SCEGGS Darlinghurst

**Name:** .....

**Teacher:** .....

**HSC Assessment 1**
**18 November, 2003**

# Mathematics

## General Instructions

- Time allowed: 60 minutes
- Weighting 15%
- This paper has **four** questions
- Attempt **all** questions and show all necessary working
- Marks will be deducted for careless or badly arranged work
- Write using blue or black pen, diagrams in pencil
- Start each question a new page
- Write your name and your teacher's name at the top of each page
- Approved calculators, mathematical templates and geometrical instruments may be used

| Questions | Marks | Communication | Reasoning | Calculus |
|---|---|---|---|---|
| 1 | 7~~8~~/10 | | | (c) ~~4~~ /5 |
| 2 | 5 /10 | (c) iii) 0 /2 | (b) 2 /2 | (c) i) ii) 2 /4 |
| 3 | 7 /10 | (a) 2 /3 | | (b) 5 /7 |
| 4 | 5 /10 | | (a) (b) (c) 5 /10 | |
| | 2~~8~~/40 | 2 /5 | 7 /12 | 10 /16 |

- Answer on the pad paper provided
- Write your name at the top of each page
- Start each question on a new page
- Clearly label each question

**Question 1** (10 marks) **Marks**

(a) For the series $5 + 11 + 17 \cdots$ find:

(i) the 16th term. **1**

(ii) the number of terms needed to give a sum of 1633. **2**

(b) Insert four terms between 8 and $\frac{1}{4}$ to form a geometric sequence. **2**

(c) (i) Differentiate $y = \sqrt{25 - x^2}$. **2**

(ii) Find the equation of the tangent to this curve at the point $x = 3$. **3**

- **Start a new page**

**Question 2** (10 marks) **Marks**

(a) Find the value of $p$ so that $p+5,\ 4p+3,\ 8p-2$ will form successive terms of an arithmetic sequence. **2**

(b) Evaluate $\sum_{n=1}^{\infty} 8\times\left(\frac{-1}{2}\right)^{n-1}$ **2**

(c) A function is given by $y = x^3 - 3x^2 - 7$.

(i) For what values of $x$ is the curve decreasing? **2**

(ii) For what values of $x$ is the curve concave up? **2**

(iii) Ethel concludes that there are therefore no points on the curve where the curve is both decreasing and concave up. **2**

She is incorrect. Explain why Ethel is wrong, stating the values of $x$ for which the curve is **both** concave up and decreasing.

- **Start a new page**

**Question 3** (10 marks) **Marks**

(a) For a particular function the following is known. **3**

$$f(0)=7$$
$$f'(2)=0$$
$$f''(2)=-10$$
$$f'(-3)=0$$
$$f''(-3)=0$$

Draw any function which satisfies all of the above conditions.

(b) For the curve $y=2x^3+3x^2-12x+7$

(i) Find the stationary points and determine their nature. **3**

(ii) Find any points of inflexion. **2**

(iii) Sketch the curve showing all important features. **2**

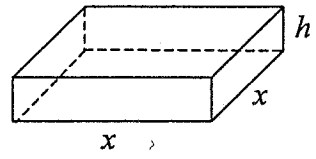
- **Start a new page**

**Question 4** (10 marks) **Marks**

(a) Explain why the series $\frac{3}{10} + \frac{7}{100} + \frac{3}{1000} + \frac{7}{10000} \cdots\cdots$ has a limiting sum. **1**

(b) Wallyville is a fast growing town. In 2003, the population is 9750. **3**
The population is increasing at a rate of 5% of the preceding year's population.
After how many years will the population first exceed 25000, if the growth rate remains constant?

(c) A sealed tin rectangular box with a lid is to have a square base and a volume of $64cm^3$. Let the length of the box be $x$ cm.




(i) Show that the total surface area is given by $A = \frac{256}{x} + 2x^2$. **2**

(ii) Find the area of the tin needed to make the box with the smallest surface area. **4**

**End of Assessment**

HSC Assessment One, 2003

1 a) $5+11+17+\ldots$ A.P.

i) $a=5,\ d=6$

$T_{16} = 5+15\times 6$

$= 95$ ✓

ii) $S_n = 1633$

$\frac{n}{2}(2\times 5+(n-1)\times 6) = 1633$

$n(10+6n-6) = 3266$

$4n+6n^2 = 3266$ ✓

$3n^2+2n-1633=0$

$n = \frac{-2\pm\sqrt{4+4\times 3\times 1633}}{6}$

$= \frac{-2\pm 140}{6}$

$= 23$ or $-23\frac{2}{3}$ ✓

as $n>0$ ∴ 23 terms are needed to give a sum of 1633.

b) $T_1 = 8 \quad T_6 = 1/4$

$a=8$

$ar^5 = 1/4$

$8r^5 = 1/4$

$r^5 = 1/32$

$r = 1/2$ ✓

∴

$8, 4, 2, 1, \frac{1}{2}, \frac{1}{4}$ ✓

ci) $y = (25-x^2)^{1/2}$

$y' = \frac{1}{2}(25-x^2)^{-1/2}\times -2x$ ✓Calc

$= \frac{-x}{\sqrt{25-x^2}}$ ✓Calc

cii) $x=3,\ y=\sqrt{25-3^2}$

$= 4$ ✓Calc

$y' = \frac{-3}{4}$ ✓Calc

$y-4 = \frac{-3}{4}(x-3)$

$4y-16 = -3x+9$

$3x+4y-25=0$ ✓Calc

2a) If AP $T_2-T_1 = T_3-T_2$

$4p+3-(p+5) = 8p-2-(4p+3)$ ✓

$4p+3-p-5 = 8p-2-4p-3$

$3p-2 = 4p-5$

$3 = p$

∴ $p=3$. ✓

b) $\sum_{n=1}^{\infty} 8\times\left(\frac{-1}{2}\right)^{n-1}$

$= 8+(-4)+(2)+\ldots$

$a=8 \quad r=-1/2$ ✓R

$S_\infty = \frac{8}{1+1/2}$

$= 5\frac{1}{3}$ ✓R

c) $y = x^3-3x^2-7$

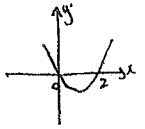
i) $y' = 3x^2-6x$

decreasing when $y'<0$

$3x^2-6x<0$ ✓Calc

$3x(x-2)<0$

$0<x<2$ ✓Calc




cii) $y'' = 6x-6$

concave up when $y''>0$

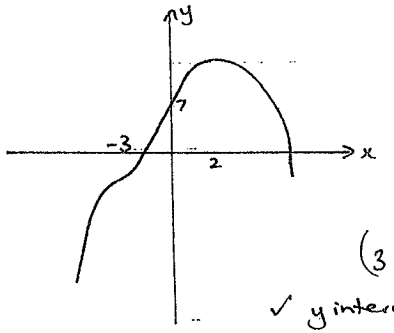
$6x-6>0$ ✓Calc

$6x>6$

$x>1$ ✓Calc

iii) Ethel is wrong because there is some overlap in those 2 answers. When $1<x<2$ the curve with both be decreasing and concave up. ✓com ✓com

3a)




(3 com)

✓ y intercept

✓ max TP

✓ Horizontal P.O.I.

b) $y = 2x^3+3x^2-12x+7$

i) $y' = 6x^2+6x-12$

TPs at $y'=0$ $\quad 6(x^2+x-2)=0$

$(x+2)(x-1)=0$

$x=-2,\ 1$ ✓Calc

$x=-2,\ y=27$

$x=1,\ y=0$

$y''=12x+6$

at $x=-2 \quad y''=12(-2)+6$

$y''<0$

∴ max TP at $(-2,27)$ ✓Calc

at $x=1 \quad y''=12(\ )+6$

$y''>0$

∴ min TP at $(1,0)$ ✓Calc

ii) pt of inflexion at $y''=0$

$12x+6=0$

$x=-1/2,\ y=$ ~~12~~ 13½ ✓Calc

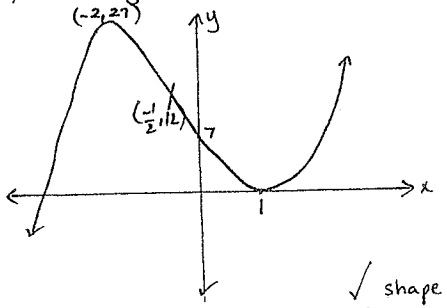
| test | $x$ | $-1$ | $-1/2$ | $0$ |
|---|---|---|---|---|
| | $y''$ | $-ve$ | $0$ | $+ve$ |
| | concavity | ∩ | | ∪ |

✓Calc

∴ concavity has changed

∴ pt inflex at $(-\frac{1}{2}, 12)$

iii) $x=0 \quad y=7$




✓ shape

✓ labels including y-intercept

4a) $\frac{3}{10} + \frac{7}{100} + \frac{3}{1000} + \frac{7}{10000} + \cdots$

$= \frac{37}{100} + \frac{37}{10000} + \cdots$

$\therefore$ GP $a = \frac{37}{100}$ $r = \frac{1}{100}$ ✓R

and as $|r| < 1$ it will have a limiting sum.

b) 9750, $9750 \times 1.05$, $9750 \times 1.05^2$, ...

GP $a = 9750$

$r = 1.05$

$T_n = ar^{n-1}$

$9750 \times 1.05^{n-1} > 25000$ ✓R

$1.05^{n-1} > \frac{100}{39}$

$n-1 \log 1.05 > \log \frac{100}{39}$

$n-1 > \frac{\log \frac{100}{39}}{\log 1.05}$ ✓R

$n-1 > 19.299$

$n > 20.299$

$\therefore n = 21$ ✓R

$\therefore$ The 21st term is the first to exceed 25000 which is 20 years after 2003.

c) i) Vol $= lbh$

$64 = x^2 h$

$h = \frac{64}{x^2}$ ✓R

$SA = 2(lb + bh + lh)$

$= 2x^2 + 4xh$

$SA = 2x^2 + 4x\left(\frac{64}{x^2}\right)$ ✓R

$\therefore A = 2x^2 + \frac{256}{x}$

ii) $A' = 4x - 256x^{-2}$

$A'' = 4 + 512x^{-3}$ ✓R

max / min $A' = 0$

$4x - \frac{256}{x^2} = 0$

$4x = \frac{256}{x^2}$

$4x^3 = 256$

$x^3 = 64$

$x = 4$ ✓R

check it's min.

$x = 4$ $A'' = 4 + \frac{512}{64}$

$A'' > 0$ ✓R

$\therefore$ min value.

When $x = 4$

$A = 2(4)^2 + \frac{256}{4}$

$= 96\ cm^2$ ✓R