)$

$= 0$ ✓

7

(d)(i)

Box1 Box2



without restriction

${}^{16}C_0 + {}^{16}C_1 + \dots + {}^{16}C_{16}$

$= \underline{2^{16}}$

Note: $(1+x)^{16}$

$= {}^{16}C_0 x^0 + {}^{16}C_1 x^1 + \dots + {}^{16}C_{16} x^{16}$

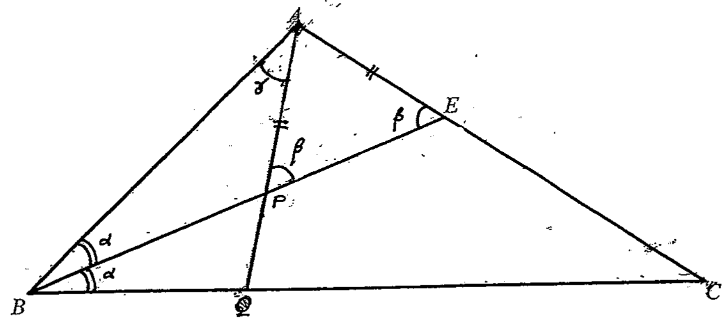
Let $x=1$

(ii) With no box empty

${}^{16}C_1 + {}^{16}C_2 + \dots + {}^{16}C_{15}$

$= \underline{2^{16} - 2}$

(3) (e)



Let $\angle ABE = \angle EBC = \alpha$ (since BE bisects $\angle ABC$)

and $\angle APE = \angle AEP = \beta$ (since $PA = AE$)

$\therefore \angle QAC = 180^\circ - 2\beta$ ($\angle$ sum of $\triangle APE$)

$\angle AQC = 2\alpha + \gamma$ (Ext $\angle$ of $\triangle ABQ$)

$\therefore \angle ACQ = 180 - \angle QAC - \angle AQC$

$= 180 - (180 - 2\beta) - (2\alpha + \gamma)$

$= \cancel{180} - \cancel{180} + 2\beta - 2\alpha - \gamma$

(but $\beta = \alpha + \gamma$ (Ext $\angle$ of $\triangle ABP$)

$= 2(\alpha + \gamma) - 2\alpha - \gamma$

$= 2\alpha + 2\gamma - 2\alpha - \gamma$

$= \gamma = \angle BAQ$ (Angle in the alt. segment)

$\therefore$ BA is the tangent to the circle passing through A, Q, C.

(3) (f) (i) Solve simultaneously: $y = x^4 - 4x^3 - 18x^2$ and

$y = mx + b$

$\therefore$ Eqn. $x^4 - 4x^3 - 18x^2 - mx - b = 0$ has roots

p, p, q, q

(α) Using $\sum \alpha = -\frac{b}{a}$

$2p + 2q = 4$

$\therefore \underline{p + q = 2}$

(β) Using $\alpha\beta\gamma\delta = \frac{e}{a}$

$\underline{p^2 q^2 = -b}$

(ii) For $y = mx + b$ to be a tangent, at two distinct points, ~~wherever~~
solve for m, b.

$\therefore \sum \alpha\beta = p^2 + pq + pq + pq + pq + q^2$

$\frac{c}{a} = 4pq + p^2 + q^2$

$\therefore -18 = 4pq + (p+q)^2 - 2pq$

$= 2pq + 4$

$\therefore 2pq = -22$

$pq = -11$

$\therefore p^2 q^2 = 121$

$\underline{b = -121}$

$\sum \alpha\beta\gamma = -\frac{d}{a} = m$

$p^2 q + pq^2 + q^2 p + qp^2 = m$

$\therefore 2p^2 q + 2q^2 p = m$

$\therefore 2pq(p+q) = m$

$4pq = m$

$pq = \frac{m}{4}$

$-44 = m.$

$\therefore \underline{y = -44x - 121}$ is the common tangent.