SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2012

HSC ASSESSMENT TASK #2

Mathematics Extension 2

General Instructions

- Reading time – 5 minutes.
- Working time – 120 minutes.
- Write using black or blue pen.
- Board approved calculators may be used.
- All necessary working should be shown in every question if full marks are to be awarded.
- Marks may **NOT** be awarded for messy or badly arranged work.
- Answers should be in simplest exact form unless specified otherwise.
- Start each **NEW** section in a separate answer booklet.
- Each section is to be returned in a separate bundle.

Total Marks - 89

- Attempt Questions 1 - 6
- All questions are NOT of equal value.

Examiner: *A. Fuller*

Section A

Question 1 (16 marks)

(a) $\int \frac{\cos x}{1+\sin x}\,dx$ **1**

(b) $\int \frac{\cos^2 x}{1+\sin x}\,dx$ **2**

(c) Given $a = 3 - 4i$ and $b = 1 + i$. **5**

Express the following in the form $x + iy$ where x and y are real numbers:

(i) $b - a$

(ii) $\overline{ab}$

(iii) $\frac{a}{b}$

(d) $P(x) = x^4 - x^3 - 2x^2 + 6x - 4$. **3**

(i) Given that $1 + i$ is a zero of $P(x)$, explain why $1 - i$ is also a zero of $P(x)$.

(ii) Hence, find all the zeros of $P(x)$.

(e) When $(1 + ax)^5 + (1 + bx)^5$ is expanded in ascending powers of x, the expansion begins $2 + 40x + 260x^2 + \ldots$ **5**

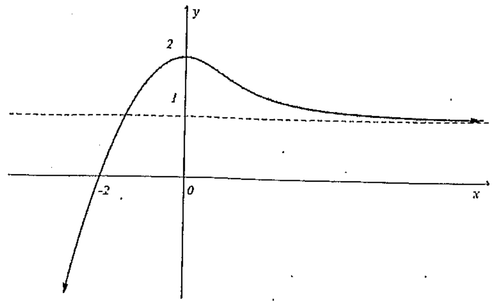
(i) Show that $a + b = 8$ and $a^2 + b^2 = 26$.

(ii) Deduce the value of ab.

(iii) Find the coefficient of x^3.

Question 2 (16 marks)

(a) Below is a sketch of $y = f(x)$. **6**



Sketch the following on separate diagrams:

(i) $y = f(|x|)$

(ii) $|y| = f(x)$

(iii) $y = [f(x)]^{-1}$

(iv) $y = 2^{f(x)}$.

(b) Plot a point A which represents the complex number z on an argand diagram given that $Re(z) < 0$, $Im(z) > 0$ and $|z| > 1$. **4**

On the same argand diagram plot the point:

(i) B representing the complex number $\bar{z}$

(ii) C representing the complex number $\frac{1}{z}$

(iii) Drepresenting the complex number $z(\cos \pi + i \sin \pi)$.

(c) $\int \ln(1 + x^2)dx$. **3**

(d) Evaluate $\int_{-1}^{1} (1 + x^3)^3 dx$ **3**

Section B **(Use a SEPARATE writing booklet)**

Question 3 (14 marks)

(a) Sketch $y^2 = (x - 2)^2(x - 1)$ without using calculus. **2**

(b) If $\arg(z + 1) = \frac{\pi}{6}$ and $\arg(z - 1) = \frac{2\pi}{3}$. **3**

Write z in Cartesian form $(x + iy)$.

(c) The equation $x^3 - 3x^2 + ax + 8 = 0$ has roots that are in arithmetic sequence. **3**

(i) Show that one of the roots is 1.

(ii) Find the value of a and solve the equation.

(d) A particle of mass 10 kg is projected vertically upwards with a velocity of $u\ m/s$. The resistive force is one-tenth of the square of its velocity. **6**

Assuming that $g = 10\ m/s^2$.

(i) Show that the particle takes $\sqrt{10}\tan^{-1}\frac{u}{10\sqrt{10}}$ seconds to reach its greatest height.

(ii) Show that the greatest height is $50\log_e\frac{1000+u^2}{1000}$ metres.

Question 4 (16 marks)

(a) (i) Show that $\int_{-a}^{a} f(x)dx = \int_0^a (f(x) + f(-x))dx$. **4**

(ii) Hence, or otherwise, evaluate

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{dx}{1+\sin x}$$

(b) The complex number $z = x + iy$, where x and y are real, is such that **5**

$|z - i| = Im(z)$.

(i) Show that the locus of z is a parabola.

(ii) Hence, find the range of possible values for $\arg(z)$.

(c) (i) Find the values of A, B and C if **7**

$\frac{-1}{x^2(x+1)} = \frac{A}{x^2} + \frac{B}{x} + \frac{C}{x+1}$.

(ii) A particle of unit mass moves on the x-axis against a resistance numerically equal to $v^2 + v^3$, where v is its velocity. Initially the particle is travelling with velocity u, where $u > 0$.

It can be proven that when the velocity is $\frac{u}{2}$ the distance X travelled by the particle is given by $X = \ln\left(\frac{2+u}{1+u}\right)$ (Do not prove this)

(α) Prove that if T is the time taken to travel the distance X then

$u(T + X) = 1$.

(β) It is alleged that if the particle started at the origin then the velocity v, displacement x, and time t are related by the equation $v = \frac{u}{ux+ut+1}$.

By finding a suitable derivative, show that this is in fact correct.

Section C **(Use a SEPARATE writing booklet)**

Question 5 (15 marks)

(a) (i) Write $(1 + i)^n$ in modulus argument form. **6**

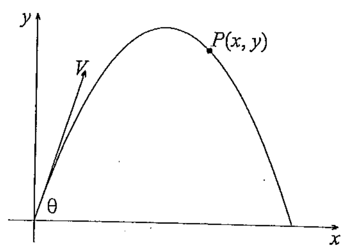
(ii) By considering the binomial expansion of $(1 + i)^n$.

Find an expression for $1 - \binom{n}{2} + \binom{n}{4} - \binom{n}{6} + \ldots$

(iii) If n is a multiple of 8.

Show that $\binom{n}{1} + \binom{n}{5} + \binom{n}{9} + \cdots = \binom{n}{3} + \binom{n}{7} + \binom{n}{11} + \cdots$

(b) **9**



A particle is projected at an angle of θ to the horizontal with velocity V. (Assume that there is no air resistance) and take gravity to be g. At time t, let x and y be the horizontal and vertical displacements respectively.

(i) Derive the equations of motion in the horizontal and vertical directions in terms of t.

(ii) It is known that at some time t during its flight, the x and y displacements of the particle are equal and the direction of motion is inclined at 45° to the downward vertical. The position of the particle at this time is marked P in the diagram above. Use this information to show that $\tan\theta = 3$.

(iii) Hence, find the range of the particle in terms of V and g.

(iv) If the speed of projection may be varied but the particle must not rise more than H above the ground. Find the maximum range in terms of H.

Question 6 (12 marks)

(a) Show that the polynomial $P(x) = x^n - x^{n-1} - 1$, where $n > 1$ cannot have a repeated real root. **3**

(b) "Words" are to be formed from the letters of the word **9**

F U N D A M E N T A L

(i) Using all of the eleven letters. How many different "words" are possible if:

(α) there is no restriction

(β) it must start and end with the same letter

(γ) **F U N** must appear together in that order

(δ) the same letter must not appear next to itself?

(ii) If I am to select five letters to form a "word". How many different five letter "words" are possible?

End of Examination

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Student No.: ____________________

Q.No	Tick	Mark
1	✓	16
2	✓	14
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Paper: EXT 2 - SBHS - 2012

Section: A (part 1)

Sheet No.: ______ of ______ for this Section.

(Q1)

(a) $\int \frac{\cos x}{1+\sin x}\,dx = \ln[1+\sin x] + C$ ✓ 1

(b) $\int \frac{\cos^2 x}{1+\sin x}\,dx = \int \frac{1-\sin^2 x}{1+\sin x}\,dx = \int 1-\sin x \; dx$ ✓

$= x + \cos x + C$ ✓ 2

(c) $a = 3-4i \quad b = 1+i$

(i) $b - a = 1+i-3+4i = -2+5i$ ✓

(ii) $ab = (3-4i)(1+i) = 3-i+4 = 7-i$ ✓

$\therefore \overline{ab} = 7+i$ ✓

(iii) $\frac{a}{b} = \frac{3-4i}{1+i} = \frac{(3-4i)(1-i)}{2} = \frac{3-4-7i}{2} = -\frac{1}{2} - \frac{7}{2}i$ ✓✓

5

(d) Conjugate root theorem: if a complex number is a root of a polynomial with real co-efficients, then the conjugate of the complex number is also a root. ✓

ie since $1+i$ is a zero, $1-i$ is also a zero

(ii) $P(x) = x^4 - x^3 - 2x^2 + 6x - 4 = 0$

$P(1) = 1-1-2+6-4 = 0$

$\therefore x-1$ is a root.

$\therefore P(x) = (x-1+i)(x-1-i)(x-1)(x+\alpha)$

where α is the other zero ✓

$P(x) = (x^2-2x+2)(x-1)(x+\alpha)$

by inspection, $\alpha = 2$.

$\therefore P(x) = (x^2-2x+2)(x-1)(x+2)$ ✓

$\boxed{\therefore \text{zeroes are } 1+i, 1-i, 1, -2}$

3

(e) $(1+ax)^5 + (1+bx)^5 \qquad {}^nC_r (1)^{n-r}(ax)^r$

$= 2 + 40x + 260x^2 + \dots = 2 + 5ax + 5bx + 10a^2 + 10b^2 \dots$

(i) equating co-eff;

$40 = 5(a+b)$ | $260 = 10(a^2+b^2)$

$\boxed{8 = a+b}$ ✓ | $\boxed{26 = a^2+b^2}$ ✓

(ii) ~~$a^2+b^2 \neq (a+b)^2 + 2ab$~~ $(a+b)^2 = a^2+b^2+2ab$

From (i),

~~$26 \neq (8)^2$~~ $(8)^2 = 26 + 2ab$

~~$2ab = 38$~~ $64 - 26 = 2ab$

~~$\boxed{ab = 19}$~~ $ab = 19$

(iii) ~~$ab = 19$ ① $a+b = 8 \Rightarrow a = 8-b$ ②~~

~~② in ①~~

~~$(8-b)b = 19$~~

~~$b^2 - 8b + 19 = 0$~~

$(1+ax)^5 = 1 + 5ax + 10a^2x^2 + 10a^3x^3 \ldots$

$(1+bx)^5 = 1 + 5bx + 10a^2x^2 + 10a^3x^3 \ldots$

$\therefore$ coeff of $x^3 = 10(a^3+b^3)$

$= 10(a+b)(a^2 - ab + b^2)$

$= 10(8)(26-19)$ (from (i) and (ii))

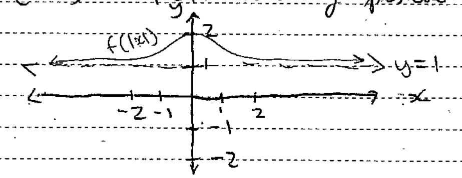
$= 560$

5

Question 2

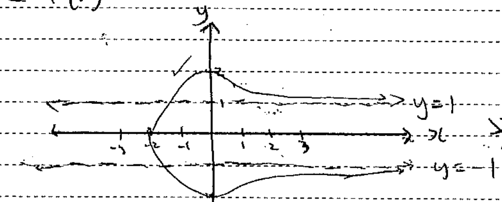
(a)

(i) $y = f(|x|)$ reflection of positive half of $y = f(x)$



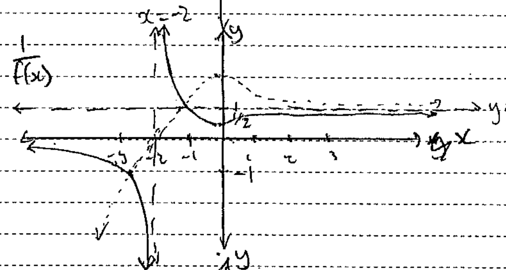
$1\frac{1}{2}$

(ii) $|y| = f(x)$



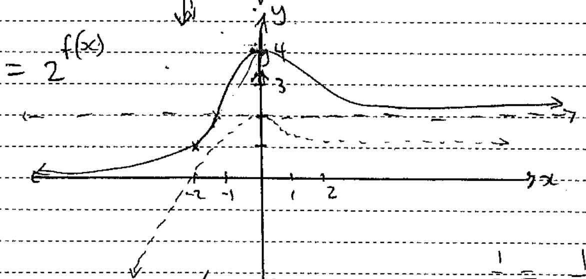
$\frac{1}{2}$

(iii) $y = \frac{1}{f(x)}$



$1\frac{1}{2}$

(iv) $y = 2^{f(x)}$



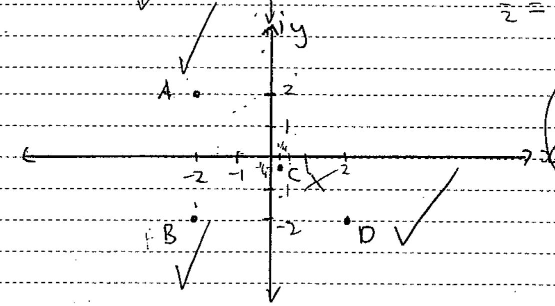
$1\frac{1}{2}$

(b)

$\frac{1}{2} = \frac{1}{2+2i} \quad \frac{1}{2}\left(\frac{1}{1+i}\cdot\frac{1-i}{1-i}\right)$

$\frac{1}{2}\left(\frac{1-i}{2}\right)$

$\frac{1-i}{4}$



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Student No.:

Paper:

Section: A (cont)

Sheet No.: ______ of ______ for this Section.

Q.No	Tick	Mark
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2(c) $I = \int \ln(1+x^2)\,dx$ put $u = \ln(1+x^2)$ $u' = \frac{2x}{1+x^2}$

$v' = 1$ $v = x$.

$\therefore I = x\ln(1+x^2) - \int \frac{2x^2}{1+x^2}\,dx$

$= x\ln(1+x^2) - 2\int \frac{x^2+1}{1+x^2} - \frac{1}{1+x^2}\,dx$

$= x\ln(1+x^2) - 2x + 2\tan^{-1}(x) + C$

3

(d) $\int_{-1}^{1} (1+x^3)^3\,dx = \int_{-1}^{1} 1 + 3x^3 + 3x^6 + x^9\,dx$

$= \left[x + \frac{3}{4}x^4 + \frac{3x^7}{7} + \frac{x^{10}}{10}\right]_{-1}^{1}$

$= \left[\frac{319}{140} - \left(-\frac{81}{140}\right)\right]$

$= \frac{20}{7} = 2\frac{6}{7}$

3

Sydney Boys' High School

004860

Student No.:

Paper:

Section: B

Sheet No.: ______ of ______ for this Section.

Q.No	Tick	Mark
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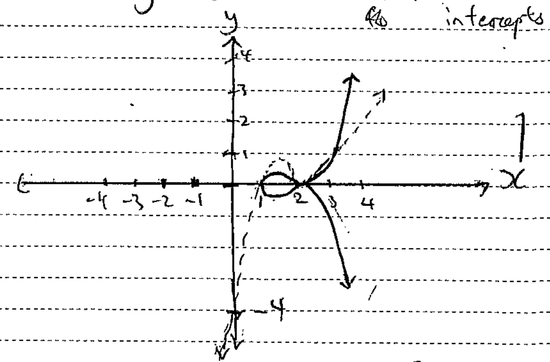
Q3

(a) first sketch $y = (x-2)^2(x-1)$ in dotted line then square. intercepts at



(b) ~~arg(z+1)~~ $\arg(z-1) - \arg(z+1) = \pi/2$

$\arg\left(\frac{z-1}{z+1}\right) = \pi/2$

let $z = x+iy$

$\arg\left(\frac{x+iy-1}{x+iy+1}\right) = \pi/2$

$$\arg\left(\frac{x+iy-1}{x+iy+1}\right) = \pi/2$$

$$\arg\left(\frac{(x+iy-1)(x-iy+1)}{(x+1)^2+y^2}\right) = \pi/2$$

$$\arg\left(\frac{(x^2-xiy+x+xiy+y^2+iy-x+iy-1)}{(x+1)^2+y^2}\right) = \pi/2$$

$$\tan^{-1}\left[\frac{\text{imaginary}}{\text{real}}\right] = \pi/2$$

(c) ~~P(x)~~ Let the roots be $\alpha-d, \alpha, \alpha+d$.

using sum of roots

$$\Sigma\alpha = \frac{-b}{a} = 3 = 3\alpha$$

$$\therefore \alpha = 1.$$

i.e. one of the roots is 1

using product of roots

$$\alpha\beta\gamma = -\frac{d}{a} = -8 = (1-d)(1+d)1$$

$$-8 = 1-d^2$$

$$d^2 = 9$$

$$d = \pm 3$$

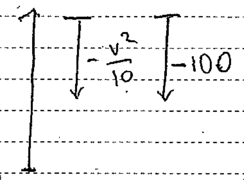
$\boxed{\therefore \text{ roots are } x=-2, 1, 5}$

$$P(1) = 0 = 1-3+a+8$$

$\boxed{\therefore a=-6}$

2

(d)



$$F = m\ddot{x} = 10\ddot{x} = -\frac{v^2}{10} - 100$$

$$\ddot{x} = -\frac{v^2}{100} - 10 = \frac{-(v^2+1000)}{100} = \frac{dv}{dt}$$

$$\frac{dt}{dv} = \frac{-100}{v^2+1000} = -100 \cdot \frac{1}{v^2+(10\sqrt{10})^2}$$

$$\therefore t = -100\left[\frac{1}{10\sqrt{10}}\tan^{-1}\frac{v}{10\sqrt{10}}\right] + C$$

at $t=0$, $v=u$

$$\therefore C = 100\left[\frac{1}{10\sqrt{10}}\tan^{-1}\frac{u}{10\sqrt{10}}\right]$$

$$\therefore t = -\sqrt{10}\tan^{-1}\frac{v}{10\sqrt{10}} + \sqrt{10}\tan^{-1}\frac{u}{10\sqrt{10}}$$

for max height let $v=0$

$$\therefore t = -\sqrt{10}\tan^{-1}(0) + \sqrt{10}\tan^{-1}\frac{u}{10\sqrt{10}}$$

$$= \sqrt{10}\tan^{-1}\left(\frac{u}{10\sqrt{10}}\right) \text{ seconds.}$$ 3

$\cancel{\text{(ii)}\ \ddot{x} = v\frac{dv}{dx} = \frac{-(v^2+1000)}{100}}$

$\cancel{\frac{dx}{dv} = \frac{-100v}{1000+v^2} = -50\times\frac{2v}{1000+v^2}}$

$\cancel{\therefore x = -50\ln(1000+v^2) + C}$

~~at x=0, v=u, at x=~~

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004117

Student No.: 23858460

Paper:

Section: B (cont) part 2

Sheet No.: ____ of ____ for this Section.

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Q3

(d ii) $\ddot{x} = v\frac{dv}{dx} = \frac{-(v^2+1000)}{100}$

$\frac{dx}{dv} = \frac{-100v}{1000+v^2} = -50 \times \frac{2v}{1000+v^2}$

$\therefore x = -50\ln[1000+v^2] + C$

at $x=0$, $v=u$

$\therefore C = 50\ln[1000+u^2]$ 3

$\therefore x = 50\ln\left[\frac{1000+u^2}{1000+v^2}\right]$

for max height, $v=0$

$x = 50\ln\left[\frac{1000+u^2}{1000}\right]$ metres.

Q4.

(a) ~~f(x)~~

(i) $LHS = \int_{-a}^{a} f(x)\,dx = \int_{-a}^{0} f(x)\,dx + \int_{0}^{a} f(x)\,dx$

$\boxed{\begin{array}{l}\text{let } x=-u \\ dx = -du.\end{array}}$

$\therefore \int_{-a}^{0} f(x)\,dx + \int_{0}^{a} f(x)\,dx$

$= \int_{a}^{0} f(u)\,-du + \int_{0}^{a} f(x)\,dx$

$= \int_{0}^{a} f(-x)\,dx + \int_{0}^{a} f(x)\,dx$ 2

$LHS = \int_{0}^{a} (f(x) + f(-x))\,dx = RHS$

(ii) $\int_{-\pi/4}^{\pi/4} \frac{dx}{1+\sin x} = \int_{0}^{\pi/4} \frac{1}{1+\sin x} + \frac{1}{1-\sin x}\,dx$

$= \int_{0}^{\pi/4} \frac{(1-\sin x)+(1+\sin x)}{1-\sin^2 x}\,dx$

$= 2\int_{0}^{\pi/4} \frac{1}{\cos^2 x}\,dx$ 2

$= 2\int_{0}^{\pi/4} \sec^2 x\,dx$

$= 2\left[\tan x\right]_{0}^{\pi/4}$

$= 2$

(b)

$|z-i| = Im(z)$

$\cancel{z}\ \sqrt{x^2+(y-1)^2} = y$

$x^2 + (y-1)^2 = y^2$

$x^2 = y^2 - (y^2-2y+1)$

$x^2 = 2y-1$

$x^2 = 2(y-\frac{1}{2})$ Z

the locus of z is clearly a parabola in the form $x^2=4a(y-k)$

(ii) $\arg z = \tan^{-1}\frac{y}{x}$

now $x^2 = 2y-1$

$\frac{x^2-1}{2} = y$

$\therefore \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{x^2-1}{2x}\right)$

$-1 < \tan^{-1}\left(\frac{x^2-1}{2x}\right) < 1$ O

(c) $\frac{-1}{x^2(x+1)} = \frac{A}{x^2} + \frac{B}{x} + \frac{C}{x+1}$

$\therefore -1 = A(x+1) + Bx(x+1) + Cx^2$

equating co-efficients of x:

$B+C=0$ (1)

$\boxed{A=-1}$ (2)

$B+A=0$ (3)

(2) into (3)

$\boxed{B=1}$ (4)

(4) into (1)

$\boxed{C=-1}$

$\therefore C=-1,\ B=1,\ A=-1$ Z

(ii) u m/s

$-v^2-v^3$

$F = m\ddot{x} = -v^2-v^3 \qquad m=1$

$\ddot{x} = \frac{dv}{dt} = -v^2-v^3$

$\frac{dt}{dv} = \frac{-1}{v^2(v+1)}$

from (i)

$\frac{dt}{dv} = \frac{-1}{v^2} + \frac{1}{v} - \frac{1}{v+1}$

$\therefore t = \frac{1}{v} + \ln v - \ln v+1 + C$

at $t=0,\ v=u.$

$\therefore C = -\frac{1}{u} - \ln u + \ln(u+1)$

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Student No.:

Paper:

Section:

Sheet No.: ____ of ____ for this Section.

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B (cont) part 3

4. c (ii) cont

$\therefore t = \frac{1}{v} - \frac{1}{u} + \ln\frac{v}{u} + \ln\frac{u+1}{v+1}$

$t = \frac{1}{v} - \frac{1}{u} + \ln\left[\frac{v(u+1)}{u(v+1)}\right]$

when $v = \frac{u}{2}$,

$t = \frac{2}{u} - \frac{1}{u} + \ln\left[\frac{\frac{u}{2}(u+1)}{u(\frac{u}{2}+1)}\right]$

$= \frac{1}{u} + \ln\left[\frac{u^2+u}{u^2+u}\right]$

$T = t = \frac{1}{u}$ $\quad \therefore uT = 1$

~~then when x = X, t = T/2~~

$\therefore X = \ln\left(\frac{2+u}{1+u}\right)$

0

Sydney Boys' High School

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Student No.:

Paper:

Section: C ~~part 2~~ part 1

Sheet No.: ____ of ____ for this Section.

Q.No	Tick	Mark
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Question 5

(a)(i) $(1+i)^n = (\sqrt{2}\,\text{cis}\,\pi/4)^n$

$= 2^{\frac{1}{2}n}\,\text{cis}(n\pi/4)$ 1

(ii) $(1+i)^n = \sum_{k=0}^{n}\binom{n}{k} i^k$

$= \binom{n}{0} + \binom{n}{1}i - \binom{n}{2} - \binom{n}{3}i \ldots + \binom{n}{n}i^n = 2^{\frac{1}{2}n}\,\text{cis}\,n\pi/4.$

equating real parts:

$\binom{n}{0} - \binom{n}{2} + \binom{n}{4} - \binom{n}{6} \pm \ldots = 2^{\frac{1}{2}n}\cos(n\pi/4)$ 3

$\boxed{1 - \binom{n}{2} + \binom{n}{4} - \binom{n}{6} + \ldots = 2^{\frac{n}{2}}\cos(n\pi/4)}$

~~if n is a multiple of~~

(iii) equating imaginary parts:

$\binom{n}{1} - \binom{n}{3} + \binom{n}{5} - \binom{n}{7} + \binom{n}{9} - \binom{n}{11} + \dots = 2^{\frac{n}{2}} \sin(n\pi/4)$

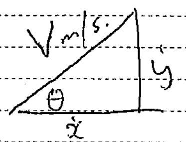
if n is a multiple of 8, ie $8k$ where k is arbitrary integer

$RHS = 2^{4k} \sin(2k\pi) = 0$.

$\therefore \binom{n}{1} - \binom{n}{3} + \binom{n}{5} - \binom{n}{7} + \binom{n}{9} - \binom{n}{11} + \dots = 0$

$\boxed{\therefore \binom{n}{1} + \binom{n}{5} + \binom{n}{9} + \dots = \binom{n}{3} + \binom{n}{7} + \binom{n}{11} + \dots}$ 2

(b)(i) at $t=0$



$\dot{y} = V\sin\theta$

$\dot{x} = V\cos\theta$

at $t=0$.

$\ddot{x} = 0$

integrating w.r.t 't'

$\dot{x} = C_1$

at $t=0$ $\dot{x} = V\cos\theta$

$\boxed{\dot{x} = V\cos\theta}$

integrate w.r.t 't'

$x = Vt\cos\theta + C_3$

at $t=0$, $x=0$

$\therefore C_3 = 0$

$\boxed{\therefore x = Vt\cos\theta}$

$\ddot{y} = -g$

$\dot{y} = -gt + C_2$

at $t=0$, $\dot{y} = V\sin\theta$

$\therefore C_2 = V\sin\theta$

$\boxed{\dot{y} = V\sin\theta - gt}$

$y = Vt\sin\theta - \frac{gt^2}{2} + C_4$

at $t=0$, $y=0$

$\therefore C_4 = 0$

$\boxed{y = Vt\sin\theta - \frac{gt^2}{2}}$

2

(ii) $x=y$ and $\alpha = 45°$

$x = Vt\cos\theta$ $\quad y = Vt\sin\theta - \frac{gt^2}{2}$

$\therefore Vt\cos 45° = Vt\sin 45° - \frac{gt^2}{2}$

$\frac{Vt}{\sqrt{2}} = \frac{Vt}{\sqrt{2}} - \frac{gt^2}{2}$

$\frac{gt^2}{2} = 0$

$\therefore t = 0$

~~$\therefore$ angle of projection $\theta = 45°$~~

$\dot{x} + \dot{y} =$

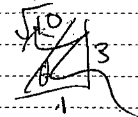
(iii) $\tan\theta = 3$.

$x = Vt\cos\theta$ $\quad y = Vt\sin\theta - \frac{gt^2}{2}$

$t = \frac{x}{V\cos\theta}$

$\therefore y = x\tan\theta - \frac{gx^2}{2V^2}(\tan^2\theta + 1)$

now $\tan\theta = 3$



$y = 3x - \frac{10gx^2}{2V^2}$

$y = 3x - \frac{5gx^2}{V^2}$

~~$V^2 = \sqrt{\dot{x}^2 + \dot{y}^2}$~~

now $V^2 = \dot{x}^2 + \dot{y}^2$

~~max~~ height when $y=0$

$0 = 3x - \frac{5gx^2}{V^2} = x\left(3 - \frac{5gx}{V^2}\right)$

but $x \neq 0$ $\quad z = \frac{5gx}{v^2}$

$$\boxed{\text{range} = \frac{3v^2}{5g}}$$

✓ 2

(iv) range $= \frac{3v^2}{5g}$

~~max range when~~

~~max range =~~

$y = Vt\sin\theta - \frac{gt^2}{2}$

$y = \frac{3Vt}{\sqrt{10}} - \frac{gt^2}{2}$

$H = \frac{3Vt}{\sqrt{10}} - \frac{gt^2}{2}$

$\frac{3Vt}{\sqrt{10}} = H + \frac{gt^2}{2}$

$V = \frac{\sqrt{10}H}{3t} + \frac{\sqrt{10}gt}{6}$

$\therefore$ range $= \frac{3v^2}{5g} = \cancel{\frac{3\left(\frac{\sqrt{10}gt}{6}\right)^2}{5g}}$

$\cancel{H = \frac{\frac{30gt}{36}}{5g}}$

Sydney Boys' High School

004858

Student No.:

Paper:

Section: ~~A~~ C (part 2)

Sheet No.: ____ of ____ for this Section.

Q.No	Tick	Mark
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

Q6

say $P(x)$ has a repeated root at $x = \alpha$,

ie $P(x) = (x-\alpha)^k \cdot Q(x)$

where k is an integer $0 < k < n$.

then $P'(x) = k(x-\alpha)^{k-1} \cdot Q(x) + (x-\alpha)^k \cdot Q'(x)$.

now $P'(x) = nx^{n-1} - (n-1)x^{n-2}$

for possible root:

$P'(x) = 0 = x^{n-2}(nx - n + 1)$

$x = 0$ or $x = \frac{n-1}{n}$

but $P(x) = x^n - x^{n-1} - 1$

clearly $x = 0$ is not a root.

$P\left(\frac{n-1}{n}\right) = \left(\frac{n-1}{n}\right)^n - \left(\frac{n-1}{n}\right)^{n-1} - 1$

$= \left(\frac{n-1}{n}\right)^{n-1}\left(\frac{n-1}{n} - 1\right) - 1$ but $n > 1$

$= \left(\frac{n-1}{n}\right)^{n-1}(-1) - 1$

ie $\left(\frac{n-1}{n}\right)^{n-1}\left(\frac{-1}{n}\right) < 0$ [positive . negative = negat.

$\therefore$ $p\left(\frac{n-1}{n}\right)$ = negative ~~...~~ $-1 \neq 0$.

$\therefore$ $P\left(\frac{n-1}{n}\right)$ is not a root.

$\therefore$ $x=0$ and $x=\frac{n-1}{n}$ are not roots.

There can be no repeated roots.

(b)

(i) (α) $\frac{11!}{2!\times 2!}$ = 9979200 "words"

(β) $\boxed{2}\ \boxed{\frac{9!}{2!}}\ \boxed{1}$

~~2 × 10!~~ $\frac{2!\times 9!}{2!}$ = 362880 ways

(γ)

$\boxed{FUN}\boxed{D}\boxed{A}\boxed{M}\boxed{E}\boxed{N}\boxed{T}\boxed{A}\boxed{L}$ 9 objects 2 A's.

$\frac{9!}{2!}$ = 181440

(δ) Total ways − (no. of ways letter next to each other)

2 A's together → $\frac{10!}{2!}$ = 1814400 ways

2N's together → $\frac{10!}{2!}$ → 1814400 ways

2A's together 2N's together → 9! → 362880 ways

from (α)

$\therefore$ ways = 9979200 − [1814400 + 1814400 + 362880]

= 5987520 ways

(ii) case: A = alike D = different.

5D: ${}^{9}C_5 \cdot 5!$ = 126×120 = 15120 ways

2A 3D: ${}^{2}C_1 \cdot {}^{1}C_1 \cdot {}^{8}C_3$ × $\frac{5!}{2!}$ = 112 × 60 = 6720 ways

4A 1D: ${}^{4}C_4 \cdot {}^{7}C_1 \times \frac{5!}{2!\times 2!}$ = 7×30 = 210 ways..

$\therefore$ ways = 15120 + 6720 + 210

= 22050 ways