SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2014
YEAR 11 2 UNIT
HALF YEARLY EXAM

Mathematics

General Instructions

- Reading Time – 5 Minutes
- Working time – 90 Minutes
- Write using black or blue pen. Pencil may be used for diagrams.
- Board approved calculators maybe used.
- Each section is to be returned in a separate bundle.
- All necessary working should be shown in every question.

Total Marks – 75

- Attempt questions 1 – 4
- All questions are not of equal value.
- Answer in simplest exact form unless otherwise stated.

Examiner: *PR Bigelow*

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n \neq -1;\ x \neq 0, \text{if } n<0$$

$$\int \frac{1}{x}\,dx = \ln x,\ x>0$$

$$\int e^{ax}dx = \frac{1}{a}e^{ax},\ a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax,\ a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\ a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,\ a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax,\ a \neq 0$$

$$\int \frac{1}{a^2+x^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\ a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\frac{x}{a},\ a>0,\ -a<x<a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}dx = \ln\left(x+\sqrt{x^2-a^2}\right),\ x>a>0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x,\ x>0$

Section A

Question 1. (19 Marks) **(Start a new booklet.)**

(a) Convert 3π radians to degrees. **1**

(b) How many significant zeroes are there in $0\cdot04007$? **1**

(c) Use a calculator to evaluate $\sqrt{\dfrac{5}{14\cdot6-11\cdot4}}$ correct to 3 significant figures. **1**

(d) Find a if: (i) $\sqrt{a}=5\sqrt{7}$ **2**

(ii) $\sqrt{54}=3\sqrt{a}$

(e) Simplify $\dfrac{16x+12}{8}$. **1**

(f) Convert 330^0 to radians. **1**

(g) Express $0\cdot0\dot{5}$ as a fraction in simplest form. **2**

(h) Solve the following pair of equations simultaneously: **2**

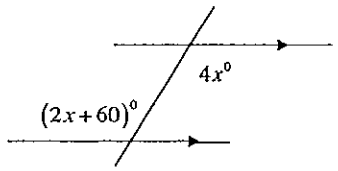
$$4x-y=7$$
$$2x+y=5$$

(i) Write down the exact values of: **2**

(i) $\sin 240^0$

(ii) $\tan -120^0$

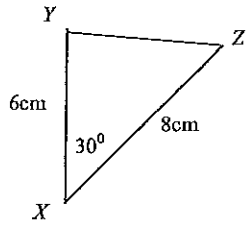
(j) Find the value of x. **2**



(k) What is the supplement of the complement of $20^0 40'$? **1**

(l) Find T_6 of the arithmetic series $5+3+1+\ldots$ **2**

(m) Write down the area of ΔXYZ. **1**



Question 2. (19 Marks)

(a) Expand then simplify: 2

$$\left(y-\frac{1}{y}\right)^2$$

(b) Express $\frac{12}{\sqrt{7}+1}$ with rational denominator, in simplest form. 2

(c) Solve $|4+x|<8$ then graph the solution on a number line.. 2

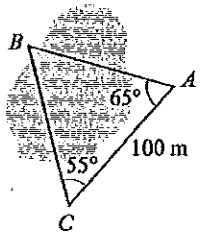
(d) Solve the following equiations: 5

(i) $a^2=2a$

(ii) $5y^2-11y+2=0$

(iii) $\frac{3x+1}{5x+1}=\frac{3x-2}{5x+2}$

(e) 2

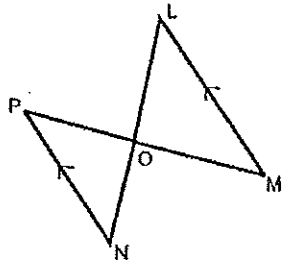


Find the length of the side BC (correct to one decimal place.)

(f) In the diagram $PN \parallel LM$, PO=20cm, OM=8cm, LO=5cm and PN=15cm. 3

(i) Copy the diagram to your answer sheet.

(ii) Find LM and NO, giving reasons.



(g) Solve the equations $a+b-c=-2$, $2a-b+2c=7$, and $3a+2b+c=3$ simultaneously. 3

Section B

Question 3. (18 Marks) **(Start a new booklet.)**

(a) Find $\sum_{r=1}^{8}(3\times 2^r)$ 2

(b) Which term of the geometric series $2+6+18+...$ is equal to 486? 2

(c) Factorise the following: 8

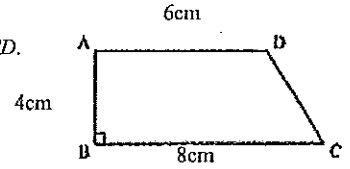
(i) x^4-y^4

(ii) $2x^3+54$

(iii) $6x^2-11xy+4y^2$

(iv) $xy-x-7y+7$

(d) Find the area of the trapezium $ABCD$. 2



(e) The diagonal of a square is $4\sqrt{2}$ cm. What is the area of the square? 2

(f) Solve $2\sin 2x=1$ for $0^0 \le x \le 360^0$. 2

Question 4. (19 Marks)

(a) The infinite geometric series $1+2x+4x^2+....$ has a sum of 6. **2**

Find the value of x.

(b) A person invests \$1750 at the beginning of each year. Interest is paid at the rate of 6% per annum, compounded annually. How much will the investment be worth after 20 years? **3**

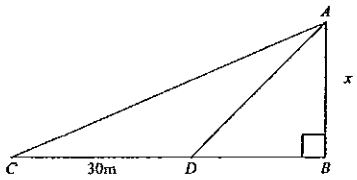
(c) The sum of a series is given by $S_n = n^2 + 5n$, where n is the number of terms. **5**

(i) Find S_6.

(ii) Find T_n.

(iii) Show that the series is arithmetic.

(d) **3**



In the diagram $\angle ACD = 20^0$ and $\angle ADB = 40^0$.

Copy the diagram to your answer sheet.

Find the value of x, correct to one decimal place.

Question continued on the next page.

(e) A woman borrows \$500 000 at an interest rate of 6% p.a. monthly reducible. It is repaid monthly over 25 years, where \$$M$ is the monthly repayment.

(i) Show that the amount owing, \$$A_1$ after the first month is given by **1**

$$A_1 = 500000(1.005) - M$$

(ii) Show that **1**

$$A_2 = 500000(1.005)^2 - M(1+1.005)$$

(iii) Write an expression for A_n. **1**

(iv) Find the value of M (to the nearest dollar). **3**

This is the end of the paper.

Q1.

(a) $3 \times 180^\circ = 540^\circ$ ①

(b) 2 ⑥

(c) 1.25 ①

(d) (i) $\sqrt{a} = 5\sqrt{7}$

$a = 25 \times 7 = 175$ ①

(ii) $3\sqrt{a} = 54$

$9a = 54$

$a = 6$ ①

(e) $2x + 1\frac{1}{2}$ ①

(f) $1^\circ = \frac{\pi}{180}$ ①

$330^\circ = \frac{330\pi}{180}$

$= \frac{11}{6}\pi$

(g) $x = 0.05 + 0.0005 + \ldots$

$a = 0.05 = \frac{5}{100}$

$r = \frac{1}{10}$

$S_\infty = \frac{a}{1-r} = \frac{5}{100} \div \frac{9}{10}$

$= \frac{5}{100} \times \frac{10}{9}$

$= \frac{1}{18}$ ②

(h) ① $4x - y = 7$

② $2x + y = 5$

①+② $6x = 12$

$x = 2$

② $4 + y = 5$, $y = 1$

$x = 2$

$y = 1$ ②

(I)

(i) $\sin 240^\circ = \sin 180^\circ + 60^\circ$

$= -\sin 60$

$= -\frac{\sqrt{3}}{2}$ ①

(ii) $\tan(-120^\circ) = +\tan 240^\circ$

$= \sqrt{3}$ ①

(j) $4x = 2x + 40$ (Alternate ∠s)

$2x = 60$

$x = 30^\circ$ ②

(k) Complement of $20^\circ 40' = 69^\circ 20'$

Supplement of $69^\circ 20' = 110^\circ 40'$ ①

(l) $5 + 3 + 1 + -1 + -3 + -5$

$T_6 = -5$ ②

(m) Area $= \frac{1}{2}ab\sin C$

$= \frac{1}{2} \times 6 \times 8 \times \sin 30^\circ$

$= 24 \times \frac{1}{2}$

$= 12\,cm^2$ ①

Q2.

(a) $y^2 - 2\left(y \times \frac{-1}{y}\right) + \frac{1}{y^2}$

$= y^2 - 2 + \frac{1}{y^2}$ ②

(b) $\frac{12}{\sqrt{7}+1} \times \frac{\sqrt{7}-1}{\sqrt{7}-1}$

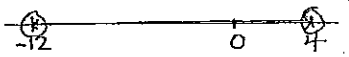
$= \frac{12(\sqrt{7}-1)}{7-1}$

$= 2(\sqrt{7}-1)$ ②

(c) $4 + x < 8$ or $-4 - x < 8$

$x < 4$ or $-x < 12$

$x > -12$



②

(d) (i) $a^2 - 2a = 0$ $a(a-2) = 0$

$a = 0, 2$ ①

(ii) $(5y-1)(y-2) = 0$

$y = \frac{1}{5}$ or $y = 2$ ①

(iii) $(2x+1)(5x+2) = (5x+1)(3x-2)$

$15x^2 + 6x + 5x + 2 = 15x^2 - 10x + 3x - 2$

$18x = -4$

$x = -\frac{2}{9}$ ③

(e) $\angle CBA = 60^\circ$

$\frac{BC}{\sin 65^\circ} = \frac{100}{\sin 60^\circ}$

$BC = \frac{100\sin 65^\circ}{\sin 60^\circ}$

$= 104.65^\circ$ ②

(f) $\angle PNO = \angle OLM$, $\angle NPO = \angle OML$

(Alternate angles in ∥ lines)

$\therefore \triangle PNO \parallel\!\parallel \triangle OLM$ (equiangular)

$\therefore \frac{PO}{OM} = \frac{20}{8} = \frac{5}{2}$

$\frac{PN}{LM} = \frac{5}{2} = \frac{15}{LM}$

$LM = 6\,cm$

$\frac{NO}{OL} = \frac{5}{2} = \frac{NO}{5}$

$NO = 12\frac{1}{2}\,cm$ ③

(g) ① $a + b - c = -2$

② $2a - b + 2c = 7$

③ $3a + 2b + c = 3$

①+② $3a + c = 5$ ④

③ $3a + c + 2b = 3$

$2b = -2$, $b = -1$

① $a - 1 - c = -2$

$a - c = -1$

①+② $3a + c = 5$ ④

④A $4a = 4$

$a = 1$

① $1 - 1 - c = -2$

$c = 2$

$a = 1$

$b = -1$

$c = 2$ ③

Section B

Question 3

a) $\sum_{r=1}^{8}(3\times 2^r) = 3\times2^1 + 3\times2^2 + 3\times2^3 + \ldots + 3\times2^8$

$= 6 + 12 + 24 + \quad + 768$

$S_n = \frac{a(1-r^n)}{1-r}$

$S_8 = \frac{6(1-2^8)}{1-2}$

$= 1530$

b) $T_n = ar^{n-1}$

$486 = 2(3)^{n-1}$

$3^{n-1} = 243$

$3^{n-1} = 3^5$

$n-1 = 5$

$n = 6$

c) i) $x^4 - y^4 = (x^2+y^2)(x^2-y^2)$

$= (x^2+y^2)(x-y)(x+y)$

ii) $2x^3 + 54 = 2(x^3+27)$

$= 2(x+3)(x^2-3x+9)$

iii) $6x^2 - 11xy + 4y^2$

x	24
+	-11
-8, -3	

$= 6x^2 - 3xy - 8xy + 4y^2$

$= 3x(2x-y) - 4y(2x-y)$

$= (2x-y)(3x-4y)$

iv) $xy - x - 7y + 7 = x(y-1) - 7(y-1)$

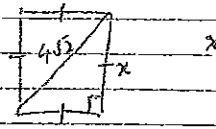
$= (y-1)(x-7)$

d) $A = \frac{1}{2}(a+b)h$

$= \frac{1}{2}(6+8)(4)$

$= 28\ cm^2$

e)



$x^2 + x^2 = (4\sqrt{2})^2$

$2x^2 = 32$

$x^2 = 16$

$A = x^2$

$= 16\ cm^2$

OR $A = \frac{1}{2}xy$

$= \frac{1}{2}(4\sqrt{2})(4\sqrt{2})$

$= 16\ cm^2$

f) $2\sin 2x = 1$ $\quad 0° \le x \le 360°$

$0° \le 2x \le 720°$

$\sin 2x = \frac{1}{2}$

S ✓	A ✓
T	C

$\sin\alpha = \frac{1}{2}$

$\alpha = 30°$

$2x = 30°, 150°, 390°, 510°$

$x = 15°, 75°, 195°, 255°$

Question 4

a) $S_\infty = \frac{a}{1-r}$

$6 = \frac{1}{1-2x}$

$1-2x = \frac{1}{6}$

$-2x = -\frac{5}{6}$

$x = \frac{5}{12}$

b) Let A_n be the amount the n^{th} \$1750 is worth at the end

$A_1 = 1750(1.06)^{20}$

$A_2 = 1750(1.06)^{19}$

$A_3 = 1750(1.06)^{18}$

$\vdots$

$A_{20} = 1750(1.06)^{1}$

$\text{Total} = 1750(1.06)^1 + 1750(1.06)^2 + \ldots + 1750(1.06)^{20}$

$S_n = \frac{a(1-r^n)}{1-r}$

$S_{20} = \frac{1750(1.06)(1-1.06^{20})}{1-1.06}$

$= \$68237.27$

c) $S_n = n^2 + 5n$

i) $S_6 = (6)^2 + 5(6)$

$= 66$

ii) $T_n = S_n - S_{n-1}$

$= n^2 + 5n - ((n-1)^2 + 5(n-1))$

$= n^2 + 5n - (n^2 - 2n + 1 + 5n - 5)$

$= 2n + 4$

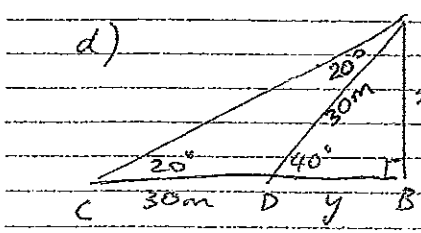
iii) $T_n - T_{n-1} = 2n+4 - (2(n-1)+4)$

$= 2n+4 - (2n - 2 + 4)$

$= 2$ which is a constant

$\therefore$ the series is arithmetic

d)



$\frac{\sin 40^\circ}{30} = \frac{x}{1}$

$x = 30 \sin 40^\circ$

$x = 19.3\text{m}$

e) i) 6% p.a $= 0.5\%$ per month

$A = P(1+r)^n$

$A_1 = 500000(1.005) - M$

ii) $A_2 = [500\,000(1.005) - M](1.005) - M$

$= 500\,000(1.005)^2 - M(1 + 1.005)$

iii) $A_3 = [500\,000(1.005)^2 - M(1 + 1.005)](1.005) - M$

$= 500000(1.005)^3 - M(1 + 1.005 + 1.005^2)$

$\vdots$

$A_n = 500\,000(1.005)^n - M(1 + 1.005 + 1.005^2 + \ldots + 1.005^{n-1})$

iv) $A_{300} = 0$

$0 = 500\,000(1.005)^{300} - M(1 + 1.005 + 1.005^2 + \ldots + 1.005^{299})$

$S_a = a\frac{(1-r^n)}{1-r}$

$S_{300} = 1\left(\frac{1 - 1.005^{300}}{1 - 1.005}\right)$

$= 692.99\ldots$ *

$0 = 500\,000(1.005)^{300} - 692.99M$

$692.99M = 500\,000(1.005)^{300}$

$M = \frac{500\,000(1.005)^{300}}{692.99}$

$= \$3221.51$

*. Use the value on calculator