Centre Number

Student Number

SCEGGS Darlinghurst

2006

HIGHER SCHOOL CERTIFICATE
TRIAL EXAMINATION

Mathematics

This is a TRIAL PAPER only and does not necessarily reflect the content or format of the Higher School Certificate Examination for this subject.

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Diagrams should be drawn in pencil
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question
- Start each question in a new booklet

Total marks – 120

- Attempt Questions 1–10
- All questions are of equal value

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

Total marks – 120
Attempt Questions 1–10
All questions are of equal value

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Marks

Question 1 (12 marks) Use a SEPARATE writing booklet.

(a) Find the value of $\dfrac{3.5^2}{\sqrt{9.4\times 3.7}}$ correct to two significant figures. **2**

(b) Factorise fully: $5x+5y+x^3+y^3$ **2**

(c) Express the recurring decimal $0.\dot{2}\dot{4}$ as a fraction in its lowest terms. **2**

(d) Find p and q if $\dfrac{4+\sqrt{2}}{2-\sqrt{2}} = p+q\sqrt{2}$ **2**

(e) Find the values of x for which **2**

$$x^2-4x-12<0$$

(f) Prove that $f(x)=xe^{x^2}$ is an odd function. **2**

Marks

Question 2 (12 marks) Use a SEPARATE writing booklet.

(a) If $f(x)=2^{x-4}$, find:

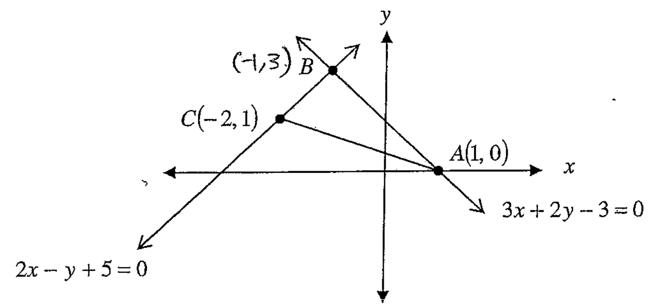
(i) $f(1)$ **1**

(ii) x if $f(x)=1$ **2**

(b) Simplify fully: $\dfrac{3\sec^2 x-3}{12\tan x}$ **2**

(c) In the diagram below, the lines $2x-y+5=0$ and $3x+2y-3=0$ intersect at the point B.

The point A has co-ordinates $(1, 0)$ and the point C has co-ordinates $(-2, 1)$.



(i) Show that the line AC has the equation $x+3y-1=0$. **2**

(ii) Find the co-ordinates of B. **2**

(iii) Find the perpendicular distance from point B to AC. Leave your answer as a surd. **1**

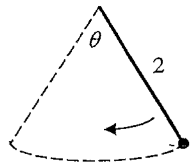
(iv) Find the length of AC. Leave your answer as a surd. **1**

(v) Hence find the area of ΔABC. **1**

Marks

Question 3 (12 marks) Use a SEPARATE writing booklet.

(a) A pendulum of length 2 metres, sweeps out an area of $\frac{\pi}{4}\,\text{m}^2$.



(i) Find the exact value of θ in radians. **1**

(ii) Hence find the length of the arc traced out by the pendulum. **1**

(b) Differentiate:

(i) $y = x^3 + 4x + \dfrac{1}{x^2}$ **1**

(ii) $y = \sqrt{x^2 + 4}$. **2**

(c) Find:

(i) $\displaystyle\int \cos(3x+2)\; dx$ **2**

(ii) $\displaystyle\int \frac{1}{2x-5}\; dx$ **2**

(d) Find the gradient of the normal to the curve $y = \log_e(\sin x)$ at the point $x = \dfrac{\pi}{4}$. **3**

Marks

Question 4 (12 marks) Use a SEPARATE writing booklet.

(a) (i) On a number plane sketch the graph of $y = |x-1|$ **1**

(ii) Solve the inequality $|x-1| < 1$ **1**

(iii) Hence, or otherwise, evaluate $\displaystyle\int_0^2 |x-1|\; dx$ **2**

(b) The common ratio r of a geometric progression satisfies the quadratic equation

$$2r^2 - 3r - 2 = 0$$

(i) Solve this equation for r. **1**

(ii) The sum to infinity of the same progression is 6. **2**

Explain why, in this case, r can only take on one value.

Hence state the common ratio r.

(iii) Show that the fifth term of this progression is $\dfrac{9}{16}$. **2**

(c) A parabola's equation is given by $8y = x^2 - 2x + 17$.

(i) Find the focal length. **1**

(ii) Determine the co-ordinates of the vertex. **1**

(iii) Find the co-ordinates of the focus. **1**

Question 5 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) In Australia, 45% of people have blood group O, 40% have blood group A and the remainder are neither.

Two people are chosen at random.

Find the probability that:

(i) both people have blood group A. **1**

(ii) one person has blood group A and one has blood group O. **2**

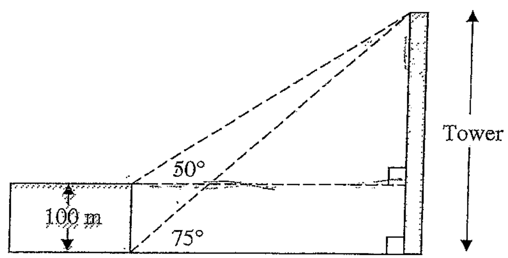
(iii) at least one person has blood group A. **2**

Solve for x: **3**

$$\log_3(x+7)-\log_3(x-1)=2$$

(c) The angles of elevation of the top of a tower from the top and bottom of a building 100 metres high are $50°$ and $75°$ respectively. **4**

Find the height of the tower, correct to the nearest metre.



Question 6 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Use Simpson's Rule with 5 function values to estimate $\int_1^3 \log_e x \; dx$. **3**

Answer correct to 2 decimal places.

(b) Consider the curve $f(x) = -xe^{2x}$.

(i) Show that $f^1(x) = -e^{2x}(2x+1)$. **1**

(ii) Find the co-ordinates of the only stationary point and determine its nature. **3**

(iii) Find the co-ordinates of any points of inflexion. **2**

(iv) What happens to $f(x)$ as $x \to -\infty$? **1**

(v) Sketch the curve $y = f(x)$, showing all important features. **2**

Question 7 (12 marks) Use a SEPARATE writing booklet. **Marks**

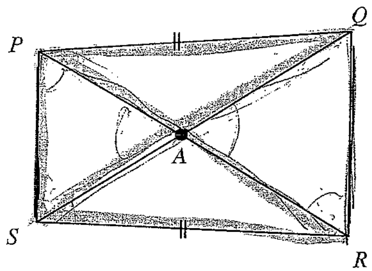
(a) The area bounded by the curve $y = x^2 + 3$, the y axis and the line $y = 4$ is rotated about the y axis. **4**

Find the exact volume of the solid formed.

(b) The roots of the equation $mx^2 - nx + 1 = 0$ are α and β. **2**

Show that $\frac{1}{\alpha} + \frac{1}{\beta} = n$.

(c) $PQRS$ is a quadrilateral with equal length diagonals that meet at A. Also, $PQ = SR$.



(i) Show that $\Delta PQS \equiv \Delta SRP$. **2**

(ii) Hence show that ΔPAS is isosceles. **1**

(iii) Hence explain why ΔQAR is also isosceles. **2**

(iv) Prove that $PS \parallel QR$. **1**

Question 8 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Louise borrows \$80 000 to start a business. The interest is calculated monthly at a rate of 2% per month. Louise intends to repay the loan with interest in 5 annual instalments of *\$M*, at the end of each year.

(i) Write an expression for A_{12} the amount Louise owes after 12 months, immediately after her first repayment. **1**

(ii) Show that **2**

$$A_{60} = 80000(1.02)^{60} - M\left(1 + 1.02^{12} + \ldots + 1.02^{48}\right)$$

(iii) Find the value of M correct to the nearest dollar. **2**

(b) Find a primitive function of 3^x. **2**

(c) A gardener found the probability that a planted passionfruit vine will eventually bear fruit was 0.26.

(i) If she planted 5 vines, what is the probability that no vines will bear fruit? **1**

(ii) If she planted n vines, what is the probability that no vines will bear fruit? **1**

(iii) How many vines must be planted to be at least 99% certain that at least one seedling will bear fruit? **3**

Marks

Question 9 (12 marks) Use a SEPARATE writing booklet.

(a) (i) Find the co-ordinates of the points of intersection of the curves $y = x^2 + 4$ and $y = x + 6$. **2**

(ii) Hence find the area bounded by these two curves. **3**

(b) Solve $2\sin^2 x + 5\sin x - 3 = 0$, $0 \le x \le 2\pi$. **3**

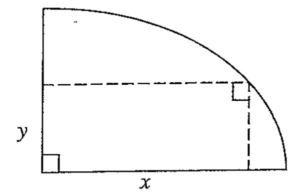
(c) (i) State the domain of the curve $y = \dfrac{x^2}{\log_e x}$. **1**

(ii) Find the exact value of the x co-ordinate where the tangent to the curve $y = \dfrac{x^2}{\log_e x}$ is horizontal. **3**

Marks

Question 10 (12 marks) Use a SEPARATE writing booklet.

(a) The logo for Livingstone's Lawns is to be made by inscribing a rectangle of maximum area inside a quadrant of fixed radius, r cm, as shown below.



The length and width of the rectangle are x and y cm respectively and $x^2 + y^2 = r^2$.

(i) Show that the area of the rectangle is given by: **1**

$$A = x\sqrt{r^2 - x^2}$$

(ii) Show that **2**

$$\frac{dA}{dx} = \sqrt{r^2 - x^2} - \frac{x^2}{\sqrt{r^2 - x^2}}$$

(iii) Hence show that the maximum area of the rectangle is $\frac{1}{2}r^2$. **4**

(b) (i) Sketch $y = 1 + \sin 2\pi x$ in the domain $0 \le x \le 3$. **1**

(ii) Show that $\int_0^n (1 + \sin 2\pi x)\, dx = n$ for all positive integers, n. **3**

(iii) Hence, shade a region on your graph in part (i) that is bounded by the curve $y = 1 + \sin 2\pi x$ and has an exact area of 1 square unit. **1**

End of Paper

MATHEMATICS TRIAL EXAMINATION

Monday 7th August 2006

Solutions + Marking Guidelines

Comments   Reas /2

QUESTION 1: (12 marks)

(a) $2.077166\ldots$ ✓

$\doteqdot 2.1$ correct to 2 sig fig ✓

Many students could not round off to 2 sig. figs.

(b) $5x+5y+x^3+y^3$

$= 5(x+y) + (x+y)(x^2-xy+y^2)$ ✓

$= (x+y)(5+x^2-xy+y^2)$ ✓

Many could not factorise sum of 2 cubes!

(c) $0.\dot{2}\dot{4} = 0.242424\ldots$

Let $x = 0.2424\ldots$

$\therefore 100x = 24.2424\ldots$

$\therefore 99x = 24$ ✓

$x = \frac{24}{99} = \frac{8}{33}$ ✓

Well done but many did not simplify the fraction

(d) $\frac{4+\sqrt{2}}{2-\sqrt{2}} \times \frac{2+\sqrt{2}}{2+\sqrt{2}} = \frac{8+4\sqrt{2}+2\sqrt{2}+2}{4-2}$ ✓

$= \frac{10+6\sqrt{2}}{2}$

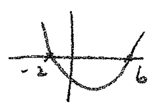
$= 5+3\sqrt{2}$ ✓

$\therefore p=5$ and $q=3$

Please remember to finish with $p=5$ and $q=3$.

(e) $x^2-4x-12<0$

$(x-6)(x+2)<0$ ✓



$\therefore -2<x<6$ ✓

Many could factorise but could not give the solution.

(f) $f(x) = xe^{x^2}$

$-f(-x) = -\left(-xe^{(-x)^2}\right)$ ✓

$= --xe^{x^2}$

$= xe^{x^2} = f(x)$ ✓ $\therefore$ An odd function

Alternatively, show $f(-x) = -f(x)$.

Don't fudge! Reas /2

QUESTION 2: (12 marks)

Comments   Reas /2

(a) $f(x) = 2^{x-4}$

(i) $f(1) = 2^{1-4}$

$= 2^{-3}$ ✓

$= \frac{1}{8}$

Accept either index notation or fraction or decimal.

A few careless errors eg $1-4=3$!

(ii) $1 = 2^{x-4}$ ✓

$\therefore x-4=0$

$x=4$ ✓

(b) $\frac{3\sec^2x-3}{12\tan x} = \frac{3(\sec^2x-1)}{12\tan x}$

$= \frac{3\tan^2x}{12\tan x}$ ✓

$= \frac{\tan x}{4}$ ✓

(b) Poorly done. Identity $\tan^2x = \sec^2x-1$ was not well known. Learn them all very carefully for the HSC. Reas /2

(c) (i) $m_{AC} = \frac{1-0}{-2-1} = -\frac{1}{3}$ ✓

$\therefore y-0 = -\frac{1}{3}(x-1)$

$3y = -x+1$

$\therefore x+3y-1=0$ ✓

(c) Overall, well done.

(ii) $\begin{cases} 3x+2y-3=0 & -① \\ 2x-y+5=0 & -② \end{cases}$

②×2: $4x-2y+10=0$ $-③$

①+③: $7x+7=0$

$\therefore x=-1$ ✓

$y=3$ ✓

(iii) $d = \frac{|-1\times1+3\times3-1|}{\sqrt{1^2+3^2}} = \frac{7}{\sqrt{10}}$ ✓

(iv) $d = \sqrt{(-2-1)^2+(1-0)^2} = \sqrt{10}$ ✓

watch minus signs in these two formulae.

(v) $A = \frac{1}{2}\times\sqrt{10}\times\frac{7}{\sqrt{10}} = 3\frac{1}{2}$ units^2 ✓

QUESTION 3: (12 marks)

Comments — Calc /10

(a) (i) $A = \frac{1}{2} r^2 \theta$

$\frac{\pi}{4} = \frac{1}{2} . 2^2 . \theta$

$\therefore \theta = \frac{\pi}{8}$ ✓

(ii) $l = r\theta$

$= 2 \times \frac{\pi}{8} = \pi/4 \text{ m}$ ✓

(b) (i) $y = x^3 + 4x + x^{-2}$

$\therefore y' = 3x^2 + 4 - 2x^{-3}$ ✓

$= 3x^2 + 4 - \frac{2}{x^3}$

Be careful of negative indices when integrating.

(ii) $y = \sqrt{x^2+4} = (x^2+4)^{1/2}$ ✓

$y' = \frac{1}{2}(x^2+4)^{-1/2} \times 2x$ ✓

$= \frac{x}{\sqrt{x^2+4}}$

Calc /3

(c) (i) $\int \cos(3x+2)\, dx$

$= \frac{1}{3} \sin(3x+2) + C$ ✓✓

One mark for $+C$.

(ii) $\int \frac{1}{2x-5}\, dx$

$= \frac{1}{2} \int \frac{2}{2x-5}\, dx$ ✓

$= \frac{1}{2} \log_e (2x-5) + C$ ✓

Calc /4

(d) $y = \log_e(\sin x)$

$y' = \frac{\cos x}{\sin x}$ ✓

At $x = \pi/4$, $y' = \frac{\cos \pi/4}{\sin \pi/4} = 1$ ✓

$\therefore$ Gradient of the normal $= -1$ ✓

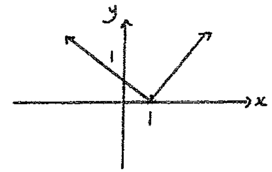
this was not a product.
More work needed on this type of question.
Don't waste time finding the equation of normal

Calc /3

QUESTION 4: (12 marks)

Comments — Com /3, Calc /2

(a) (i)



✓

(ii) $0 < x < 2$ ✓

(iii) $\int_0^2 |x-1|\, dx$

$= 2 \times \left(\frac{1}{2} \times 1 \times 1\right)$ ✓

$= 1 \text{ unit}^2$. ✓

*(a) This question was poorly done. ai) Com /1

(i) When graphing – always use a ruler & label all intercepts.

(ii) The answer to this question is NOT: '$x > 0$, $x < 2$' or '$x > 0$ or $x < 2$'

If you are going to split it up they need to be joined by an 'AND' ie. $x > 0$ AND $x < 2$. aii) Calc /2

(iii) The absolute value signs are there for a reason. You can't just get rid of them. You need to recall the relationship between integrals & areas.

(b) (i) $2r^2 - 3r - 2 = 0$

$(2r+1)(r-2) = 0$

$\therefore r = -\frac{1}{2}$ or 2 ✓

(ii) If a limiting sum exists, then $|r| < 1$. $\therefore r \neq 2$. ✓

$\therefore$ Common ratio is $-\frac{1}{2}$. ✓

*(b) (i) Well done

(ii) Be careful
$|r|<1$ or $-1<r<1$ is NOT the same as $0<|r|<1$ or $|r| \le 1$ or $-1 \le r \le 1$. bii) Com /2

(iii) $T_5 = ar^4$

$S_\infty = \frac{a}{1-r}$

$\therefore 6 = \frac{a}{1 - -\frac{1}{2}}$ ✓

$\therefore a = 9$

$\therefore T_5 = 9 \times \left(-\frac{1}{2}\right)^4 = \frac{9}{16}$ ✓

(iii) You cannot start with what you are trying to prove! You must start with what you are given (ie. $S_\infty = 6$ & $r = -1/2$)

(c) $8y - 17 = x^2 - 2x$

$\therefore 8y - 16 = (x-1)^2$

$4.2.(y-2) = (x-1)^2$

(i) Focal length is 2 ✓

(ii) Vertex is $(1, 2)$ ✓

(iii) Focus is $(1, 4)$ ✓

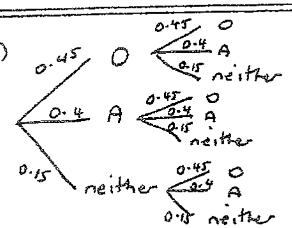
*(c) Poorly done.

(i) When finding the focal length you must take out all factors
$(x -)^2 = 4 \times a \times (y -)$
coeff of $y = 1$.

(ii) If you draw a picture you are less likely to get the x & y coordinates mixed up.

QUESTION 5: (12 marks)	Comments Reas /3

(a)



It is a lot easier if you draw a probability tree and work in decimals!

(i) $0.4 \times 0.4 = 0.16$ ✓

(ii) $P(A,O) + P(O,A)$ ✓

$= 0.4 \times 0.45 + 0.45 \times 0.4$

$= 0.36$ ✓

(iii) $P(O,A) + P(A, \text{anything}) + P(\text{neither}, A)$

$= 0.18 + 0.4 + 0.15 \times 0.4$ ✓

$= 0.64.$ ✓

Without the tree this part was very difficult.

(b) $\log_3(x+7) - \log_3(x-1) = 2$

$\log_3\left(\frac{x+7}{x-1}\right) = 2$ ✓

$\therefore \frac{x+7}{x-1} = 3^2$ ✓

$x + 7 = 9x - 9$

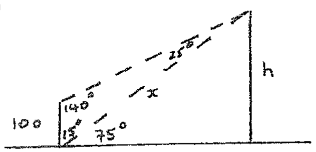
$8x = 16$

$\therefore x = 2$ ✓

Learn the log laws!

(b) Reas /3

(c)



✓ angles labelled

$\frac{x}{\sin 140} = \frac{100}{\sin 25}$

$\therefore x = \frac{100 \sin 140}{\sin 25} \doteq 152.1$ ✓

$\therefore \sin 75 = \frac{h}{152.1}$ ✓

$\therefore h = 146.9139\ldots$

$\doteq 147$ metres ✓

This part was very badly done! It is only question 5 so it was not nearly as difficult as many made it. Always redraw the diagram and label with your letters you use for your solution

QUESTION 6: (12 marks)	Comments Com /2 Calc /7

(a)

x:	1	1.5	2	2.5	3
$f(x)$:	0	0.405	0.693	0.916	1.099
k:	1	4	2	4	1
$k \times f(x)$:	0	1.62	1.386	3.664	1.099

Table method was most successful. Some incorrect values in this row.

$\Sigma \doteq 7.769$ ✓

$\therefore \int_1^3 \log_e x\,dx \doteq \frac{1}{3} \times 0.5 \times 7.769$ ✓

$\doteq 1.29$ ✓ (or 1.30)

Watch values for 'h' – width of each subinterval.

(b) $f(x) = -xe^{2x}$

NB Product Rule!!!

(i) $u = -x \quad v = e^{2x}$

$u' = -1 \quad v' = 2e^{2x}$

$\therefore f'(x) = -e^{2x} + (2e^{2x} \times -x)$

$= -e^{2x} - 2xe^{2x}$ ✓

$= -e^{2x}(1 + 2x)$

i Calc /1

(ii) Stationary pts $\Rightarrow f'(x) = 0$

method usually correct although some careless miscalculations

$0 = -e^{2x}(1+2x)$

$\therefore x = -\frac{1}{2}$ ✓

$f(x) = \frac{1}{2e}$ $(\doteq 0.1839\ldots)$

$f''(x) = -2e^{2x}(1+2x) - e^{2x} \times 2$

$= -4e^{2x}(1+x)$ ✓

when $x = -\frac{1}{2}$, $f''(x) = -4 \cdot e^{-1}\left(\frac{1}{2}\right)$

< 0

→ or use of table:
* make sure you indicate which derivative you are substituting into!

$\therefore \left(-\frac{1}{2}, \frac{1}{2e}\right)$ is a max t.p. ✓

ii Calc /3

Qu 6 (cont):

(iii) Possible POI $\Rightarrow f''(x) = 0$

$0 = -4e^{2x}(1+x)$

$\therefore x = -1$, $f(x) = \frac{1}{e^2}$ $(\doteqdot 0.135)$ ✓

x	-1.1	-1	-0.9
$f''(x)$	$+$	0	$-$

$\therefore$ Change in concavity ✓

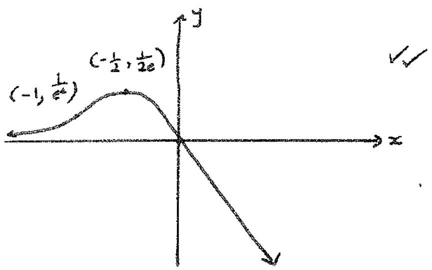
$\therefore$ $(-1, \frac{1}{e^2})$ is a poi

Many students forget to demonstrate a change in concavity! (iii) Calc /2

(iv) As $x \to -\infty$, $f(x) \to 0$ ✓

poorly done. (iv) Calc /1

(v) when $x=0$, $y=0$.

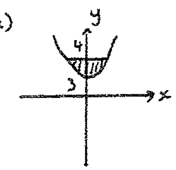


✓✓

Many graphs did not agree with previously drawn conclusions eg if you have only found one max t.p. in part (ii), make sure there is only one max t.p. on your graph! Com /2

QUESTION 7: (12 marks) | Comments | Com /2, Calc /4, Reas /6

(a)



$$V = \pi\int_3^4 x^2\,dy$$

$$= \pi\int_3^4 y-3\,dy$$ ✓✓

$$= \pi\left[\frac{y^2}{2} - 3y\right]_3^4$$ ✓

$$= \pi\left[(8-12) - (\tfrac{9}{2} - 9)\right]$$

$$= \frac{\pi}{2} \text{ units}^3$$ ✓

Draw a graph to obtain correct limits!

correct limits → 1
correct expression → 1

Read the question carefully. This question involved rotation about the y axis. Calc /4

(b) $mx^2 - nx + 1 = 0$

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta}$$ ✓

$$= \frac{\frac{n}{m}}{\frac{1}{m}}$$ ✓

$$= n$$

Reas /2

(c) (i) $PQ = SR$ (given)

$PR = SQ$ (given - diagonals equal) ✓

PS is common

$\therefore \Delta PQS \equiv \Delta SRP$ (SSS) ✓

c i) /2 Com

(ii) $\angle PSQ = \angle SPR$ (corresp. angles in congruent Δ are =)

$\therefore \Delta PAS$ is isosceles ✓

(iii) Diagonals are equal: $PR = SQ$

and $PA = SA$ ($\angle$ opp sides in isos Δ =) ✓

$\therefore QA = AR$

$\therefore \Delta QAR$ is isosceles ✓

(iv) $\angle PAS = \angle QAR$ (vert. opp $\angle$ =)

$\therefore \angle ASP = \angle APS = \angle AQR = \angle ARQ$ (two isos. Δs and $\angle$ sum Δ is $180°$) ✓

$\therefore PS \parallel QR$ as alt. angles are =. (eg $\angle SPA = \angle ARQ$).

The diagonals were given equal. This does not mean that A is the midpoint. This assumption meant that it was difficult to score marks.

c(ii) - (iv) /4 Reas

QUESTION 8: (12 marks)

Comments — Com /3, Calc /2, Reas /3

(a)(i) $A_{12} = 80000(1.02)^{12} - M$ ✓

Read the question carefully! Interest gets paid monthly & payments are made yearly

(ii) $A_{24} = A_{12} \times (1.02)^{12} - M$ ✓

$= (80000(1.02)^{12} - M)\, 1.02^{12} - M$

$= 80000(1.02)^{24} - M(1.02)^{12} - M$

$A_{36} = A_{24} \times (1.02)^{12} - M$

$= (80000(1.02)^{24} - M(1.02)^{12} - M) \times (1.02)^{12} - M$

$= 80000(1.02)^{32} - M(1.02)^{24} - M(1.02)^{12} - M$ ✓

$\vdots$

$A_{60} = 80000(1.02)^{60} - M(1.02)^{48} - M(1.02)^{32} - \dots - M$

$= 80000(1.02)^{60} - M(1.02^{48} + 1.02^{32} + 1.02^{24} + 1.02^{12} + 1)$

$= 80000(1.02)^{60} - M(1 + 1.02^{12} + \dots + 1.02^{48})$

You are not SHOWING anything if you simply copy the pattern given for A_{60} for A_{12}, A_{24} etc.

Neither are you showing anything if you simply write a formula for A_n & plug in 60.

To show it to be true, you need to show how the terms given in the question relate to the expressions written on your page.

a(i)(ii) Com /3

(iii) $A_{60} = 0$

$\therefore 0 = 80000(1.02)^{60} - M(1 + 1.02^{12} + \dots + 1.02^{48})$

$M(1 + 1.02^{12} + \dots + 1.02^{48}) = 80000(1.02)^{60}$

$M\left[\dfrac{1((1.02^{12})^5 - 1)}{(1.02^{12}) - 1}\right] = 80000(1.02)^{60}$

$M = \dfrac{80000(1.02)^{60}}{8.5036\dots}$

$= \$30867.083\dots$

$\doteqdot \$30867$ (to nearest \$)

Part (ii) was very poorly done & was marked leniently — don't expect the marking scheme to be nice in the HSC!

(iii) The series $1 + 1.02^{12} + \dots + 1.02^{48}$ has a ratio of $\underline{1.02^{12}}$ & has $\underline{5}$ terms.

Many people would have done better if they simply added the five terms on their calculator! [this is not such a stupid idea]

Qu 8 (cont):

(b) $\dfrac{dy}{dx} = 3^x$

$y = \dfrac{3^x}{\log_e 3} (+C)$ ✓✓

or: $\dfrac{dy}{dx} = 3^x$

$= (e^{\log_e 3})^x$

$= e^{x \log_e 3}$ ✓

$y = \dfrac{e^{x\log_e 3}}{\log_e 3} (+C)$

$= \dfrac{3^x}{\log_e 3} (+C)$ ✓

b) Calc /2

This question was very poorly done.

* 'Find a primitive' does NOT mean differentiate

* $\int 3^x\,dx$ does not equal $\dfrac{3^{x+1}}{x+1} + C$

& it certainly does not equal $x3^{x-1}$!!!

neither does it equal any of these other interesting responses:

$\log_e 3 \times 3^x + C$

$3^x + C$

$\log_x 3 + C$

$\dfrac{3^{2x}}{2} + C$

$\dfrac{x^2}{2} \times 3^x + C$

$\frac{1}{4} 3^x + C$

$\dfrac{1}{x} + C$

$\log_e 3 + C$

$\dfrac{\log_e 3^x}{3^x} + C$

$\vdots$

& the list goes on ... & on.

Qu 8 (cont):

(c) 0.26 Fruit < F, NF

0.74 no fruit < F, 0.74 NF ...

(i) $(0.74)^5$ ✓

(ii) $(0.74)^n$ ✓

(iii) P(at least one bears fruit)

$= 1 - P(\text{none bear fruit})$

$\therefore 1 - (0.74)^n \geq 0.99$ ✓

$(0.74)^n \leq 0.01$

$\therefore n(\ln 0.74) \leq \ln(0.01)$

$\therefore n \geq \dfrac{\ln(0.01)}{\ln(0.74)}$ ✓

$\geq 15.294...$

∴ 16 vines must be planted to be at least 99% certain. ✓

(i) & (ii) done well.

(iii) • $1-(0.74)^n \neq 0.26^n$

• Many people who used the inequality sign got confused when & when not to flip it. In particular, $\log 0.74 < 0$ & so when you divide by it you need to flip the sign!

• Many people who solved the equality incorrectly rounded the answer of 15.294... down to 15 in the last step.

c iii) Reas /3

QUESTION 9: (12 marks)

Comments

Calc /3
Reas /4

(a) (i) $y = x^2+4$

$y = x+6$

$\therefore x^2+4 = x+6$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

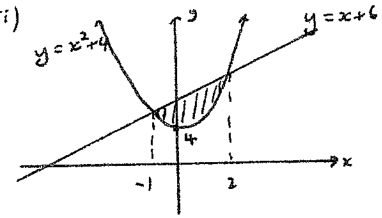
$\therefore x = -1,\ 2$ ✓

$y = 5,\ 8$

$\therefore$ PoI are $(-1,5)$ and $(2,8)$ ✓

* Remember to state the coordinates of the points

(ii)



Draw a diagram and use it to answer the question!

Area $= \int_{-1}^{2} (x+6) - (x^2+4)\,dx$ ✓

Many subtracted in the wrong order

$= \int_{-1}^{2} x+6-x^2-4\,dx$

$= \int_{-1}^{2} x+2-x^2\,dx$

$= \left[\dfrac{x^2}{2} + 2x - \dfrac{x^3}{3}\right]_{-1}^{2}$ ✓

$= \left(2+4-\frac{8}{3}\right) - \left(\frac{1}{2} - 2 + \frac{1}{3}\right)$

$= 3\frac{1}{3} + 1\frac{1}{6}$

$= 4\frac{1}{2}$ units². ✓

a(ii) Calc /3

[OR $A_1 = \dfrac{39}{2}$ ✓

$A_2 = \left[\dfrac{x^3}{3} + 4x\right]_{-1}^{2} = 15$ ✓

$\therefore A_1 - A_2 = 4\frac{1}{2}$ units²] ✓

Qu 9 (cont):

(b) $2\sin^2 x + 5\sin x - 3 = 0$

let $m = \sin x$

$2m^2 + 5m - 3 = 0$

$(2m-1)(m+3) = 0$

$\therefore m = \frac{1}{2}$ or -3

$\therefore \sin x = \frac{1}{2}$ or $\sin x = -3$ → no sol^n

$\therefore x = \frac{\pi}{6}$ or $\frac{5\pi}{6}$

S | A
T | C

Very difficult to solve if you don't reduce to a quadratic.

(c) (i) $x > 0$

(ii) $y = \frac{x^2}{\log_e x}$ $u = x^2$ $v = \ln x$ $u' = 2x$ $v' = \frac{1}{x}$

$\frac{dy}{dx} = \frac{2x\ln x - x}{(\ln x)^2}$

Horizontal tangent means $\frac{dy}{dx} = 0$

$\therefore 0 = \frac{x(2\ln x - 1)}{(\ln x)^2}$

$\therefore 0 = x(2\ln x - 1)$

$\therefore x = 0$ or $\ln x = \frac{1}{2}$

$\therefore x = e^{1/2}$

but domain is $x > 0$

$\therefore$ Tangent is horizontal at $x = \sqrt{e}$

Very poorly answered.

* $(\ln x)^2 \neq 2\ln x$

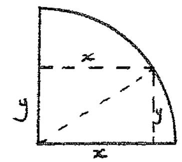
Learn log laws.

(c) Reas /4

Must indicate 2 solns to $0 = x(2\ln x - 1)$ but because of the domain $x = \sqrt{e}$

QUESTION 10: (12 marks) | Comments | Com /2 Calc /2 Reas /4

(a)



(i) $x^2 + y^2 = r^2$ (given)

$\therefore y = \sqrt{r^2 - x^2}$

Area $= xy$
$= x\sqrt{r^2 - x^2}$

*Some good attempts made at this question, although many students need to work on their time management!

(i) Com /1

(ii) Product rule

$u = x$ $v = (r^2 - x^2)^{1/2}$

$u' = 1$ $v' = \frac{1}{2}(r^2 - x^2)^{-1/2} \times -2x$ ← a few minus signs missing

$= \frac{-x}{\sqrt{r^2 - x^2}}$

$\therefore \frac{dA}{dx} = 1 \times (r^2 - x^2)^{1/2} + x \times \frac{-x}{\sqrt{r^2 - x^2}}$

$= \sqrt{r^2 - x^2} - \frac{x^2}{\sqrt{r^2 - x^2}}$

(ii) Calc /2

(iii) Max $\Rightarrow \frac{dA}{dx} = 0$

$0 = \sqrt{r^2 - x^2} - \frac{x^2}{\sqrt{r^2 - x^2}}$

$\frac{x^2}{\sqrt{r^2 - x^2}} = \sqrt{r^2 - x^2}$

$x^2 = r^2 - x^2$

$2x^2 = r^2$

$x^2 = \frac{r^2}{2}$

$x = \frac{r}{\sqrt{2}}$ (since $x > 0$ because it is a length)

Correct method, although some careless errors with $\sqrt{}$

Testing:

x	$0.6r$	$\frac{\sqrt{2}r}{2}$ $(0.7r)$	$0.8r$
$\frac{dA}{dx}$	$0.8r - \frac{0.36r^2}{0.8r}$ $= 0.35r$ > 0	0	$0.6r - \frac{0.64r^2}{0.6r}$ $= -0.4\dot{6}r$ < 0

/ — \

$\therefore x = \frac{r}{\sqrt{2}}$ is a max t.p.

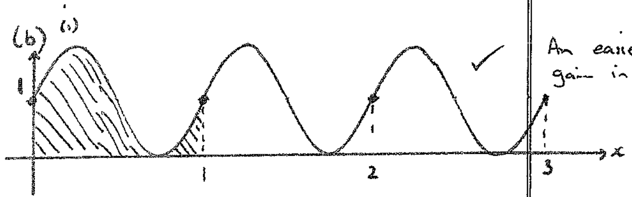
$$\text{Area} = x\sqrt{r^2 - x^2}$$

$$= \frac{r}{\sqrt{2}}\sqrt{r^2 - \frac{r^2}{2}}$$

$$= \frac{r}{\sqrt{2}} \times \sqrt{\frac{r^2}{2}} \quad ✓$$

$$= \frac{r^2}{2}$$

(b) (i)



✓ An easier mark to gain in this qu.

(b i) Com /1

(ii) $\int_0^n (1 + \sin 2\pi x)\,dx$

$$= \left[x - \frac{1}{2\pi}\cos 2\pi x\right]_0^n \quad ✓$$

$$= \left(n - \frac{1}{2\pi}\cos 2\pi n\right) - \left(0 - \frac{1}{2\pi}\cos 0\right) \quad ✓$$

$$= n - \frac{1}{2\pi} + \frac{1}{2\pi} \quad ✓$$

$$= n$$

(iii) Shading ✓

(b ii) (iii) Reas /4