SYDNEY BOYS HIGH SCHOOL

PRELIMINARY MATHEMATICS

Half Yearly Examination 2001

Time Allowed: 1 hour (plus 5 minutes reading time)

Total Marks: 60

Examiner: Mr P Bigelow

INSTRUCTIONS:

- Attempt *all* questions.
- *All* questions are of equal value.
- All necessary working should be shown in every question. Full marks may not be awarded if work is careless or badly arranged.
- Approved calculators may be used.
- Return your answers in 5 booklets, 1 for each question. Each booklet must show your name.
- If required, additional Writing Booklets may be obtained from the Examination Supervisor upon request.

Question One [13 marks]

(a) Express $0 \cdot 00639$ in scientific notation.

(b) Use a calculator to evaluate $\dfrac{40 \cdot 7}{9 \cdot 6 \times 5 \cdot 3}$ correct to 2 decimal places.

(c) Simplify (i) $2x^2 + 3x^2$

(ii) $\dfrac{4a+8}{12}$

(d) Simplify (i) $\sqrt{7} \times 3\sqrt{7}$

(ii) $\sqrt{90} - \sqrt{40}$

(e) If $F = \dfrac{9}{5}C + 32$ Find (i) F when $C = 20$

(ii) C when $F = 113$

(f) Factorise (i) $a^2 - 64$

(ii) $y^2 - 5y - 14$

(g) Expand then simplify $(4c-1)(2c-3)$

(h) Write down the value of a if $x^2 - 22x + a$ is a perfect square.

Question Two [12 marks]

(a) How many zeros are significant in $0 \cdot 0407$?

(b) Evaluate $3 - |4 - 5| - 2$

(c) Write down the exact values of

(i) $\sin 60^\circ$

(ii) $\tan \frac{5\pi}{3}$

(d) Write $0 \cdot 0\dot{4}\dot{1}$ as a fraction in simplest form.

(e) Given the set $\left\{\frac{\pi}{6}, 0 \cdot \dot{8}, \sin 30^\circ, \frac{1}{\sqrt{2}}, 37\%\right\}$

(i) Which numbers are rational?

(ii) Write the numbers in ascending order of magnitude.

(f) Express $\frac{4}{3\sqrt{6}}$ with a rational denominator in simplest form.

(g) Solve (i) $14 - 3x < x - 2$.

(ii) $2x(x+4) \geq 0$

Question Three [11 marks]

(a) Expand then simplify

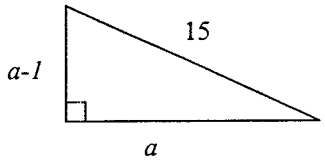
(i) $(\sqrt{3}+1)^2$

(ii) $(2\sqrt{5}-\sqrt{3})(2\sqrt{5}+\sqrt{3})$

(b) Factorise (i) $2x-6y-xy+3y^2$

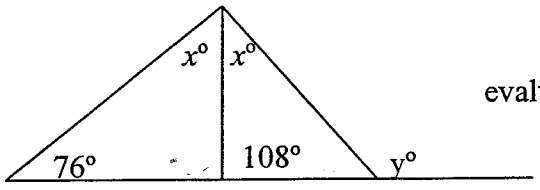
(ii) $6a^2-7a-10$

(c)



find a by forming
then solving an equation. (Express your answer correct to 1 decimal place)

(d)



evaluate x and y

(e) Evaluate $a+b$ if $\frac{a}{b}=-1$

(f) If $\frac{1}{x+1}=\frac{2}{3}$ write down the value

of $\frac{1}{x-1}$

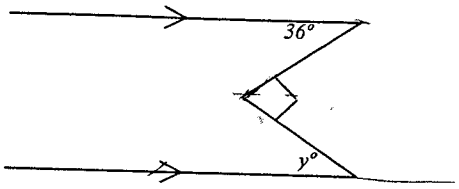
Question Four [12 marks]

(a) Solve (i) $|1-a|=4$

(ii) $a^2 = 2a$

(b) How many sides has a regular polygon if each interior angle is 168°?

(c)



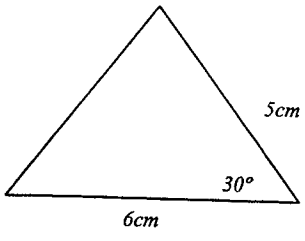
Find y

(d) Simplify $\dfrac{\tan\theta}{\sec\theta}$

(e) Solve the simultaneous equations

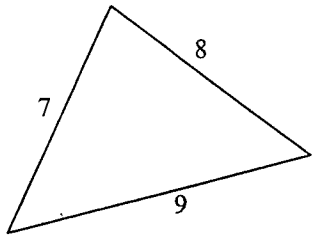
$3x+2y-10=0$
$4x+3y-13=0$

(f) Find the area of the triangle



Question Five [12 marks]

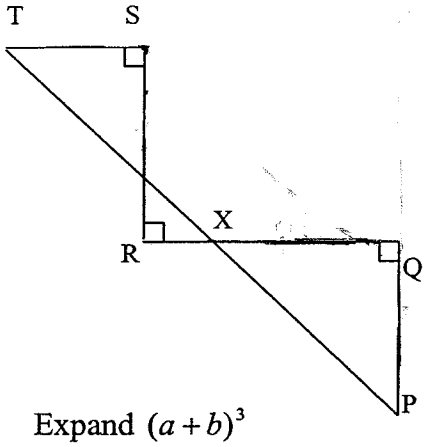
(a) Find the size of the smallest angle in the triangle (to the nearest degree)



(b) Express as a single fraction in simplest form

$$\frac{5x}{x^2-16}-\frac{4}{x+4}$$

(c)



In the diagram
$PQ = QR = RS = 8$
and $ST = 2$

Find the length of QX

(d) (i) Expand $(a+b)^3$

(ii) Given that $x+\frac{1}{x}=-1$ evaluate $x^3+\frac{1}{x^3}$

SBHS Yearly — PREL. 2001

Q1

a) 6.39×10^{-3}

f) i) $a^2 - 64 = (a+8)(a-8)$

ii) $y^2 - 5y - 14$

$(y-7)(y+2)$ ✓

$y = 7$

g) $(4c-1)(2c-3)$

$8c^2 - 14c + 3$

$2(4c^2 - 7c + 2)$

h) $x^2 - 22x + a$

If it is a perfect square

$(-11)^2 = a$

$a = 121$ ✓

2. a) 1

b) $3 - |4-5| - 2$

$= 0$ ✓

c) i) $\frac{\sqrt{3}}{2}$ ✓

ii) $\frac{22}{7}$

$= \frac{110}{7} \times \frac{3}{1} = \frac{330}{7}$

$= \tan \frac{330}{7} = 88°47'$ (n.m.

d) Let $0.4\dot{1}$ be x

$100x = 41.4\dot{1}$

$99x = 41$

$x = \frac{41}{99}$ ✓

e) i) 37%, $0.\dot{3}$ ✓ $\sin 30° = \frac{1}{2}$

ii) ~~+0.55+~~ $0.3333\ldots$, $.5$, $.7071106\ldots$, $.37$

0.5235

$= .\dot{3}, 37\%, \sin 30°, \frac{\pi}{6}, \frac{1}{\sqrt{2}}$ ✓

f) $\frac{\sqrt{6}}{\sqrt{6}} \times \frac{4}{3\sqrt{6}} \times \frac{-3\sqrt{6}}{-3\sqrt{6}} = \frac{-12\sqrt{24}}{-18}$

$\frac{4}{3\sqrt{6}+1} \times \frac{3\sqrt{6}-1}{3\sqrt{6}-1}$

$= \frac{2 \times \sqrt{6}}{3 \times 3}$ $= \frac{12\sqrt{6}}{18}$

$= \frac{2\sqrt{6}}{9}$ $= \frac{2\sqrt{6}}{3}$

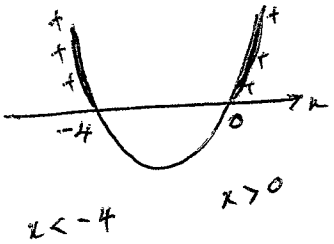
g) i) $14 - 3x < x - 2$

~~ii) $2x(x+4) \geq 0$~~

$16 < 4x$

$4 < x$ $\quad x > 4$ ✓

ii) $2x(x+4) > 0 \Rightarrow$



$x < -4$ $\quad x > 0$

$2x^2 + 8 > 0$

$x^2 + 4 > 0$

$x^2 > -4$

not real

Q3

a) i) $(\sqrt{3}+1)^2 = 3 + 2\sqrt{3} + 1$

$= 4 + 2\sqrt{3}$ ✓

ii) $(2\sqrt{5} - \sqrt{3})(2\sqrt{5} + \sqrt{3}) = 20 - 3$

$= 17$ ✓

b) i) $x(2-y) + 3y(-2+y)$

$= (x - 3y)(2-y)$ ✓

ii) $\frac{(6a-12)(6a+5)}{6}$

$(a-2)(6a+5)$ ✓

c) $2a^2 - 2a + 1 = 225$ ✓

$2a^2 - 2a - 224 = 0$

$a^2 - a - 112 = 0$

$a = \frac{1 \pm \sqrt{449}}{2} = \frac{1 \pm 16\sqrt{7}}{2} = 21.7$ or $\underbrace{-20.7}_{\text{not valid}}$

d) $x + 76 = 108°$ (exterior opp. ∠'s = ext. ∠)

$x = 32°$ ✓

$y = 2x + 76°$ ✓ (sum of int. ∠'s = ext ∠)

$y = 140°$.

e) if $a + b = -1$, $a = -b$ or $-a = b$ and ~~$a = b$~~

$a + -a = 0$ ⇒ implies $a + b = 0$

$a + b = 0$ ✓

f) $\frac{1}{x+1} = \frac{2}{3}$ → $3 = 2x + 2$

$\therefore 2x = 1$

$x = \frac{1}{2}$

$\therefore \frac{1}{x-1} = \frac{1}{\frac{1}{2}-1}$

$= \frac{1}{-\frac{1}{2}}$

$\frac{2}{3x+3}$ $\therefore 3x + 3 = 3$

$x = 0$

$\frac{1}{-1} = -1 = \frac{-2}{}$.

Q4

a) i) $|1-a| = 4$

$a = \mp 3$ or 5 ✓

ii) $a^2 = 2a$

$a(2+a) = 0$

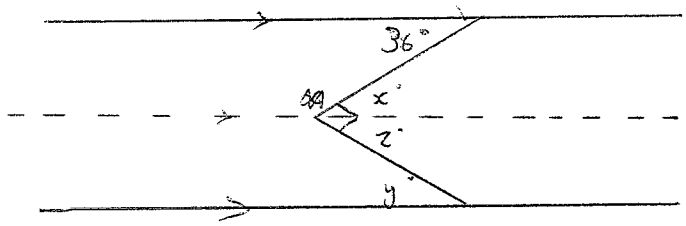
$a = 0$ ✓ or $a = -2$ ✓

b) ext. $\angle = 12°$

$12n = 360$

$\therefore n = 30$ ✓ $\therefore$ there are 30 sides

c)



$x° = z°$ (adj. $\angle$'s = in bisected angle) Not given.

$x = 36°$ (alt $\angle$'s in $\parallel$ lines) but $x + y = 90°$ (given)

$\therefore y = 36°$ (alt $\angle$'s in $\parallel$ lines) $36 + y = 90°$

$\therefore y = 54°$

d) $\dfrac{\tan\theta}{\frac{1}{\cos\theta}} = \tan\cos\theta$

$= \dfrac{\sin}{\cos}\cdot\cos\theta$

$= \sin\theta$ ✓

e) $3x + 2y - 10 = 0$

$4x + 3y - 13 = 0$

$4x + \frac{3}{2}(10 - 3x) - 13 = 0$

$2 - \frac{1}{2}x = 0$ ✓

$x = 4$

$y = -1$ ✓

f) $\frac{ab}{2}\sin C = 15\sin 30°$

$= 7.5\,cm^2$ ✓

Q5

a) $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

$\cos A = \dfrac{64 + 81 - 49}{144}$ ✓

$= \dfrac{2}{3}$

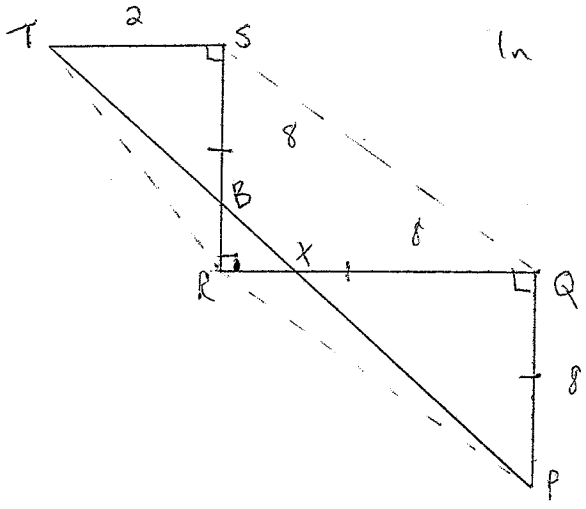
$A^\circ = 48^\circ 11'$ ✓

b) $\dfrac{5x}{(x+4)(x-4)} - \dfrac{4}{x+4} = \dfrac{5x(x-4) - 4}{(x+4)}$

$= \dfrac{5x - 4(x-4)}{(x+4)(x-4)}$ ✓

$= \dfrac{x+16}{(x+4)(x-4)}$ ✓

c)



In $\triangle$s TBS and XBR,

$\angle R = \angle S$ (given)

$\angle RBX = \angle SBT$ (v.o. $\angle$'s =)

$\therefore \triangle TBS \,|||\, \triangle XBR$ (equiangular). ✓

In $\triangle$s XBR and XPQ,

$\angle BRX = \angle PQX$ (given)

$\angle BXR = \angle PXQ$ (v.o. $\angle$s =) ✓

$\therefore \triangle XBR \,|||\, \triangle XPQ$ (equiangular)

$\therefore \triangle XPQ \,|||\, \triangle TBS$ ✓

$\dfrac{8-x}{2} = \dfrac{y}{8-y}$

$\dfrac{8-x}{x} = \dfrac{y}{8}$ ✓

$\dfrac{8-x}{2} = \dfrac{64-8x}{x}$

$8 - \left(\dfrac{64-8x}{x}\right)$ ✓

$\frac{64 - 8x}{x}$

$8 - \frac{64 - 8x}{x}$

$\frac{64 - 8x}{x} \times \frac{x}{-64} = \frac{64x - 8x^2}{-64x} = \frac{64 - 8x}{-64} = \frac{8 - x}{8}$ ✓

$\frac{8 - x}{2} = \frac{64 - 8x}{64}$ $\frac{8 \cdot x}{8}$

$8 - x = 128$ $\frac{8 - x}{4}$ /

$32 - 4x = 8 - x$

$24 = 3x$

$x = 8$ ✓

d) i) $(a+b)^3 = (a^2 + b^2 + 2ab)(a+b)$

$= a^3 + b^3 + 2a^2b$ $a^3 + a^2b + ab^2 + b^3 + 2a^2b + 2ab^2$

$= a^3 + 3a^2b + 3ab^2 + b^3$

ii) $x + \frac{1}{x} = -1$

$x^2 + x + 1 = 0$

$\left(x + \frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3}$

$\therefore (-1)^3 = -1$

$\therefore x^3 + \frac{1}{x^3} = -1$

$\left(x + \frac{1}{x}\right)^3 = x^3 + 3x^2\left(\frac{1}{x}\right) + 3x\left(\frac{1}{x^2}\right) + \frac{1}{x^3}$

$(-1)^3 = x^3 + \frac{1}{x^3} + 3x + \frac{3}{x}$

$= x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right)$

$= x^3 + \frac{1}{x^3} + 3(-1)$

$-1 + 3 = x^3 + \frac{1}{x^3}$

$2 = x^3 + \frac{1}{x^3}$