Natasha

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|---|---|---|---|---|

Centre Number

| 1 | 5 | 3 | 7 | 2 | 6 | 4 | 8 | |
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Student Number

2.

SCEGGS Darlinghurst

**2005**
**Higher School Certificate**
**Trial Examination**

# Mathematics Extension 1

This is a TRIAL PAPER only and does not necessarily reflect the content or format of the Higher School Certificate Examination for this subject.

**General Instructions**

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

**Total marks – 84**

- Attempt Questions 1–7
- All questions are of equal value

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**Total marks – 84**
**Attempt Questions 1–7**
**All questions are of equal value**

Answer each question in a SEPARATE writing booklet

Marks

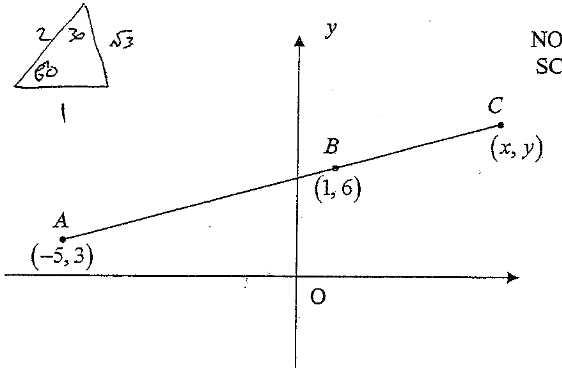
**Question 1** (12 marks)

(a) Find $\frac{d}{dx}\left(\tan^{-1} 2x\right)$ **2**

(b) Find the obtuse angle between the two straight lines $y = x - 1$ and $2x + y = 1$. Answer correct to the nearest degree. **2**

(c) Evaluate $\int_0^{\frac{\pi}{6}} \sec 2x \tan 2x \, dx$ **3**

(d) **2**




Given that $AC : CB = 5 : 2$, find the co-ordinates of the point $C$.

(e) Use the substitution $u = x^2 - 6x + 7$ to find the exact value of **3**

$$\int_0^1 \frac{x-3}{x^2-6x+7}\, dx .$$



**Question 2** (12 marks) Begin a NEW writing booklet

Marks

(a) How many different positive integers can be formed from the digits 1, 3, 5, 7 if a digit cannot be used more than once in a particular number? **2**

(b) Consider $P(x) = 2 + 3x - 3x^2 - 2x^3$

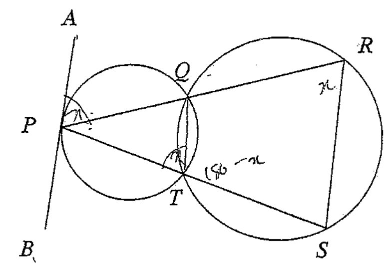
(i) Prove $P(1) = 0$. **1**

(ii) Solve $P(x) \le 0$. **3**

(c) Find the term independent of $x$ in the expansion of **3**

$$\left(2x^2 - \frac{1}{2x}\right)^6$$

(d) **3**

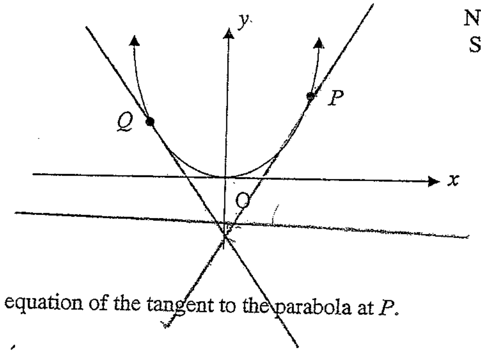



The two circles intersect at $Q$ and $T$.
$AB$ is a tangent to the smaller circle at $P$.
$PQR$ and $PTS$ are straight lines.

Prove that the tangent at $P$ is parallel to the chord $RS$.

**Question 3** (12 marks) Begin a NEW writing booklet **Marks**

(a) Two points $P\left(2ap, ap^2\right)$ and $Q\left(2aq, aq^2\right)$ lie on the parabola $x^2 = 4ay$ with vertex $(0, 0)$ as shown below.




(i) Find the equation of the tangent to the parabola at $P$. **1**

(ii) Hence, prove that the tangents at $P$ and $Q$ intersect at the point $R\ \left(a(p+q), apq\right)$. **3**

(iii) State the condition that the tangents intersect on the directrix. **1**

(b) (i) Prove that there is a solution to the equation $2\sin\frac{\pi}{2}x - 2x + 3 = 0$ between $x = 1.5$ and $x = 2$ where $x$ is measured in radians. **2**

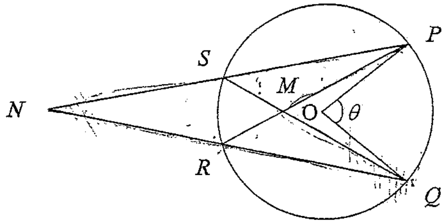
(ii) Using an initial approximation of $x = 1.75$ and one application of Newton's Method, find a better approximation correct to 4 significant figures. **2**

(c) Find the exact volume formed when the region bounded by $y = 1 + \sin\frac{x}{2}$, the $x$ axis and $x = 0$ to $x = \frac{\pi}{2}$ is rotated about the $x$ axis. **3**



**Question 4** (12 marks) Begin a NEW writing booklet **Marks**

(a)




NOT TO SCALE

In the diagram, $P$, $Q$, $R$ and $S$ are points on the circle centre O. $\angle POQ = \theta$
The straight lines $PS$ and $QR$ intersect at $N$ and $PR$ and $QS$ intersect at $M$.

(i) Prove $\angle PRN = 180° - \frac{1}{2}\theta$ **1**

(ii) Prove $\angle PMQ + \angle PNQ = \theta$ **2**

(b) (i) Express $\sin 2x - 2\cos 2x$ in the form $A\sin(2x - \alpha)$ for $A > 0$ and $0 \le \alpha \le 90°$. **2**

(ii) Hence solve $\sin 2x - 2\cos 2x = 1$ for $0 \le x \le 180°$. **2**

(c) Consider $f(x) = \dfrac{2x}{x-1}$ :

(i) Sketch the hyperbola $y = f(x)$ showing important features. **2**

(ii) Find $y = f^{-1}(x)$. **1**

(iii) State the domain and range of $y = f^{-1}(x)$. **2**

Marks

**Question 5** (12 marks) Begin a NEW writing booklet

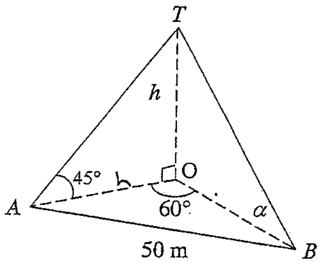
(a) Prove that $\int_0^1 \frac{dx}{\sqrt{4-3x^2}} = \frac{\pi}{3\sqrt{3}}$. **3**

(b) Use Mathematical Induction to prove that **4**

$$\frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \dots + \frac{n}{2^n} = 2 - \frac{n+2}{2^n}$$

for an integer $n > 0$.

(c)




NOT TO SCALE

In the diagram, the points $A$, $B$ and O are in the same horizontal plane.
$A$ and $B$ are 50m apart and $\angle AOB = 60°$. $OT$ is a vertical tower of height $h$ metres.
The angles of elevation of $T$ from $A$ and $B$ respectively are $45°$ and $\alpha$. ($\alpha$ is acute.)

(i) Prove $AO = h$. **1**

(ii) Prove $h^2 \cot^2 \alpha - h^2 \cot \alpha + h^2 = 50^2$ **2**

(iii) Given the tower is 30 m high, find the angle $\alpha$ correct to the nearest degree. **2**

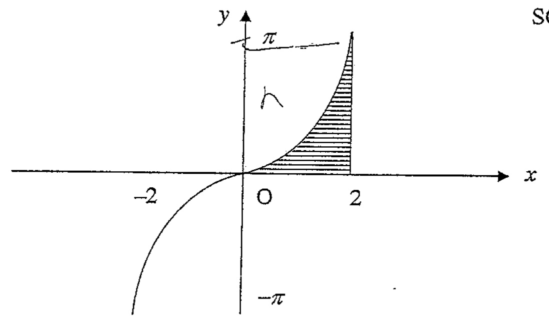


Marks

**Question 6** (12 marks) Begin a NEW writing booklet

(a) The curve shown is $y = 2\sin^{-1} Bx$.

NOT TO SCALE




(i) Evaluate $B$. **1**

(ii) Find the exact value of the shaded area. **3**

(b) A ball is thrown at ground level with an initial velocity $Vms^{-1}$ and an angle of projection of $\alpha$ with the horizontal.

You may assume the equations of motion.

$\ddot{x} = 0$ $\qquad \ddot{y} = -10$

$\dot{x} = V \cos\alpha$ $\qquad \dot{y} = -10t + V\sin\alpha$

$x = Vt\cos\alpha$ $\qquad y = -5t^2 + Vt\sin\alpha$

(i) Prove that the horizontal range is $\frac{V^2}{10}\sin 2\alpha$. **2**

(ii) Explain why the maximum horizontal range occurs when $\alpha = 45°$. **1**

(iii) Find the maximum horizontal range where $V = 30ms^{-1}$. **1**

(iv) How much further can the ball be thrown under these conditions if it is projected from a platform 10m above the ground? **4**

**Question 7** (12 marks) Begin a NEW writing booklet | **Marks**

(a) Use the substitution $u = \tan x$ to evaluate **4**

$$\int_0^{\frac{\pi}{4}} \frac{dx}{9\cos^2 x + 25\sin^2 x}$$

(b) Given $(1+x)^n = \binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \cdots + \binom{n}{n}x^n$,

prove:

(i) $\binom{n}{1} + 3\binom{n}{2} + 9\binom{n}{3} + \cdots + 3^{n-1}\binom{n}{n} = \frac{1}{3}\left(2^{2n}-1\right)$ **2**

where $n$ is a positive integer.

(ii) $\binom{n}{0} + \binom{n}{2} + \binom{n}{4} + \cdots + \binom{n}{n} = 2^{n-1}$ **2**

where $n$ is an even integer.

(c) A class of 20 students consists of 12 girls and 8 boys. For a discussion session, 4 students are chosen at random to form a committee. The committee then chooses 1 of these 4 students at random to be the chairman.

How many of these committees:

(i) have 4 female members? **1**

(ii) have at least 1 male member? **1**

(iii) have a male chairman? **2**

**End of Paper**

Extension 1 Trial 2005.

**1 a)** $\frac{d}{dx}(\tan^{-1} 2x) = \frac{2}{1+4x^2}$

Look at formula sheet

**b)** $m_1 = 1, \quad m_2 = -2$

if $\theta$ acute, $\tan\theta = \left| \frac{1+2}{1-2} \right| = 3.$

$\theta = 72°$ (nearest degree)

$\therefore$ obtuse angle is $108°$ (nearest degree)

Careful of the sign in the formula.

**c)** $\int_0^{\frac{\pi}{6}} \sec 2x \tan 2x . dx = \frac{1}{2}\left[\sec 2x\right]_0^{\frac{\pi}{6}}$

$= \frac{1}{2}\left(\sec\frac{\pi}{3} - \sec 0\right)$

$= \frac{1}{2}(2-1) = \frac{1}{2}.$

Look at formula sheet

**d)** External division $5 : -2$.

$x = \frac{-5\times -2 + 1\times 5}{3} \qquad y = \frac{3\times -2 + 6\times 5}{3}$

$= 5 \qquad\qquad = 8$

$C$ is $(5, 8)$

Well done.

**e)** $\frac{du}{dx} = 2x - 6, \quad$ if $x = 1, u = 2$

$x = 0, u = 7.$

$\therefore \int_0^1 \frac{x-3}{x^2-6x+7} . dx = \int_7^2 \frac{x-3}{u} . \frac{du}{2x-6}$

$= \frac{1}{2}\int_7^2 \frac{du}{u}$

$= \frac{1}{2}\left[\log_e u\right]_7^2$

$= \frac{1}{2}\log_e \frac{2}{7}$

Watch where new limits go.

$\int \frac{1}{u} du = \log_e u + c$

**2 a)** One digit numbers $= 4$

two " " $= 4\times 3 = 12$

three " " $= 4\times 3\times 2 = 24$

four " " $= 4\times 3\times 2\times 1 = 24$

$\therefore$ Total number $= 64$.

Numbers have a max$^m$ of 4 digits So can have 1, 2, 3 or 4 digits.

**b) (i)** $P(1) = 2 + 3 - 3 - 2 = 0$

Well done.

**(ii)**

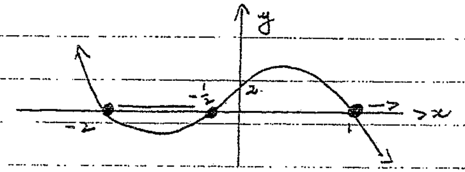
$$\begin{array}{r} -2x^2 - 5x - 2 \\ x - 1 \overline{\big) -2x^3 - 3x^2 + 3x + 2} \\ -2x^3 + 2x^2 \\ \hline -5x^2 + 3x \\ -5x^2 + 5x \\ \hline -2x + 2 \\ -2x + 2 \\ \hline \end{array}$$

$\therefore P(x) = (x-1)(-2x^2 - 5x - 2)$

$= -(x-1)(2x^2 + 5x + 2)$

$= -(x-1)(2x+1)(x+2)$

A 'natural' follow on from part (i)




Solution: $-2 \le x \le -\frac{1}{2}$ and $x \ge 1$

A clue that this is shape of the graph is that the y intercept is 2. Check your zeroes are correct.

**c)** $T_{k+1} = \binom{6}{k}(2x^2)^{6-k}\left(-\frac{1}{2}x^{-1}\right)^{k}$

$= \binom{6}{k} 2^{6-k} \left(-\frac{1}{2}\right)^{k} x^{12-2k} x^{-k}$

$\therefore$ if independent of $x$, $12 - 3k = 0$

$k = 4.$

Term is $\binom{6}{4} 2^2 \times \left(-\frac{1}{2}\right)^4 = \frac{15}{4}$

Know the formula

**d)** Construction: Join QT

Proof: $\angle QPA = \angle PTQ$ ($\angle$ between tangent and chord $= \angle$ in alternate segment)

$\angle PTQ = \angle QRS$ (ext. $\angle$ of cyclic quad. = interior opposite $\angle$)

$\therefore \angle APQ = \angle QRS$

But these angles are in the alternate position with

Well done.

Careful: $\angle APR \neq \angle RSP$.

3) a) (i) $y = \frac{x^2}{4a}$

$\frac{dy}{dx} = \frac{2x}{4a}$

at P, gradient is $\frac{4ap}{4a} = p$.

$\therefore$ equation of tangent is: $y - ap^2 = p(x - 2ap)$

$= px - 2ap^2$.

$\therefore$ $px - y - ap^2 = 0$ is tangent at P.

*must find the gradient at P!*

(ii) tangent at Q: $qx - y - aq^2 = 0$

Subtracting: $px - qx = ap^2 - aq^2$

$(p-q)x = a(p-q)(p+q)$

$x = a(p+q)$

$y = pa(p+q) - ap^2$

$= ap^2 + apq - ap^2$

$= apq$

$\therefore$ R is $(a(p+q), apq)$

*no need to do this again, just put q for p.*

iii) Directrix is $y = -a$

$\therefore apq = -a$

condition is $pq = -1$

*This is the condition. There are other properties*

)(i) Let $P(x) = 2\sin\frac{\pi}{2}x - 2x + 3$

*Radians!*

$P(1.5) = 2\sin\frac{\pi}{2}\times 1.5 - 3 + 3 = 1.4142\ldots$

$P(2) = 2\sin\frac{\pi}{2}\times 2 - 4 + 3 = -1$

since the sign has changed and the function is continuous there is a solution between $x = 1.5$ and $2$

*must say continuous*

(ii) $P(1.75) = 2\sin\frac{\pi}{2}\times 1.75 - 3.5 + 3 = 0.265366\ldots$

$P'(x) = 2\times\frac{\pi}{2}\cos\frac{\pi}{2}x - 2$

$P'(1.75) = \pi\cos\frac{\pi}{2}\times 1.75 - 2 = -4.90245\ldots$

*differentiate carefully.*

new estimate $= 1.75 - \frac{0.2653\ldots}{-4.902}$

$= 1.804$ (4 s.f.)

c) $V = \pi\int_0^{\frac{\pi}{2}} \left(1 + \sin\frac{x}{2}\right)^2 dx$.

$= \pi\int_0^{\frac{\pi}{2}} 1 + 2\sin\frac{x}{2} + \sin^2\frac{x}{2}\, dx$

$= \pi\int_0^{\frac{\pi}{2}} 1 + 2\sin\frac{x}{2} + \frac{1}{2} - \frac{1}{2}\cos x\, dx$

$= \pi\left[\frac{3x}{2} - 4\cos\frac{x}{2} - \frac{1}{2}\sin x\right]_0^{\frac{\pi}{2}}$

$= \pi\left[\frac{3\pi}{4} - 4\cos\frac{\pi}{4} - \frac{1}{2}\sin\frac{\pi}{2} - 0 + 4\cos 0 + \frac{1}{2}\sin 0\right]$

$= \pi\left[\frac{3\pi}{4} - \frac{4}{\sqrt{2}} - \frac{1}{2} + 4\right]$

Volume $= \pi\left[\frac{3\pi}{4} - 2\sqrt{2} + \frac{7}{2}\right] u^3$.

*Watch $y^2$ for volume.*

*This question required care and attention to small details.*

④ (i) $\angle PRQ = \frac{1}{2}\angle POQ$ (angle at centre is twice the angle at the circumference subtended by the same arc)

$= \frac{1}{2}\theta$.

$\angle PRN = 180° - \frac{1}{2}\theta$ (straight angle $\angle NRQ$ is 180°)

*well done.*

(ii) Similarly $\angle NSM = 180° - \frac{1}{2}\theta$.

$\angle PNQ + \angle SMR + \angle PRN + \angle NSM = 360°$ (angle sum of quad. is 360°)

$\therefore \angle PNQ + \angle SMR + 360° - \theta = 360°$

$\therefore \angle PNQ + \angle SMR = \theta$

but $\angle SMR = \angle PMQ$ (vertically opp. angles are =)

$\therefore \angle PNQ + \angle SMR = \theta$.

*Don't give up. Try to find more information so marks can be allocated.*

b) (i) $A\sin(2x - \alpha) = A\sin 2x\cos\alpha - A\cos 2x\sin\alpha$

$= \sin 2x - 2\cos 2x$

$A = \sqrt{4+1} = \sqrt{5} \quad (A>0)$

$\cos\alpha = \frac{1}{\sqrt{5}}$

$\alpha = 63°26'$ (nearest minute) ($\alpha$ acute)

$\therefore \sin 2x - 2\cos 2x = \sqrt{5}\sin(2x - 63°26')$

*Well done.*

(ii) $\therefore \sin(2x - 63°26') = \frac{1}{\sqrt{5}}$

*Well done.*

Acute angle $= 26°34'$, 1st 2nd quadrants

$\therefore 2x - 63°26' = 26°34'$, or $153°26'$

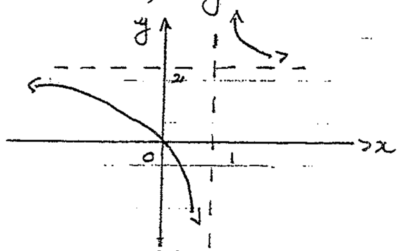
$2x = 90°, \ 216°52'$

$x = 45°, \ 108°26'$. (nearest minute)

c) (i) $f(x) = \frac{2x}{x-1} = \frac{2x-2+2}{x-1}$

$y = 2 + \frac{2}{x-1}$

asymptotes : $x=1, \ y=2$.




*Problems with horizontal asymptote.*

*Curve passes through (0,0)*

(ii) $y = \frac{2x}{x-1}$

$\therefore x = \frac{2y}{y-1}$

$xy - x = 2y$

$y(x-2) = x$

$y = \frac{x}{x-2}$

$\therefore f^{-1}(x) = \frac{x}{x-2}$

*(ii) Must rewrite until you have y = ...*

(iii) Domain: all real $x$ except $x=2$

Range: all real $y$ except $y=1$

*(iii) the reverse of (i) which gives a clue to the graph of (i).*

(5) a) $\int_0^1 \frac{dx}{\sqrt{4-3x^2}} = \frac{1}{\sqrt{3}}\int_0^1 \frac{dx}{\sqrt{\frac{4}{3}-x^2}}$

$= \frac{1}{\sqrt{3}}\left[\sin^{-1}\frac{\sqrt{3}}{2}x\right]_0^1$

$= \frac{1}{\sqrt{3}}\left(\sin^{-1}\frac{\sqrt{3}}{2} - \sin^{-1}0\right)$

$= \frac{1}{\sqrt{3}} \times \left(\frac{\pi}{3} - 0\right)$

$= \frac{\pi}{3\sqrt{3}}$

*Well done.*

b) Consider $n=1$,

LHS $= \frac{1}{2}$ RHS $= 2 - \frac{3}{2} = \frac{1}{2}$

$\therefore$ true for $n=1$

Assume true for $n=k$

i.e. Assume $\frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \cdots + \frac{k}{2^k} = 2 - \frac{k+2}{2^k}$

Consider $n=k+1$

LHS $= \frac{1}{2} + \frac{2}{2^2} + \cdots + \frac{k}{2^k} + \frac{k+1}{2^{k+1}}$

$= 2 - \frac{k+2}{2^k} + \frac{k+1}{2^{k+1}}$

$= 2 - \frac{2(k+2) - k - 1}{2^{k+1}}$ (note signs here!)

$= 2 - \frac{2k+4-k-1}{2^{k+1}}$

$= 2 - \frac{k+3}{2^{k+1}} = 2 - \frac{(k+1)+2}{2^{k+1}}$

$=$ RHS if $n=k+1$

$\therefore$ If true for $n=k$, the statement is true for $n=k+1$. But it is true for $n=1$ and thus is true for $n=2, 3, \ldots$ etc. i.e. true for all $n$ integer $>0$.

*Too many errors with negative signs. Knowing the answer needed then trying to achieve it by dubious means is really obvious.*

c) (i) $\tan 45^\circ = \frac{h}{AO} = 1$

$\therefore AO = h$

✓ well done.

(ii) $\tan\alpha = \frac{h}{OB}$

$\therefore OB = h\cot\alpha$.

using cosine rule:

$AB^2 = AO^2 + OB^2 - 2\times AO\times OB\times \cos\angle AOB$

$50^2 = h^2 + h^2\cot^2\alpha - 2\times h\times h\cot\alpha\times\cos 60^\circ$

$\cos 60^\circ = \frac{1}{2}$

$\therefore h^2\cot^2\alpha - h^2\cot\alpha + h^2 = 50^2$

well done.

(iii) $900(\cot^2\alpha - \cot\alpha + 1) = 2500$

$\therefore 900\cot^2\alpha - 900\cot\alpha - 1600 = 0$

$9\cot^2\alpha - 9\cot\alpha - 16 = 0$

$\cot\alpha = \frac{9 \pm \sqrt{81+576}}{18}$

$= \frac{9 \pm 25.632..}{18}$

$\cot\alpha = \frac{34.632..}{18}$ ($\alpha$ is acute)

$\alpha = 27^\circ$ (nearest degree)

Don't be afraid of the quadratic formula.

Explain answers that are discarded.

6 a) (i) $B = \frac{1}{2}$

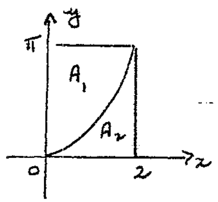
Well done.

(ii) $y = 2\sin^{-1}\frac{x}{2}$

$\frac{y}{2} = \sin^{-1}\frac{x}{2}$

$\therefore \frac{x}{2} = \sin\frac{y}{2}$

$x = 2\sin\frac{y}{2}$




$A_1 = \int_0^{\pi} 2\sin\frac{y}{2}\,dy$

$= 2\left[-2\cos\frac{y}{2}\right]_0^{\pi}$

Cannot integrate $\sin^{-1}x$ at Extension 1 level. So you must find area between curve and y axis. Then subtract from the area of rectangle

$A_1 = -4\left(\cos\frac{\pi}{2} - \cos 0\right)$

$= 4u^2$.

$\therefore$ Shaded area $= (2\pi - 4)u^2$

b) (i) Horizontal range is $x$ when $y = 0$ for the second time.

Well done.

$t(-5t + V\sin\alpha) = 0$

$t = 0$ or $\frac{V\sin\alpha}{5}$ ($t = 0$ is initial value)

horizontal range $= \frac{V\cos\alpha \times V\sin\alpha}{5}$

$= \frac{V^2 \times 2\sin\alpha\cos\alpha}{10}$

$= \frac{V^2\sin 2\alpha}{10}$

(ii) maximum value of $\sin 2\alpha$ is 1

$\therefore$ maximum value for horizontal range is $\frac{V^2}{10}$ which occurs when $2\alpha = 90^\circ$

i.e. when $\alpha = 45^\circ$. ($\alpha$ acute)

Explain carefully.

(iii) maximum range is $\frac{30^2}{10} = 90\,m$

Well done.

(iv) If projected 10 m above ground; $V = 30$, $\alpha = 45^\circ$

$\ddot{x} = 0$ $\quad\quad \ddot{y} = -10$

$\dot{x} = \frac{30}{\sqrt{2}}$ $\quad\quad \dot{y} = -10t + \frac{30}{\sqrt{2}}$

$x = \frac{30}{\sqrt{2}}t$ $\quad\quad y = -5t^2 + \frac{30}{\sqrt{2}}t + 10$

if $y = 0$, $\quad 5t^2 - \frac{30t}{\sqrt{2}} - 10 = 0$

$t^2 - \frac{6t}{\sqrt{2}} - 2 = 0$

$t = \frac{\frac{6}{\sqrt{2}} \pm \sqrt{18+8}}{}$

Quadratic formula again

$t = 4.6708301$ or $-0.4281\ldots$

but $t \geq 0$, $x = \frac{30}{\sqrt{2}} \times 4.6708301$

$= 99.08\ldots$ m.

$\therefore$ can be thrown approximately 9m further.

*Explain discarded answer.*

⑦ a) $u = \tan x$    if $x = \frac{\pi}{4}$, $u = 1$

$\frac{du}{dx} = \sec^2 x$    $x = 0$, $u = 0$.

*Not well done*

$dx = \cos^2 x \, du$.

$$\int_0^{\frac{\pi}{4}} \frac{dx}{9\cos^2 x + 25\sin^2 x} = \int_0^1 \frac{\cos^2 x \, du}{9\cos^2 x + 25\sin^2 x} \quad \begin{matrix} \div \cos^2 x \\ \div \cos^2 x \end{matrix}$$

*This line was good. You know $u = \tan x$ so you must divide top and bottom by $\cos^2 x$ (not hard from there)*

$$= \int_0^1 \frac{du}{9 + 25\tan^2 x}$$

$$= \int_0^1 \frac{dx}{9 + 25u^2}$$

$$= \frac{1}{25}\int_0^1 \frac{du}{\frac{9}{25} + u^2}$$

$$= \frac{1}{25} \times \frac{5}{3}\left[\tan^{-1}\frac{5u}{3}\right]_0^1$$

$$= \frac{1}{15}\tan^{-1}\frac{5}{3}$$

b)(i) $(1+x)^n = \binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \cdots + \binom{n}{n}x^n$

if $x = 3$, $4^n = \binom{n}{0} + \binom{n}{1}.3 + \binom{n}{2}3^2 + \cdots + \binom{n}{n}3^n$

*The increasing powers of 3 mean $x = 3$.*

now $\binom{n}{0} = 1$, $\therefore 3\binom{n}{1} + 3^2\binom{n}{2} + 3^3\binom{n}{3} + \cdots + 3^n\binom{n}{n} = 4^n - 1$

*$4^n = 2^{2n}$ !*

*$\binom{n}{0} = 1$*

dividing by 3 and noting $2^{2n} = 4^n$

$$\binom{n}{1} + 3\binom{n}{2} + 9\binom{n}{3} + \cdots + 3^{n-1}\binom{n}{n} = \frac{1}{3}\left(2^{2n} - 1\right)$$

(ii) if $x = 1$, $2^n = \binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \cdots + \binom{n}{n}$

if $x = -1$, $0 = \binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \cdots + (-1)^n\binom{n}{n}$

*every second term means substitution of $x = 1$ and $x =$ and adding*

if $n$ is even $(-1)^n$ is 1

Adding: $2\binom{n}{0} + 2\binom{n}{2} + 2\binom{n}{4} + \cdots + 2\binom{n}{n} = 2^n$

dividing by 2, $\binom{n}{0} + \binom{n}{2} + \binom{n}{4} + \cdots + \binom{n}{n} = 2^{n-1}$.

c) (i) $\binom{12}{4} = 495$

*Committees are usually combinations*

(ii) $\binom{20}{4} - \binom{12}{4} = 4845 - 495$

$= 4350$

(iii) Committee of 4 males must have a male chairman: $\binom{8}{4} = 70$.

Committee of 3 males 1 female has $\frac{3}{4}$ chance of a male chairman $\binom{8}{3} \times \binom{12}{1} \times \frac{3}{4} = 504$.

Committee of 2 males 2 females has $\frac{1}{2}$ chance of a male chairman:

$\binom{8}{2}\binom{12}{2} \times \frac{1}{2} = 924$

Committee of 1 male 3 females has $\frac{1}{4}$ chance of a male chairman

$\binom{8}{1} \times \binom{12}{3} \times \frac{1}{4} = 440$

Total number $= 70 + 504 + 924 + 440$

$= 1938$.