SYDNEY BOYS HIGH
MOORE PARK, SURRY HILLS

2012
Year 11 ACCELERANTS
YEARLY EXAMINATION

Mathematics

General Instructions:

- Reading time – 5 minutes
- Working time – 90 minutes
- Write using black or blue pen Black pen is preferred
- Board approved calculators may be used
- A table of standard integrals is provided on the back of the Multiple Choice answer sheet
- All necessary working should be shown in every question if full marks are to be awarded
- Marks may NOT be awarded for messy or badly arranged work
- Answer in simplest exact form unless otherwise stated

Total marks – 70 Marks
Section I Pages 2–5
10 marks

- Attempt Questions 1–10
- Answer on the Multiple Choice answer sheet provided.
- Allow about 15 minutes for this section

Section II Pages 6–9
60 marks

- Attempt Questions 11–14
- Allow about 75 minutes for this section
- For Questions 11–14, start a new answer booklet per question

Examiner: Mr D. Hespe

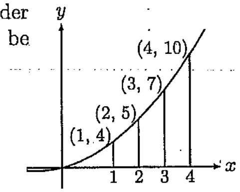
Section I— 10 marks

Select the alternative A, B, C, or D that best answers the question.
Fill in the response oval on your multiple choice answer sheet.

Marks

1. The graph of $x^3 + 2x^2 + x - 2$ has: $\boxed{1}$
   (A) 2 points of inflexion
   (B) 1 turning point and 1 point of inflexion
   (C) 3 turning points
   (D) 2 turning points

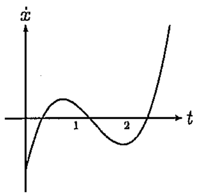
2. A student is using the trapezoidal rule to find the area under the curve at right from $x = 1$ to $x = 4$. His answer should be approximately equal to: $\boxed{1}$
   (A) 26 sq. units
   (B) 22 sq. units
   (C) 19 sq. units
   (D) 16 sq. units



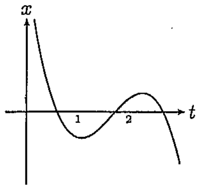
3. $\sin y + \sin(x - y) = \sin x$ for all y provided x is $\boxed{1}$
   (A) $\dfrac{\pi}{3}$
   (B) $\dfrac{\pi}{2}$
   (C) π
   (D) 2π

4. If $y = 5e^{-6x}$ then $\dfrac{dy}{dx}$ is equal to $\boxed{1}$
   (A) $-6e^{-6x}$
   (B) $5e^{-6x-1}$
   (C) $-30e^{-6x}$
   (D) $5e^{-7x}$

5. The graph to the right shows how the velocity of a particle varies with time. Its displacement-time graph is best given by $\boxed{1}$



(A)



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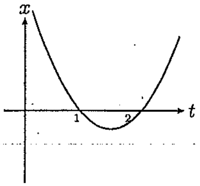
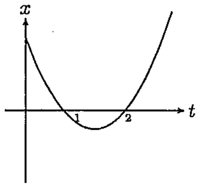
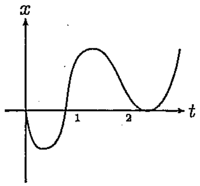


(C)



(D)



6. The area bounded by the graph of $f(x) = e^x - 1$, the x-axis, and the line $x = 2$ is equal to: $\boxed{1}$

(A) $e^2 - 1$

(B) $e^2 - 3$

(C) $e^2 + 1$

(D) $e^2 + 3$

7. The graph of $g(x)$ has the following properties: $\boxed{1}$

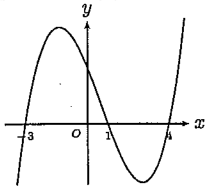
i $g'(x) = 0$ if $x = -3$, 1 and 4

ii $g'(x) < 0$ if $x < -3$ and $1 < x < 4$

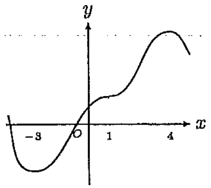
iii $g'(x) > 0$ for all other x

then the graph of $g(x)$ could be:

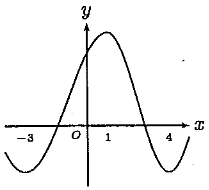
(A)



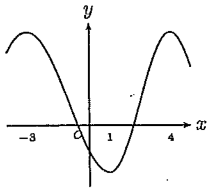
(B)



(C)



(D)



8. The indefinite integral $\int \left(\cos\frac{x}{3} - 3\sin 3x\right)\,dx$ is equal to: $\boxed{1}$

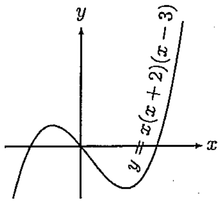
(A) $\frac{1}{3}\sin\frac{x}{3} + 9\cos 3x + c$

(B) $3\sin\frac{x}{3} - \cos 3x + c$

(C) $-3\sin\frac{x}{3} + \cos 3x + c$

(D) $3\sin\frac{x}{3} + \cos 3x + c$

9. The area bounded by the curve on the graph at right and the x-axis is equal to: $\boxed{1}$
   (A) $20\frac{5}{12}$ sq. units
   (B) $21\frac{1}{12}$ sq. units
   (C) $10\frac{5}{12}$ sq. units
   (D) $-10\frac{5}{12}$ sq. units



10. If $\frac{dR}{dt} = kR$ then which of the following is *not* possible? $\boxed{1}$
   (A) $R = ke^{10t}$
   (B) $R = 10ke^{kt}$
   (C) $R = 20e^{kt}$
   (D) $R = -20e^{kt}$

End Multiple Choice Questions

Section II— 60 marks

Marks

Question 11 (15 marks) (use a separate answer booklet)

(a) Find $\frac{dy}{dx}$ if $y = e^{\cos x}$. $\boxed{1}$

(b) Find the indefinite integrals:
   (i) $\int \frac{dx}{3x-2}$ $\boxed{1}$
   (ii) $\int \frac{6}{e^{2x}}\,dx$ $\boxed{1}$

(c) The gradient function of a curve is $7-4x$ and the curve passes through the point (1, 10). Find its equation. $\boxed{2}$

(d) Find the area enclosed by the curve $y = 6x - x^2$ and the x-axis. $\boxed{2}$

(e) A minor segment of a circle has an area of 50 cm^2 and subtends a central angle of $\frac{\pi}{5}$ radians. Find the radius of the circle correct to one decimal place. $\boxed{2}$

(f) A particle moves in a straight line and its displacement from the origin is given by $s = 5 - 6t + t^2$, find:
   (i) the distance of the particle from the origin after 2 seconds, $\boxed{1}$
   (ii) the times when the particle is at the origin, $\boxed{2}$
   (iii) at what instant the velocity is zero, $\boxed{2}$
   (iv) the acceleration of the particle. $\boxed{1}$

Marks

Question 12 (15 marks) (use a separate answer booklet)

(a) Differentiate and simplify:

(i) $2xe^{2x}$, $\boxed{1}$

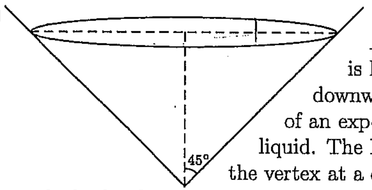
(ii) $\dfrac{\cos x}{1-\sin x}$. $\boxed{2}$

(b) Find correct to four significant figures $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\cos x}{2+\sin x}\,dx$. $\boxed{2}$

(c) Evaluate $\displaystyle\int_0^2 |x^2+2x-3|\,dx$ (HINT: sketch the curve first). $\boxed{2}$

(d) Find a primitive of the function $\tan^2\theta$. $\boxed{2}$

(e) $\boxed{6}$



A hollow cone of semi-vertical angle 45° is held with its axis vertical and vertex downwards (see diagram). At the beginning of an experiment, it is filled with 390 cm^3 of liquid. The liquid runs out through a small hole at the vertex at a constant rate of 2 cm^{3}s^{-1}. Find the rate at which the depth of the liquid is decreasing 3 minutes after the start of the experiment. Give your answer correct to 3 significant figures.

Marks

Question 13 (15 marks) (use a separate answer booklet)

(a) Find where the tangent to x^3+2x+1 at the point where $x=-1$ meets the curve again. $\boxed{4}$

(b) Find the volume of the solid formed when the region enclosed by the graph of $y=\ln x$, the x-axis, the y-axis and the line $y=\ln 3$ is rotated about the y-axis. Give your answer in exact form. $\boxed{4}$

(c) The equation of a curve C is given as $y=\dfrac{x^2}{x+\lambda}$ where λ is a non-zero constant.

(i) Write down the equations of the asymptotes of C. $\boxed{3}$

(ii) Draw, on separate diagrams, a sketch of C for the cases where $\boxed{4}$

(α) $\lambda>0$

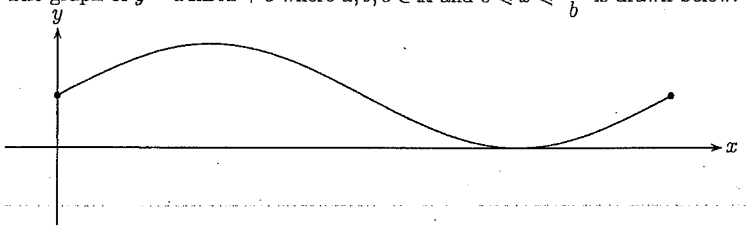
(β) $\lambda<0$.

Marks

Question 14 (15 marks) (use a separate answer booklet)

(a) It is given that $\ln a = x$ and $\ln b = y$. Express $\ln\left(\dfrac{a^2b}{e}\right)$ in terms of x and y. $\boxed{2}$

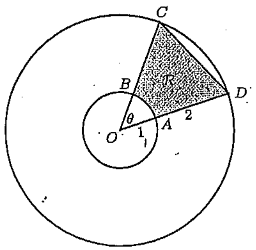
(b) The graph of $y = a\sin bx + c$ where $a, b, c \in \mathbb{R}$ and $0 \leqslant x \leqslant \dfrac{2\pi}{b}$ is drawn below. $\boxed{3}$



Copy the diagram to your answer booklet. On the same set of axes, sketch the graph of $y = -2a\sin\left(\frac{b}{2}x\right)$.

(c) To historically date objects less than 50 000 years old, carbon 14 dating is used. Carbon 14 has a half-life of 5370 years. Before death animals and plants have a reading for carbon 14 of 12.5 counts per minute on a radiation counter. Show that the decay rate for carbon 14 is -1.29×10^{-4}. If a piece of wood from an excavation site has a reading of 7 counts per minute, show that the wood's age is approximately 4500 years. $\boxed{4}$

(d)



The diagram shows two circles, of radii 1 and 3, each with centre O. The angle between the lines OAD and OBC is θ radians. The shaded region $\mathcal{R}$ is bounded by the minor arc AB and the lines BC, CD, and DA.

(i) Find the area of $\mathcal{R}$. $\boxed{2}$

(ii) Find the value of θ for which the area of $\mathcal{R}$ is greatest. $\boxed{2}$

(iii) Find the greatest value of θ which ensures that the whole of the line segment CD lies between the two circles. $\boxed{2}$

End of Paper

Student Number:______________________________

Mathematics

~~Trial HSC 2012~~

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Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9
A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ☒ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word correct and drawing an arrow as follows.

A ☒ B ☒ (correct) C ○ D ○

Section I: **Multiple choice answer sheet.**

Completely colour the cell representing your answer. Use black pen.

1. A ○ B ○ C ○ D ●
2. A ○ B ○ C ● D ○
3. A ○ B ○ C ○ D ●
4. A ○ B ○ C ● D ○
5. A ○ B ○ C ○ D ●
6. A ○ B ● C ○ D ○
7. A ○ B ○ C ● D ○
8. A ○ B ○ C ○ D ●
9. A ○ B ● C ○ D ○
10. A ● B ○ C ○ D ○

Year 11 Accelerated Mathematics Exam Soln:

Question 11:

a)

$$y = e^{\cos x}$$
$$\frac{dy}{dx} = -\sin x\, e^{\cos x}$$

b)

(i)

$$\int \frac{dx}{3x-2}$$
$$= \frac{1}{3}\int \frac{3.dx}{3x-2}$$
$$= \frac{1}{3}\ln(3x-2) + C$$

(ii)

$$\int \frac{6}{e^{2x}}.dx$$
$$= \int 6\,e^{-2x}.dx$$
$$= \frac{6e^{-2x}}{-2} + C$$
$$= -3e^{-2x} + C$$

c) $y' = 7 - 4x$

$y = 7x - 2x^2 + C$

At $(1, 10)$

$10 = 7 - 2 + C$

$\therefore C = 5$

$y = 7x - 2x^2 + 5$

d)

$$A = \int_0^6 6x - x^2 .dx$$
$$= \left[3x^2 - \frac{x^3}{3}\right]_0^6$$
$$A = 36\,u^2$$

e)

$A = \frac{1}{2}r^2(\theta - \sin\theta)$

$50 = \frac{1}{2}r^2\left(\frac{\pi}{5} - \sin\frac{\pi}{5}\right)$

$r^2 = \frac{100}{\frac{\pi}{5} - \sin\frac{\pi}{5}}$

$r = 49.7\ cm$ (nearest 1 d.p.)

f) $s = 5 - 6t + t^2$

(i) When $t = 2$

$s = 5 - 6(2) + 2^2$

$s = -3$

Distance = 3 units

(ii) When $s = 0$

$5 - 6t + t^2 = 0$

$(t-5)(t-1) = 0$

$t = 1, \quad 5$

(iii) $\frac{ds}{dt} = -6 + 2t$

When $\frac{ds}{dt} = 0$

$-6 + 2t = 0$

$2t = 6$

$t = 3$

(iv) $\frac{d^2s}{dt^2} = 2$

QUESTION 12.

(a) (i) $y = 2x e^{2x}$

$y' = 4x.e^{2x} + 2e^{2x}$

$= 2e^{2x}(2x+1)$

(ii) $y = \frac{\cos x}{1 - \sin x}$

$y' = \frac{(1-\sin x)(-\sin x) - \cos x \times -\cos x.}{(1-\sin x)^2}$

$= \frac{-\sin x + \sin^2 x + \cos^2 x}{(1-\sin x)^2}$

$= \frac{1 - \sin x}{(1-\sin x)^2}$

$= \frac{1}{1-\sin x}.$

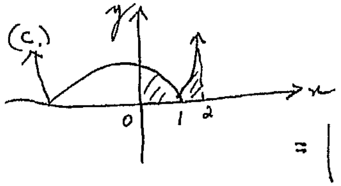
(b). $\int_0^{\frac{\pi}{2}} \frac{\cos x}{2+\sin x}.dx = \left[\ln(2+\sin x)\right]_0^{\frac{\pi}{2}}$

$= \ln(2+1) - \ln(2+0)$

$= \ln 3/2.$

$\doteqdot 0.4055.$

(c.)



$\int_0^2 |x^2+2x-3|\,dx.$

$= \left|\int_0^1 (x^2+2x-3)\,dx\right| + \int_1^2 (x^2+2x-3)\,dx.$

$= \left|\left[\frac{x^3}{3}+x^2-3x\right]_0^1\right| + \left[\frac{x^3}{3}+x^2-3x\right]_1^2$

$= \left|\frac{1}{3}+1-3\right| + \frac{8}{3}+4-6-\left(\frac{1}{3}+1-3\right)$

$= 1\frac{2}{3} + \frac{2}{3} + 1\frac{2}{3}$

$= 4.$

(d) Let $f'(\theta) = \tan^2\theta$

$= \sec^2\theta - 1$

$\therefore f(\theta) = \tan\theta - \theta + c.$

(e).

NB $h = r$.

$\therefore V = \frac{1}{3}\pi r^2 h$

$\Rightarrow V = \frac{1}{3}\pi h^3$

$\therefore \frac{dV}{dh} = \pi h^2$.

[now after 3 minutes there is 30 cm³ remaining. at which time.

$30 = \frac{1}{3}\pi h^3$.

$\therefore \frac{90}{\pi} = h^3$

$\therefore h = \left(\frac{90}{\pi}\right)^{\frac{1}{3}}$ — Ⓐ]

$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$

$= \frac{1}{\pi h^2} \times -2$.

$= \frac{-2}{\pi\left(\frac{90}{\pi}\right)^{2/3}}$

$\doteqdot -0.0680$ cm/s.

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Question 13

(a) $y = x^3 + 2x + 1$

$y' = 3x^2 + 2$

$y'(-1) = 3 + 2$ $\quad y(-1) = -1 - 2 + 1$

$= 5$ $\quad = -2$

∴ Tangent

$y + 2 = 5(x + 1)$

$= 5x + 5$

$y = 5x + 3$

To find intersections, substitute for y:

$x^3 + 2x + 1 = 5x + 3$

$x^3 - 3x - 2 = 0$

Two solutions are $x = -1$

By division

$$\begin{array}{r} x^2 - x - 2 \\ x+1 \overline{) x^3 \quad - 3x - 2} \\ \underline{x^3 + x^2} \\ -x^2 - 3x \\ \underline{-x^2 - x} \\ -2x - 2 \end{array}$$

$x^2 - x - 2 = 0$

$(x - 2)(x + 1) = 0$

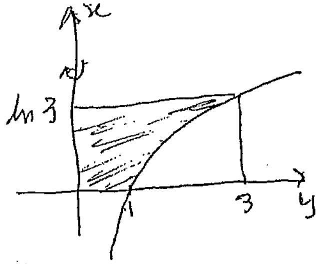
$x = 2, -1$

∴ Next meets curve when

$x = 2, y = 13$

ie $(2, 13)$ [4]

(b)



$y = \ln x$

$x = e^y$

$V = \pi \int_c^d x^2\,dy$

$= \pi \int_0^{\ln 3} e^{2y}\,dy$

$= \pi \left[\frac{1}{2}e^{2y}\right]_0^{\ln 3}$

$= \frac{\pi}{2}\left[(e^{\ln 3})^2 - e^0\right]$

$= \frac{\pi}{2}[9 - 1]$

$= 4\pi$ [4]

(c) $y = \frac{x^2}{x + \lambda}$

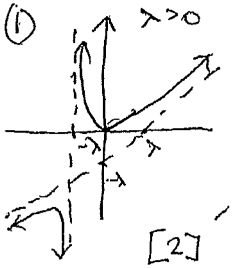
Asymptotes: $\underline{x = -\lambda}$ Vertical [3]

Oblique: $y = \frac{(x^2 - \lambda^2) + \lambda^2}{x + \lambda}$

$= (x - \lambda) + \frac{\lambda^2}{x + \lambda}$

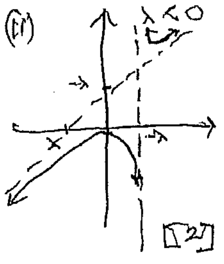
As $x \to \pm\infty$, $\underline{y \to x - \lambda}$

(i)



[2]

(ii)



[2]

Question 14

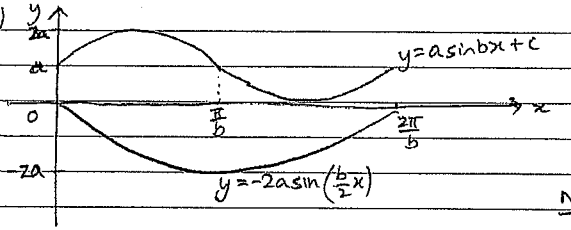
(a) $\ln\left(\frac{a^2b}{e}\right) = \ln(a^2b) - \ln e$

$= \ln a^2 + \ln b - \ln e$

$= 2\ln a + \ln b - \ln e$

$= 2x + y - 1$

(b)



Note: $c=a$

(c) $P = Ae^{kt}$

when $t = 5370$

$P = \frac{A}{2}$

$\frac{A}{2} = Ae^{5370k}$

$\frac{1}{2} = e^{5370k}$

$5370k = \ln\left(\frac{1}{2}\right)$

$k = \frac{1}{5370} \cdot \ln\left(\frac{1}{2}\right)$

$\underline{k \doteqdot -1.29 \times 10^{-4}}$ which is the growth rate.

$P = Ae^{kt}$

when $t=0$

$P = 12.5$

$12.5 = Ae^0$

$A = 12.5$

$P = 12.5e^{kt}$

when $P=7$

$7 = 12.5e^{kt}$

$e^{kt} = 0.56$

$kt = \ln(0.56)$

$t = \frac{\ln(0.56)}{k}$, where $k = -1.29 \times 10^{-4}$

$t \doteqdot 4500$ years

d) i) $A = \frac{1}{2}(3)(3)\sin\theta - \frac{1}{2}(1)^2\theta$

$= \frac{9}{2}\sin\theta - \frac{\theta}{2}$

ii) $\frac{dA}{d\theta} = \frac{9}{2}\cos\theta - \frac{1}{2}$

let $\frac{dA}{d\theta} = 0$ for stat. points

$\frac{9}{2}\cos\theta - \frac{1}{2} = 0$

$\frac{9}{2}\cos\theta = \frac{1}{2}$

$\cos\theta = \frac{1}{9}$

$\theta = \cos^{-1}\left(\frac{1}{9}\right)$

≈ 1.46

$\frac{d^2A}{d\theta^2} = -\frac{9}{2}\sin\theta$

when $\theta = 1.46$

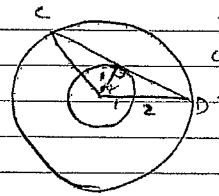
$\frac{d^2A}{d\theta^2} = -4.47\ldots$

< 0 ∩

$\therefore$ Max. Area when $\theta = \cos^{-1}\left(\frac{1}{9}\right)$

$\approx 1.46.$

iii)



The greatest value of θ will involve considering when CD is a tangent to smaller circle.

$\cos\alpha = \frac{1}{3}$

$\alpha = \cos^{-1}\left(\frac{1}{3}\right)$

$2\alpha = 2\cos^{-1}\left(\frac{1}{3}\right)$

≈ 2.46

For CD to lie between the two circles

$0 < \theta < \underline{2.46}$