Student Number

SCEGGS Darlinghurst

2013

HSC Assessment 2
19 March 2013

Mathematics

Assessment Outcomes: P2, P3, P4, P8, H2, H4, H5, H8, H9
Weighting: 20%

General Instructions

- Time allowed – 70 minutes
- This paper is in **two** sections
- Write your Student Number on the front of each writing booklet
- Attempt **all** questions and show all necessary working
- Marks may be deducted for careless or badly arranged work
- Mathematical templates and geometrical equipment and scientific calculators may be used

Total marks – 41
Assessment Weighting – 20%

Section I

4 marks

- Attempt Questions 1 – 4
- Allow about 6 minutes for this section
- Answer on the separate Objective Response Answer Sheet provided

Section II

37 marks

- Attempt Questions 5 – 7
- Allow about 64 minutes for this section
- ALL working must be shown
- Answer in the writing booklets provided
- Start each question in a **new booklet**

Question	Integration		Series and Sequences		Trigonometry		Total
1 – 4		/2		/1		/1	/4
5		/6		/7		/1	/14
6		/9				/2	/11
7		/6		/3		/3	/12
Total		/23		/11		/7	/41

Section I

4 marks
Attempt Questions 1–4
Allow about 6 minutes for this section
Use the objective-response answer sheet

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2+4=$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word ***correct*** and drawing an arrow as follows.

correct ↓

A ⊗ B ● C ○ D ○

Question 1

What is the value of $\sum_{r=1}^{3} 2^{r-1}$?

A. $\frac{3}{4}$

B. 1

C. 6

D. 7

Question 2

Find $\int 2x^3\,dx$.

A. $2x^4 + C$

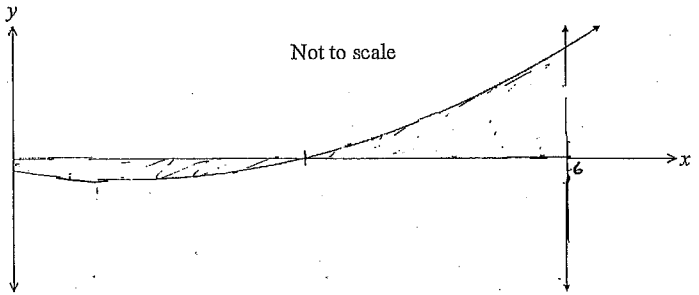
B. $6x^2 + C$

C. $\frac{1}{2}x^4 + C$

D. $\frac{2}{3}x^4 + C$

Question 3

The diagram below shows the graph of $y = x^2 - 2x - 8$.



What is the correct expression for the area bounded by the x axis and the curve $y = x^2 - 2x - 8$ between $x = 0$ and $x = 6$?

A. $Area = \int_0^2 x^2 - 2x - 8\,dx + \left| \int_2^6 x^2 - 2x - 8\,dx \right|$

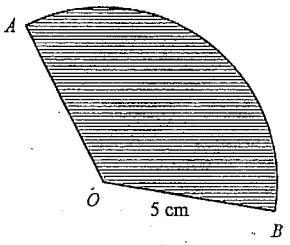
B. $Area = \left| \int_0^2 x^2 - 2x - 8\,dx \right| + \int_2^6 x^2 - 2x - 8\,dx$

C. $Area = \int_0^4 x^2 - 2x - 8\,dx + \left| \int_4^6 x^2 - 2x - 8\,dx \right|$

D. $Area = \left| \int_0^4 x^2 - 2x - 8\,dx \right| + \int_4^6 x^2 - 2x - 8\,dx$

Question 4

AOB is a sector of a circle, centre O and radius 5 cm. The sector has an area of $10\pi \text{cm}^2$.



Not to scale

What is the arc length of the sector?

A. 2π cm

B. 4π cm

C. 6π cm

D. 10π cm

Section II

37 marks
Attempt Questions 5 – 7
Allow about 64 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 5 – 7, your responses should include relevant mathematical reasoning and/or calculations.

Question 5 (14 marks) Use a SEPARATE writing booklet.

(a) What is the exact value of $\cos \frac{\pi}{6}$? 1

(b) Consider the series $252 + 261 + 270 + \ldots$

(i) Find the 10^{th} term. 2

(ii) Find the sum of the first 10 terms. 1

(iii) Find the first term greater than 1000. 2

(c) Consider the series $0.8 - 0.4 + 0.2 - \ldots$

(i) Explain why this series has a limiting sum. 1

(ii) What is the limiting sum for this series. 1

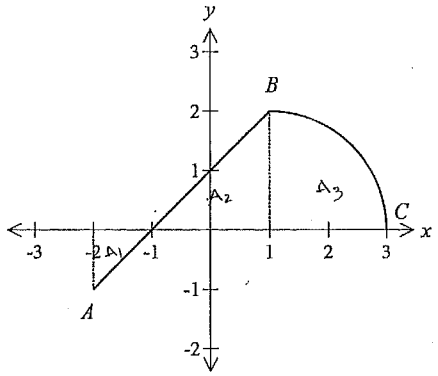
(d) (i) Evaluate $\int_0^3 4t + 3 \, dt$. 2

(ii) Find $\int \frac{1}{(5x-4)^3} \, dx$. 2

(e) Find the area of the region bounded by the x axis, the parabola $y = x^2 + 2$ and the lines $x = -1$ and $x = 2$. 2

Question 6 (11 marks) Use a SEPARATE writing booklet.

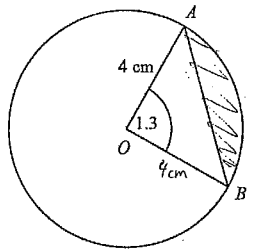
(a) The graph below shows $y = f(x)$ from $x = -2$ to $x = 3$. 2
AB is a straight line and BC is an arc of a circle.



Use this graph to evaluate $\int_{-2}^{3} f(x)\,dx$.

(b) The diagram shows a circle with centre O and radius 4 centimetres. 2
The points A and B lie on the circumference of the circle and $\angle AOB = 1.3$ radians.

Find, to the nearest square centimetre, the area of the minor segment bounded by the chord AB and the arc AB.



Not to scale

Question 6 continues on page 8

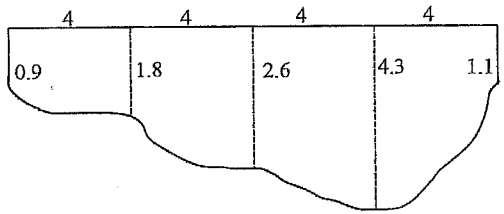
Question 6 (continued)

(c) If $\int_{0}^{a} (4 - 2x)\,dx = 4$, find the value of a. 2

(d) The area bounded by $y = x^2 + 1$ and $y = 2$ is rotated about the y axis. 3

Find the exact volume of the solid generated.

(e) A river is 16 metres wide. The depth of the river in metres has been measured at 4 metre intervals. The cross section is shown below: 2



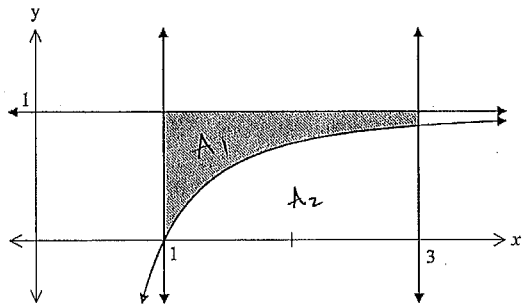
Use Simpsons Rule with 5 depth measurements to calculate the approximate area of the cross-section.

End of Question 6

Question 7 (12 marks) Use a SEPARATE writing booklet.

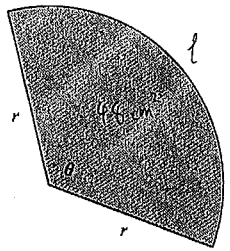
(a) Find $\int_0^3 \frac{x+1}{\sqrt{x}}\,dx$. Give your answer as a simplified surd. 3

(b) The diagram below shows a portion of the curve $y = 1 - \frac{1}{x^2}$. 3



Calculate the shaded area.

(c) The diagram below shows a sector with radius r and angle θ. 3
The total distance around the sector is 28 cm and the area of the sector is 48 cm^2.



Show that there are two possible sectors that satisfy these conditions.
Find the size of the radius of each.

Question 7 continues on page 10

Question 7 (continued)

(d) At the beginning of the year, Karl started his own ΣUMS club that would meet once a week at lunchtime. It was not very popular at first – Karl was the only person at the first meeting. However, each week, one more person joined the club. So in the second week there were 2 members, in the third week there were 3 members etc. Now, after the nth week, there are n members. 3

At each meeting, the ΣUMS club completed sums. At the first meeting Karl, on his own, could only manage to do $k+1$ sums, but each week each member would complete one more sum than in the previous week. So in the second week members completed $k+2$ sums each, in the third week, $k+3$ sums, etc.

Show that the total number of sums completed by the ΣUMS club after their nth meeting is given by:

$$\frac{1}{6}n(n+1)(3k+2n+1)$$

[You may assume, without proof, that $1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{1}{6}n(n+1)(2n+1)$]

End of paper

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Qu 1 $\sum_{r=1}^{3} 2^{r-1} = 2^0 + 2^1 + 2^2 = 7$ D

Qu 2 $\int 2x^3\,dx = \frac{2x^4}{4} + C = \frac{1}{2}x^4 + C$ C

Qu 3 $x^2 - 2x - 8 = (x-4)(x+2)$

Curve cuts x axis at -2 and 4 D

Intercept shown on graph must be $x = 4$

$\therefore$ Required area = D

incorrect factorisation cost some students this mark.

Qu 4 $\frac{1}{2}r^2\theta = 10\pi = \frac{1}{2} \times 25 \times \theta$

$\theta = \frac{10\pi}{\frac{1}{2}r^2} = \frac{4\pi}{5}$ B

$l = r\theta = 5 \times \frac{4\pi}{5} = 4\pi$

5 (a) $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$ ✓

These ratios must be learned

(b) $a = 252 \quad d = 9$ ✓

(i) $T_{10} = 252 + 9(9)$ ✓

$= 333$

(ii) $S_{10} = 5[2(252) + 9(9)]$

$= 2925$ ✓

(iii) $T_n \geq 1000$

$252 + 9(n-1) \geq 1000$ ✓

$9n \geq 757$

$n \geq 84\frac{1}{9}$

$n = 85$

$T_n = 252 + 84(9)$

$= 1008$ ✓

Must read the question carefully. The term was required.

(c)(i) $|r| < 1$ ✓

The condition must be clearly stated

(ii) $S_\infty = \frac{a}{1-r} = \frac{0.8}{1-(-0.5)} = \frac{8}{15}$ ✓

(d)(i) $\int_0^3 4t+3\,dt = \left[\frac{4t^2}{2} + 3t\right]_0^3$ ✓

$= 18 + 9 - 0$

$= 27$ ✓

Both parts well done.

(ii) $\int \frac{1}{(5x-4)^3}\,dx = \int (5x-4)^{-3}\,dx$

$= \frac{(5x-4)^{-2}}{-2 \times 5} + C$ ✓

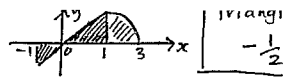
$= \frac{-1}{10(5x-4)^2} + C$ ✓

5 (e) $A = \int_{-1}^{2} x^2 + 2 \, dx = \left[\frac{x^3}{3} + 2x\right]_{-1}^{2}$ ✓

$= \left(\frac{8}{3} + 4\right) - \left(\frac{-1}{3} - 2\right)$

$= 9 \text{ u}^2$ ✓

Well done.



Triangle + Triangle + quarter circle

$-\frac{1}{2} + 2 + \pi$

$\int_{-2}^{3} f(x)\,dx$ means evaluate not find the area so the area part below the x-axis stays negative in sign in the calculation.

6 (a) $\int_{-2}^{3} f(x)\,dx$ = Trapezium OR $\frac{1}{2}(1+2) + \frac{1}{4} \times \pi \times 2^2$ ✓

$= \frac{3}{2} + \pi$ ✓

(b) Area $= \frac{1}{2} r^2 (\theta - \sin\theta)$

Area $= \frac{1}{2} \times 4^2 (1.3 - \sin 1.3)$ ✓

$= 2.6915\ldots \text{ cm}^2$ ✓

$= 3 \text{ cm}^2$ (nearest cm^2)

Use 1.3 radians in the calculation.

Do not try to convert the angle in any way

(c) $\int_0^a 4 - 2x \, dx = 4$

$\left[4x - x^2\right]_0^a = 4$ ✓

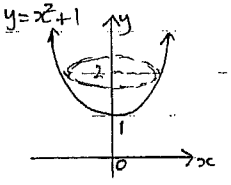
$4a - a^2 = 4$

$a^2 - 4a + 4 = 0$

$(a-2)^2 = 0$

$a = 2$ ✓

Rearrange this quadratic to make the squared term a positive coefficient then it's easy to factorise and solve

(d) 

Volume about the y-axis

$V = \pi \int x^2 \, dy$ ✓

$= \pi \int_1^2 y - 1 \, dy$

$= \pi \left[\frac{y^2}{2} - y\right]_1^2$ ✓

$= \pi \left[(2 - 2) - \left(\frac{1}{2} - 1\right)\right]$

$= \frac{\pi}{2} \text{ u}^3$ ✓

Make x or x^2 the subject

$y = x^2 + 1$

$x^2 = y - 1$

Use y values on the integral sign $y=1$ & $y=2$

This is a very easy question when rotated the right way.

Make sure you read the question very carefully and know the rule for the specified axis. Draw a clear diagram to help sort out your logic.

(e)

x	0	4	8	12	16	
y	0.9	1.8	2.6	4.3	1.1	
fact	1	4	2	4	1	Σ
prod	0.9	7.2	5.2	17.2	1.1	31.6

✓ $A \doteq \frac{h}{3} \times \Sigma$

$\doteq \frac{4}{3} \times 31.6$ ✓

$\doteq 42\frac{2}{15}$

or $\frac{4}{3}\left[0.9 + 1.1 + 4(1.8 + 4.3) + 2(2.6)\right]$

Well done. Just a few calculation errors that were minor.

Question 7

(a) $\int_0^3 \frac{x+1}{\sqrt{x}}\,dx = \int_0^3 \frac{x}{\sqrt{x}} + \frac{1}{\sqrt{x}}\,dx$ ✓

$= \int_0^3 x^{\frac{1}{2}} + x^{-\frac{1}{2}}\,dx$

$= \left[\frac{2x^{\frac{3}{2}}}{3} + 2x^{\frac{1}{2}}\right]_0^3$ ✓

$= \frac{2}{3}(3)^{\frac{3}{2}} + 2(3)^{\frac{1}{2}}$

$= \frac{2\sqrt{27} + 2\sqrt{3}}{3}$

$= \frac{2 \times 3\sqrt{3} + 2\sqrt{3}}{3}$

$= 4\sqrt{3}$

Generally well done, however too many girls did not simplify the expression properly first.

(b) shaded $= 2 \times 1 - \int_1^3 (1 - x^{-2})\,dx$

$= 2 - \left[x + x^{-1}\right]_1^3$ ✓

$= 2 - \left[(3 + \frac{1}{3}) - (1 + 1)\right]$ ✓

$= 2 - (1\frac{1}{3})$

$= \frac{2}{3}$ ✓

Some girls found the wrong area, or did not subtract from the rectangle!

or

$A = \int_1^3 1 - (1 - x^{-2})\,dx = \int_1^3 x^{-2}\,dx$ ✓

$= \left[\frac{x^{-1}}{-1}\right]_1^3 = -\frac{1}{3} - (-1) = \frac{2}{3}$ ✓

(c) $P = r + r + r\theta = 28 \cdots (i)$

$A = \frac{1}{2}r^2\theta = 48 \cdots (ii)$ ✓

From (ii) $\theta = \frac{96}{r^2}$

Sub in (i) $2r + r \cdot \frac{96}{r^2} = 28$ ✓

$2r^3 + 96r = 28r^2$

$2r(r^2 - 14r + 48) = 0$

$2r(r-6)(r-8) = 0$

$\therefore r = 0$ invalid

$r = 6$ cm ✓

$r = 8$ cm

Too many students did not recognize that they had simultaneous equations, in quadratic form. Expect quadratics!

(d)

Week	1	2	3	4	...	n
# People	1	2	3	4		n
# sums/person	k+1	k+2	k+3	k+4	...	k+n
# total sums	k+1	2(k+2)	3(k+3)	4(k+4)	...	n(k+n)

Total # of sums

$= k+1 + 2(k+2) + 3(k+3) + \cdots n(k+n)$

$= k+1 + 2k + 2^2 + 3k + 3^2 + \cdots nk + n^2$

$= k + 2k + 3k + \cdots nk + 1 + 2^2 + 3^2 + \cdots n^2$

$= k(1 + 2 + 3 + \cdots n) + \frac{1}{6}(n)(n+1)(2n+1)$

$= k\left(\frac{n}{2}\right)(1+n) + \frac{1}{6}(n)(n+1)(2n+1)$

$= \frac{1}{6}n(n+1)(3k + 2n + 1)$

Very few girls recognize that these were two sequences in this expression ✓