



SCEGGS Darlinghurst

2008

HIGHER SCHOOL CERTIFICATE
TRIAL EXAMINATION

Mathematics

This is a TRIAL PAPER only and does not necessarily reflect the content or format of the Higher School Certificate Examination for this subject.

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Diagrams should be drawn in pencil
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question
- Start each question in a new booklet

Total marks – 120

- Attempt Questions 1–10
- All questions are of equal value

Centre Number									
Student Number									

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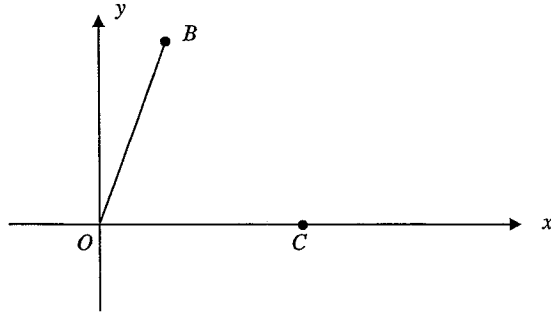
Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

	Marks
Question 1 (12 marks)	
(a) Find the value of $\sqrt{\frac{19}{4\pi}}$ correct to two decimal places.	2
(b) Solve $x - 3(2 - x) = 0$.	2
(c) Factorise $a^3 - 27$.	2
(d) Write $0.5\dot{2}\dot{6}$ in the form of $\frac{a}{b}$.	2
(e) Kate invests \$5500 for 3 years. The investment earns 7.5% p.a. compounded annually. How much interest does Kate's investment earn after 3 years?	2
(f) Find a primitive function of $4 - \frac{1}{x^2}$.	2

Marks

Question 2 (12 marks) Use a SEPARATE writing booklet.

(a)



O is the origin, B is the point (1, 6) and C is the point (5, 0).

Copy this diagram into your writing booklet.

- (i) Find the co-ordinates of the point P such that OBPC is a parallelogram. Label this point on your diagram. 1
- (ii) Find, in its simplified form, the exact length of BC. 2
- (iii) Find the area of the parallelogram OBPC. 1
- (iv) Hence or otherwise, find the perpendicular distance from P to the diagonal BC. 2
- (v) Find $\angle BCO$ to the nearest degree. 2

(b) The quadratic equation $4x^2 - 3x + 1 = 0$ has roots α and β . Find:

- (i) $\alpha + \beta$ 1
- (ii) $\alpha\beta$ 1
- (iii) $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$ 2

Marks

Question 3 (12 marks) Use a SEPARATE writing booklet.

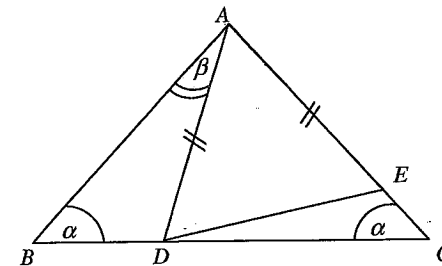
(a) Find:

- (i) $\frac{d}{dx}(\sin 3x)$ 2
- (ii) $\frac{d}{dx}(x^2 e^x)$ 2

(b) Evaluate each of the following correct to 3 decimal places:

- (i) $\int_0^1 e^{-4x} dx$ 2
- (ii) $\int_1^5 \frac{dx}{\sqrt{x}}$ 2

(c)



In the isosceles triangle ABC, $\angle ABC = \angle ACB = \alpha$. The points D and E lie on BC and AC, so that $AE = AD$. Let $\angle BAD = \beta$.

- (i) Explain why $\angle ADC = \alpha + \beta$. 1
- (ii) Find $\angle DAC$ in terms of α and β . Give reasons. 1
- (iii) Hence, or otherwise, find $\angle EDC$ in terms of β . 2

Marks

Question 4 (12 marks) Use a SEPARATE writing booklet.

- (a) The movie theatre on the QEII is specially designed. The floor narrows from front to back so that each row of seats behind the first has two seats less than the row in front of it. The front row has 37 seats.
- (i) How many seats are in the n^{th} row? 2
- (ii) What is the greatest value n can take? 1
- (iii) The movie theatre seats 360 people. How many rows of seats are there? 3
- (b) A function is defined as $y = \ln x$.
- (i) Copy the table into your writing book and complete it giving answers to 3 decimal places. 1
- | | | | | |
|-------------|---|---|---|---|
| x | 2 | 3 | 4 | 5 |
| $y = \ln x$ | | | | |
- (ii) Using the trapezoidal rule, find an approximation, to 1 decimal place, of $\int_2^5 \ln x \, dx$ using 4 function values. 3
- (iii) Sketch the graph of $y = \ln x$ and use it to explain why your approximation in part (ii) will be less than the exact value of the integral. 2

Marks

Question 5 (12 marks) Use a SEPARATE writing booklet.

- (a) (i) Sketch $y = 2 \sin 2x$ for $0 \leq x \leq 2\pi$. 2
- (ii) Calculate the area bounded by the curve $y = 2 \sin 2x$ and the x -axis between $x = 0$ and $x = \frac{3\pi}{4}$. 3
- (b) A box contains 4 red and 5 green marbles. Anna randomly selects 3 marbles one at a time and without replacement.
- (i) What is the probability that she selects green, red, green in that order? 1
- (ii) What is the probability that she selects a majority of green marbles? 2
- (c) (i) If $f(x) = \log_e \sqrt{x}$ evaluate $f(e^2)$. 1
- (ii) Find the equation of the tangent to the curve $y = \log_e \sqrt{x}$ at the point where $x = e^2$. 3

Marks

Question 6 (12 marks) Use a SEPARATE writing booklet.

- (a) For the curve $y = x^3 - 3x$:
- (i) Find the stationary points and determine the nature of these points. 3
- (ii) Show that $(0, 0)$ is a point of inflexion. 1
- (iii) Sketch the curve for $-2 \leq x \leq 3$. 2
- (iv) Hence, or otherwise, evaluate $\int_{-1}^1 x^3 - 3x \, dx$ 1

- (b) Carl the caterpillar woke up one morning and he was feeling very hungry. So he found himself a big leaf and started munching. After 10 minutes he had eaten 5 cm of the leaf. Since his tummy was starting to fill up, in the next 10 minutes he ate 3.75 cm of the leaf. In the next 10 minutes he ate 2.8125 cm of the leaf and so on.
- If the leaf was 25 cm long, would Carl finish eating the leaf for breakfast? Give reasons.

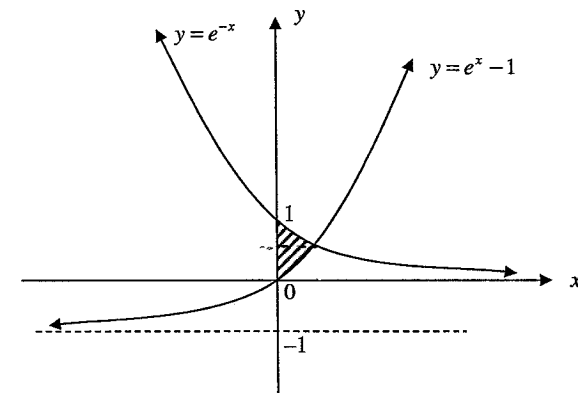
- (c) (i) Find $\frac{d}{dx}(e^{\tan 2x})$. 1
- (ii) Hence, evaluate $\int_0^{\frac{\pi}{6}} (\sec^2 2x) e^{\tan 2x} \, dx$. Leave your answer in exact form. 2



Marks

Question 7 (12 marks) Use a SEPARATE writing booklet.

- (a) For the parabola $y^2 - 2y - 4x - 7 = 0$, find:
- (i) the co-ordinates of the vertex. 2
- (ii) the co-ordinates of the focus. 1
- (iii) the equation of the directrix. 1
- (iv) Sketch $y^2 - 2y - 4x - 7 = 0$, labelling the vertex, focus and directrix only. 2
- (b) The diagram shows the graph of $y = e^x - 1$ and $y = e^{-x}$.

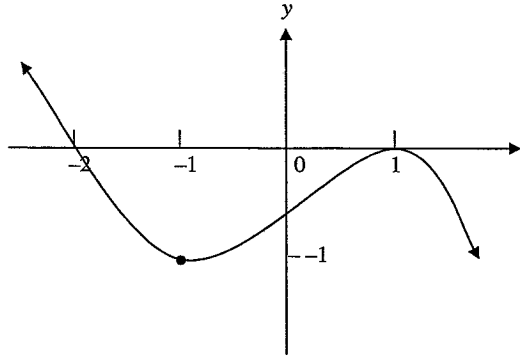


- (i) Show that the curves intersect when: 2
- $$e^{2x} - e^x - 1 = 0$$
- (ii) Use the results in part (i) and the substitution $m = e^x$, to show that the x - co-ordinate of the point of intersection of the curves is approximately 0.481. 2
- (iii) Find the area of the shaded region to 1 decimal place. 2

Marks

Question 8 (12 marks) Use a SEPARATE writing booklet.

(a) The graph below shows $y = f'(x)$.



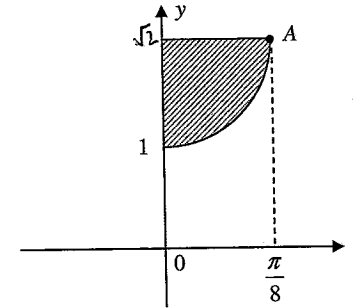
Copy the graph into your writing booklet.

- (i) For which values of x is the curve of $y = f(x)$ increasing. 1
- (ii) On the same axes sketch $y = f(x)$ clearly labelling the important features at $x = -2$, $x = -1$ and $x = 1$. 3
- (b) (i) For what values of k does the quadratic equation $kx^2 + (k + 3)x - 1 = 0$ have no real roots. 3
- (ii) Hence, explain why $y = kx^2 + (k + 3)x - 1$ can never be positive definite. 1

Question 8 continues on page 10

Question 8 (continued)

(c) The graph is $y = \sec 2x$ from $x = 0$ to $x = \frac{\pi}{8}$.



- (i) The x -coordinate of A is $\frac{\pi}{8}$. Find the y -coordinate. 1
- (ii) The shaded region is rotated around the x -axis. Show that the volume generated by this rotation is given by: 3

$$\frac{\pi^2}{4} - \pi \int_0^{\frac{\pi}{8}} \sec^2 2x \, dx$$

and hence find the exact volume.

Marks

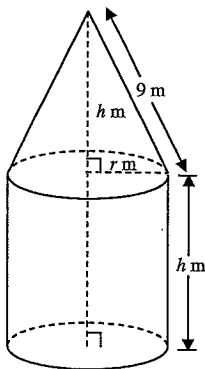
Marks

Question 9 (12 marks) Use a SEPARATE writing booklet.

(a) Solve $2\sin^2 x - 3\cos x = 0$ for $0 \leq x \leq 2\pi$.

3

(b) A grain silo is constructed from a right circular cone and cylinder of equal heights " h " m. The slant edge of the cone is 9 m and the radius of the cylinder is " r " m.



(i) Write down the equation linking r and h .

1

(ii) Show that the volume of the silo is given by:

2

$$V = 108\pi h - \frac{4}{3}\pi h^3$$

(iii) Find the height of the silo that gives a maximum volume.

3

(c) In the Jackpot Lottery, the probability of the Jackpot prize being won in any draw is approximately 1 in 50.

(i) What is the probability that the Jackpot prize will be won in each of three consecutive draws?

1

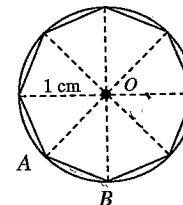
(ii) How many consecutive draws must be made for there to be a 50% chance that at least one Jackpot prize will have been won?

2

Marks

Question 10 (12 marks) Use a SEPARATE writing booklet.

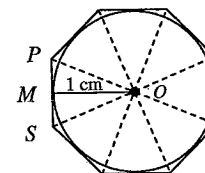
(a) A regular octagon is drawn inside a circle with centre O so that its vertices lie on the circumference. The circle has radius 1 cm.



(i) Find the area of $\triangle AOB$ and hence find the area of this octagon. (Leave your answer in surd form.)

2

Another regular octagon is drawn outside the circle. The altitude OM of $\triangle OPS$ is 1 cm.



(ii) Find the area of $\triangle OPS$ and hence find the area of this outer octagon.

2

(iii) By considering the results in parts (i) and (ii), show that:

1

$$\sqrt{2} < \frac{\pi}{2} < 4 \tan \frac{\pi}{8}$$

Question 10 continues on page 13

Marks

Question 10 (continued)

- (b) Miss Toolan decides to borrow \$400 000 to buy an apartment. Interest is calculated monthly on the balance still owing, at a rate of 9.12% p.a. The loan is to be repaid at the end of 25 years with equally monthly repayments of \$ M . Let \$ A_n be the amount owing after the n th repayment.
- (i) Derive an expression for A_{300} . 2
- (ii) Find the value of M . 2
- (iii) Calculate the amount still owing, to the nearest dollar, after 10 years of payments at this rate. 1
- (iv) At the end of 10 years, the interest rate is increased to 12% p.a. and Miss Toolan changes her repayments to \$3800 per month. 2
- How many months are needed to pay off the remainder of the loan?

End of Paper

2008 Trial HSC MATHEMATICS - Solutions

Q1 a) 1.23 ✓ \$1 for correct rounding.

Done very well

$$\begin{aligned} \text{b) } x - 3(2-x) &= 0 \\ x - 6 + 3x &= 0 \quad \checkmark \\ 4x - 6 &= 0 \\ 4x &= 6 \\ x &= \frac{6}{4} \\ &= \frac{3}{2} \quad \checkmark \end{aligned}$$

Quite a few students made silly errors. The most annoying was thinking it was a quadratic $(x-3)(2-x)=0$

$$\text{c) } a^3 - 27 = (a-3)(a^2 + 3a + 9) \quad \checkmark \quad \checkmark$$

Done fairly well, most common mistake $a^2 + 6a + 9$

$$\begin{aligned} \text{d) let } x &= 0.5262626\dots \\ 10x &= 5.262626\dots \quad \textcircled{1} \quad \checkmark \\ 1000x &= 526.262626\dots \quad \textcircled{2} \\ \textcircled{2} - \textcircled{1} \quad 990x &= 521 \\ x &= \frac{521}{990} \quad \checkmark \end{aligned}$$

Done well. Most errors came from not aligning the recurring decimals
i.e. 52.6....
526.2....

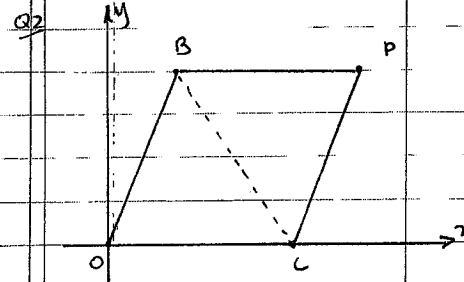
$$\begin{aligned} \text{e) } A &= P \left(1 + \frac{r}{100}\right)^n \\ &= 5500 \left(1 + \frac{7.5}{100}\right)^3 \quad \checkmark \\ &= 6832.63 \\ \text{Int} &= 6832.63 - 5500 \\ &= 1332.63 \\ \text{Kate receives } &\$1332.63 \text{ interest.} \quad \checkmark \end{aligned}$$

Most common mistake was not reading the question. It asked for the interest not how much it was worth

$$\begin{aligned} \text{f) } 4x + x^{-1} \\ &= 4x + \frac{1}{x} + c \\ &\quad \checkmark \quad \checkmark \end{aligned}$$

Calc - 2

Done well



i) $P(6, 6) \quad \checkmark$

$$\begin{aligned} \text{ii) } BC &= \sqrt{(5-1)^2 + (0-6)^2} \quad \checkmark \\ &= \sqrt{4^2 + (-6)^2} \\ &= \sqrt{16 + 36} \\ &= \sqrt{52} \\ &= 2\sqrt{13} \text{ units} \quad \checkmark \end{aligned}$$

$$\begin{aligned} \text{iii) } A &= l \times h \\ &= 5 \times 6 \\ &= 30 \text{ units}^2 \quad \checkmark \end{aligned}$$

$$\begin{aligned} \text{iv) Area of } \parallel\text{ogram } OBPC &= \text{Area } \triangle OBC + \text{Area } \triangle CBP \\ 30 &= 2 \times \text{Area } \triangle CBP \quad \triangle OBC \cong \triangle CBP \\ 30 &= 2 \times \frac{1}{2} \times BC \times h \text{ dist from } P \quad \checkmark \\ h \text{ dist from } P &= \frac{30}{2} \\ &= \frac{15 \cdot 2\sqrt{13}}{\sqrt{13}} \text{ units.} \quad \checkmark \quad \text{Reas - 2} \end{aligned}$$

$$\begin{aligned} \text{v) } \sin \angle CBP &= \frac{15}{\sqrt{13}} \quad \checkmark \\ &= \frac{15}{\sqrt{13}} \\ &= 56^\circ \text{ (nearest degree)} \quad \checkmark \\ \angle BCO &= 56^\circ \text{ (alternate angles in parallel lines)} \quad \checkmark \end{aligned}$$

Reas - 2

Many students did not update their diagram at a This is why students are asked to copy in the first place! Many students also they were dealing with a rhombus (because it looks like one) The diagonals of the shape do NOT bisect at right angles. A few students did not understand that the vertices should be listed cyclically and place P in the wrong position.

Many responses were very long and complicated. Be guided by the marking scheme.

Some students who used the distance or other formulas, made careless substitution errors

Many students introduced an angle θ without updating it on their diagram. The marker has no knowledge of where θ is. Similarly, very few students correctly used notation and identify the acute angle

Q2 cat b) i) $\alpha + \beta = \frac{-b}{a}$
 $= \frac{-(-3)}{4}$
 $= \frac{3}{4} \checkmark$

ii) $\alpha\beta = \frac{c}{a}$
 $= \frac{1}{4} \checkmark$

iii) $\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2 + \beta^2}{\alpha^2\beta^2}$
 $= \frac{(\alpha + \beta)^2 - 2\alpha\beta}{(\alpha\beta)^2} \checkmark$
 $= \frac{(\frac{3}{4})^2 - 2 \times \frac{1}{4}}{(\frac{1}{4})^2}$
 $= \frac{\frac{9}{16} - \frac{1}{2}}{\frac{1}{16}}$
 $= 1 \checkmark$

Q3 a) i) $\frac{d}{dx} (\sin 3x)$
 $= 3 \cos 3x \checkmark$

ii) $\frac{d}{dx} (x^2 e^x) = x^2 e^x + 2x e^x \checkmark$
 $= e^x (x^2 + 2x) \checkmark$ Calc - 4

b) i) $\int_0^1 e^{-4x} dx = \left[-\frac{1}{4} e^{-4x} \right]_0^1 \checkmark$
 $= -\frac{1}{4} e^{-4} + \frac{1}{4} e^0$
 $= \frac{-1}{4e^4} + \frac{1}{4}$
 $= 0.245 \checkmark$

ii) $\int_1^5 \frac{dx}{\sqrt{x}} = \int_1^5 x^{-1/2} dx$
 $= \left[2x^{1/2} \right]_1^5 \checkmark$
 $= \left[2\sqrt{x} \right]_1^5 = 2\sqrt{5} - 2\sqrt{1}$
 $= 2.472 \checkmark$ Calc - 4

Few students could handle the algebra. Many had learned off that $(\alpha + \beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$ but had no idea how to apply it!

On the whole well done. Some mix up with integration. Funny to see "+c" at this question. Well done.

Did not evaluate to 3 decimal places. Only 1 mark lost if done in both parts.

Identify $\frac{1}{\sqrt{x}} = x^{-1/2}$. Some careless errors with simplifying.

c) i) exterior angle of a triangle equals the sum of the two opposite interior angles
 Comm-1

ii) $\angle DAC = 180 - (\alpha + \beta + \alpha)$ (angle sum of a triangle is 180°)
 $= 180 - (2\alpha + \beta) \checkmark$
 Comm-1

iii) $\angle ADE = \frac{180 - \angle DAC}{2}$
 $= \frac{180 - [180 - (2\alpha + \beta)]}{2} \checkmark$
 $= \frac{2\alpha + \beta}{2}$
 $= \alpha + \frac{\beta}{2}$
 $\therefore \angle EDC = \angle ADC - \angle ADE$
 $= \alpha + \beta - (\alpha + \frac{\beta}{2})$
 $= \beta - \frac{\beta}{2}$
 $= \frac{\beta}{2} \checkmark$ Reas-2

Needed to say the two opposite to get the mark. Other variations of the answer the mark was awarded as long as it was explained.

ii) Well done as long as correct reason was given and it was attempted. iii) In terms of β mean the soldier should have this term.

Also of non-attempts. 'Hence' indicates one can use earlier parts to help with the solution.

Q4 a) i) No of seats = $37 + 35 + 33 + \dots$
 this is an A.P.
 with $a = 37 \checkmark$
 $d = -2$

$\therefore T_n = a + (n-1)d$
 $= 37 + (n-1)(-2)$
 $= 37 - 2n + 2$
 $= 39 - 2n \checkmark$ Comm-2

$\therefore n^{\text{th}}$ row has $39 - 2n$ seats

ii) find n $T_n \geq 0$
 $39 - 2n \geq 0$
 $2n \leq 39$
 $n \leq 19.5$
 \therefore greatest value n can be is 19. \checkmark Reas-1

Well done - be careful $d = -2$

Be careful with greatest value of n

iii) find n when $S_n = 360$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$\therefore 360 = \frac{n}{2} [2 \times 37 + (n-1) \times 2] \quad \checkmark$$

$$720 = n(74 - 2n + 2)$$

$$720 = n(76 - 2n)$$

$$720 = 76n - 2n^2$$

$$2n^2 - 76n + 720 = 0$$

$$n^2 - 38n + 360 = 0$$

$$(n-18)(n-20) = 0 \quad \checkmark$$

$$n=18 \quad n=20$$

Since the greatest value for n is 19

then $n=18$

\therefore 18 rows of seats. \checkmark Reas-3

b) i)

x	2	3	4	5
$y = \ln x$	0.693	1.099	1.386	1.609

Well done

$$\text{ii) } \int_2^5 \ln x \, dx \approx \frac{3-2}{2} [f(2)+f(3)] + \frac{4-3}{2} [f(3)+f(4)] + \frac{5-4}{2} [f(4)+f(5)] \quad \checkmark$$

$$= \frac{1}{2} [f(2) + 2f(3) + 2f(4) + f(5)]$$

$$= \frac{1}{2} [0.693 + 2 \times 1.099 + 2 \times 1.386 + 1.609] \quad \checkmark$$

$$= 3.6 \text{ (to 1 d.p.)} \quad \checkmark \quad \text{Calc-3}$$

Don't confuse Trapezoidal rule with Simpson's

2x rest does not mean $2 \times 1.386 \times 1.609$

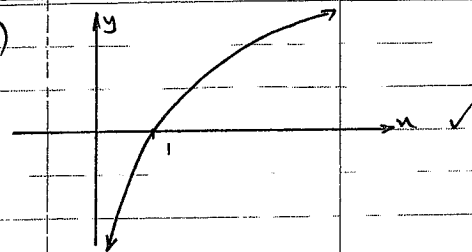
one decimal place.

- Learn formula

Be careful of substitution.

Algebra - factorising let abt of students down

iii)

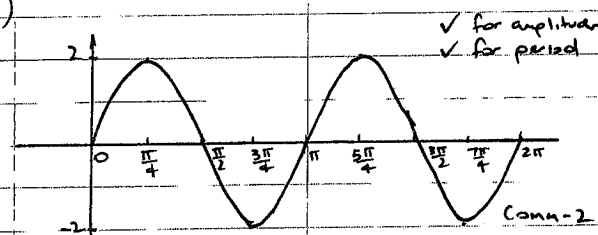


The graph of $y = \ln x$ is concave down, therefore the area of the trapezium will be less than the actual area. This leads to an underestimation of the integral. \checkmark Comm-2

Most students could sketch this

Students who made reference to their sketch or displayed their reason on their sketch got the mark. Must be trapezium though!

or a) i)



$$\text{ii) } A = \int_0^{\frac{\pi}{2}} 2 \sin 2x \, dx + \left| \int_{\frac{\pi}{2}}^{\frac{3\pi}{4}} 2 \sin 2x \, dx \right| \quad \checkmark$$

$$= \left[-\cos 2x \right]_0^{\frac{\pi}{2}} + \left| \left[-\cos 2x \right]_{\frac{\pi}{2}}^{\frac{3\pi}{4}} \right| \quad \checkmark$$

$$= (-\cos \pi + \cos 0) + \left| -\cos \frac{3\pi}{2} + \cos \pi \right|$$

$$= (1+1) + |0-1|$$

$$= 3 \text{ units}^2 \quad \checkmark \quad \text{Calc-3}$$

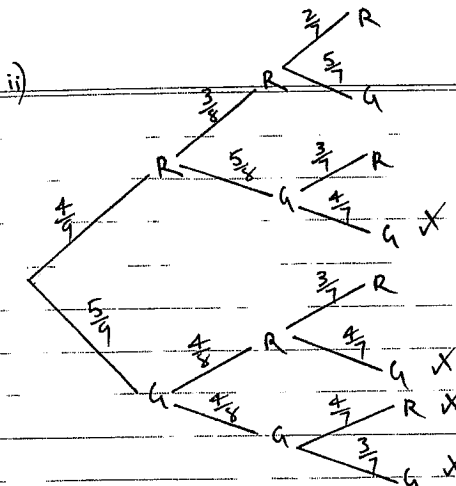
You must clearly label the range values on the y-axis. Don't assume the lines on your page count as one unit.

Poorly done. The total area should be split into pieces with absolute value around any area that lies below the x-axis.

Check the standard integrals when finding $\int 2 \sin 2x \, dx$

$$\text{b) i) } P(\text{GRA}) = \frac{5}{9} \times \frac{4}{8} \times \frac{4}{7} \quad \checkmark$$

$$= \frac{10}{63}$$



$$P(\text{Majority of green}) = P(2GR) + P(3G)$$

$$= 3 \times \left(\frac{5}{9} \times \frac{4}{8} + \frac{4}{7} \right) + \frac{5}{9} \times \frac{4}{8} + \frac{4}{7}$$

$$= \frac{3 \times 10}{63} + \frac{5}{42}$$

$$= \frac{25}{42}$$

Comm-2

c): i) $f(x^2) = \log_e \sqrt{x^2}$
 $= \log_e x$
 $= 1$ ✓

ii) $y = \log_e \sqrt{x}$
 $= \frac{1}{2} \log_e x$ ✓
 $\frac{dy}{dx} = \frac{1}{2} \times \frac{1}{x}$ ✓
 $= \frac{1}{2x}$ ✓

\therefore Tangent = $\frac{1}{2e^2}$ at $(e^2, 1)$

\therefore using $y - y_1 = m(x - x_1)$

$$y - 1 = \frac{1}{2e^2} (x - e^2)$$

$$2e^2 y - 2e^2 = x - e^2$$

$$x - 2e^2 y + e^2 = 0$$
 ✓

Calc-3

The students who drew a clear tree diagram had the greatest success in this part. Don't forget $0 \leq P(E) \leq 1$ so think about the validity of your answer

Calculator errors not penalised here but if you got it wrong, please practise your calculator steps.

Very poorly done, which was surprising.
 $\frac{d}{dx} (\log_e f(x)) = \frac{1}{f(x)} \times f'(x)$

$$\frac{d}{dx} (\log_e x^{1/2}) = \frac{1}{x^{1/2}} \times \frac{1}{2} x^{-1/2}$$

$$= \frac{1}{2x}$$

Q6 a): i) $y = x^3 - 3x$
 $\frac{dy}{dx} = 3x^2 - 3$
 \therefore for stationary points $\frac{dy}{dx} = 0$

$$3x^2 - 3 = 0$$

$$3(x^2 - 1) = 0$$

$$3(x-1)(x+1) = 0$$
 ✓

$$x = 1 \quad x = -1$$

$$y = -2 \quad y = 2$$

$$\frac{d^2y}{dx^2} = 6x$$

test $(1, -2)$ $\frac{d^2y}{dx^2} = 6 > 0$
 $\therefore (1, -2)$ is a min. t.p. ✓

test $(-1, 2)$ $\frac{d^2y}{dx^2} = -6 < 0$
 $\therefore (-1, 2)$ is a max. t.p. ✓

Calc-3

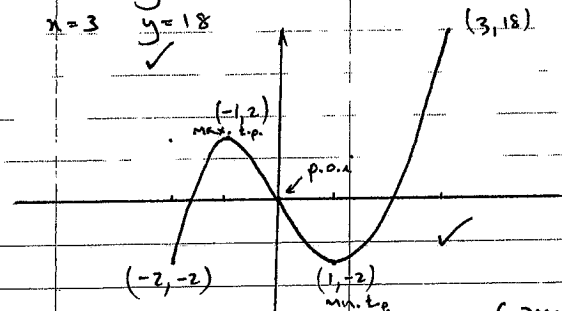
ii) when $x = 0$ $\frac{d^2y}{dx^2} = 6 \times 0 = 0$

x	-1	0	1
$\frac{d^2y}{dx^2}$	-6	0	6

Calc-11

\therefore change of concavity ✓
 $\therefore (0, 0)$ is a point of inflexion.

iii) $x = -2 \quad y = -2$
 $x = 3 \quad y = 18$ ✓



Comm-2

Several students did not find the y-coordinate. A point requires a pair of coordinates.

Many students appear to not understand that the second derivative shows

concavity. Students who write $x = -1 \quad 0 \quad 1$
 $\frac{d^2y}{dx^2} = -6 \quad 0 \quad 6$
 this is not a p.a.i.

show poor understanding and make the examiner very suspicious.

Students must resist the temptation to add extra points in their graph. Always draw smooth curves not a straight line. Also evaluate endpoints. Do diagrams in pencil.

iv) Since $y = x^3 - 3x$ is an odd function
 $\int_{-1}^1 x^3 - 3x \, dx = 0$ ✓
 Recs - 1

b) Amount eaten = $5 + 3 \cdot 75 + 2 \cdot 8125 + \dots$
 $\underbrace{\hspace{10em}}_{\text{AP } a=5, r=\frac{3}{4}}$
 $\therefore S_{20} = \frac{5}{1 - \frac{3}{4}}$
 $= 20$
 \therefore Since $20 < 25$ Cal will not finish the leaf.
 Recs - 2

c) i) $\frac{d}{dx} (e^{\tan 2x})$
 $= 2 \sec^2 2x e^{\tan 2x}$ ✓

ii) $\int_0^{\frac{\pi}{6}} (\sec^2 2x) e^{\tan 2x} \, dx$
 $= \frac{1}{2} \int_0^{\frac{\pi}{6}} (2 \sec^2 2x) e^{\tan 2x} \, dx$
 $= \frac{1}{2} e^{\tan 2x} \Big|_0^{\frac{\pi}{6}}$
 $= \frac{1}{2} (e^{\tan \frac{\pi}{3}} - e^{\tan 0})$
 $= \frac{1}{2} (e^{\sqrt{3}} - e^0)$
 $= \frac{e^{\sqrt{3}} - 1}{2}$
 Calc - 3

Very few students understood the difference between an integral and an area. Most students incorrectly found the area.

Best answers used limiting sum rather than answers that showed supporting calculations were acceptable.

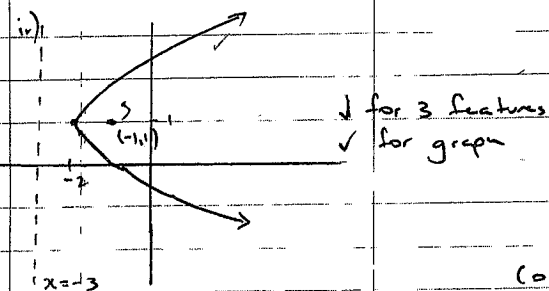
Many students did not pay adequate attention to detail and omitted 2 in $2 \sec^2 2x e^{\tan 2x}$.

Many could not relate the parts and incorrectly found $2e^{\tan 2x}$ instead of $\frac{1}{2} e^{\tan 2x}$.

Q7 a) i) $y^2 - 2y - 4x - 7 = 0$
 $y^2 - 2y = 4x + 7$
 $y^2 - 2y + 1 = 4x + 8$ ✓
 $(y-1)^2 = 4(x+2)$ ✓
 \therefore vertex $(-2, 1)$ ✓

ii) $4a = 4$
 $a = 1$
 \therefore focus $(-1, 1)$ ✓

iii) eqn. of directrix $x = -3$ ✓



b) i) to find the point of intersection solve simultaneously $y = e^{-x}$
 $y = e^x - 1$
 $\therefore e^{-x} = e^x - 1$ ✓
 $1 = e^{2x} - 1$
 $e^{2x} = 2$
 $1 = e^{2x} - e^x$
 $e^{2x} - e^x - 1 = 0$ ✓ (Comm - 2)

Careless mistakes with completing the square. Some students had no idea. Marks carried forward from mistakes made in i)

Most label 3 features and have correct direction of graph.

(Comm - 2)

First mark most students got, however most 'fudged' to show equation resulting in no 2nd mark. No idea what to do after letting 2 equations equal each other.

ii) let $m = e^x$

$$m^2 - m - 1 = 0$$

$$\therefore m = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \times 1 \times -1}}{2 \times 1} \quad \checkmark$$

$$= \frac{1 \pm \sqrt{1+4}}{2}$$

$$= \frac{1 \pm \sqrt{5}}{2}$$

since $e^x > 0$

$$\text{then } e^x = \frac{1 + \sqrt{5}}{2}$$

$$\therefore x = \ln\left(\frac{1 + \sqrt{5}}{2}\right) \quad \checkmark$$

Recs-2

$$= 0.481$$

iii) $A = \int_0^{0.481} e^{-x} - e^x + 1 \, dx$

$$= \left[-e^{-x} - e^x + x \right]_0^{0.481} \quad \checkmark$$

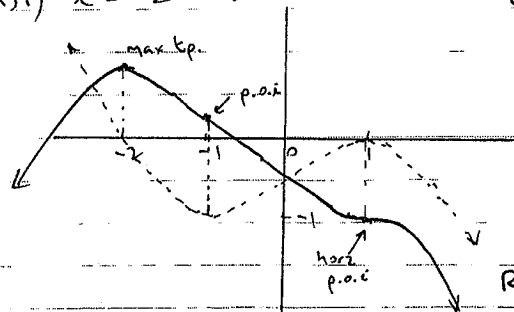
$$= \left(-e^{-0.481} - e^{0.481} + 0.481 \right) - \left(-e^0 - e^0 + 0 \right)$$

$$= 0.2 \text{ units}^2 \quad \checkmark$$

Calc-2

Q8 a) i) $x < -2$ \checkmark

Calc-1



Recs-3

\checkmark for correct concavity

\checkmark for important features

Basic use of 'b'
formula + many
students made
careless errors

only one soln. as

$e^x > 0$

students did not
find $\ln\left(\frac{1 + \sqrt{5}}{2}\right)$

This part could be
completed without answers

above parts correctly

Be careful of signs

$$e^{-x} - (e^x - 1)$$

$$= e^{-x} - e^x + 1$$

learn how to integrate
exponentials.

Need to be careful
with accuracy of
the sketches.

A few students drew
the point of inflexion
at $x = -1$ as a horizontal
p.o.i.

A few students
drew the derivative
graph.

b) i) for no real roots $\Delta < 0$

$$\Delta = b^2 - 4ac$$

$$= (k+3)^2 - 4 \times k \times -1 \quad \checkmark$$

$$= k^2 + 6k + 9 + 4k$$

$$= k^2 + 10k + 9$$

$$\therefore k^2 + 10k + 9 < 0 \quad \checkmark$$

$$(k+9)(k+1) < 0$$

$$\therefore -9 < k < -1 \quad \checkmark$$

Comm-3

ii) for positive definite $\Delta < 0$ and $a > 0$

now $a = k$

since k has to lie between -9 and -1

for $\Delta < 0$ it will never be positive

$$\therefore a \neq 0 \quad \checkmark \quad \text{Comm-1}$$

\therefore it can never be positive definite

c) i) $x = \frac{\pi}{8}$ $y = \sec \frac{\pi}{4}$

$$= \frac{1}{\cos \frac{\pi}{4}}$$

$$= \frac{1}{\frac{1}{\sqrt{2}}}$$

$$= \sqrt{2} \quad \checkmark$$

$$\therefore A\left(\frac{\pi}{8}, \sqrt{2}\right)$$

ii) Volume of solid = Volume of disc - $\pi \int_a^b [f(x)]^2 \, dx$

$$= \pi r^2 h - \pi \int_0^{\frac{\pi}{8}} \sec^2 2u \, du$$

$$= \pi \times (\sqrt{2})^2 \times \frac{\pi}{8} - \pi \int_0^{\frac{\pi}{8}} \sec^2 2u \, du \quad \checkmark$$

$$= \frac{\pi^2}{4} - \pi \int_0^{\frac{\pi}{8}} \sec^2 2u \, du$$

Done fairly well.

Although some student
solved $(k+9)(k+1) < 0$

$$\text{as } k < -1 \quad k < -9$$

Very poorly explained!

Most students didn't

give enough detail.

You had to relate
a to k.

Done well!

Most students didn't

know how to derive

this equation, but it

was pleasing to see

that they then went

on to get the

correct volume

$$= \frac{\pi^2}{4} - \pi \left[\frac{1}{2} \tan 2x \right]_0^{\pi/4} \checkmark$$

$$= \frac{\pi^2}{4} - \pi \left[\frac{1}{2} \tan \frac{\pi}{4} - \frac{1}{2} \tan 0 \right]$$

$$= \frac{\pi^2}{4} - \pi \left(\frac{1}{2} - 0 \right)$$

$$= \frac{\pi^2}{4} - \frac{\pi}{2}$$

$$= \frac{\pi^2 - 2\pi}{4} \text{ units}^3 \checkmark \quad \text{Calc-3}$$

Q9 a) $2 \sin^2 x - 3 \cos x = 0$

$$2(1 - \cos^2 x) - 3 \cos x = 0 \checkmark$$

$$2 - 2 \cos^2 x - 3 \cos x = 0$$

$$2 \cos^2 x + 3 \cos x - 2 = 0$$

$$(2 \cos x - 1)(\cos x + 2) = 0$$

$$\cos x = \frac{1}{2} \quad \cos x = -2 \checkmark$$

no solution as $-1 \leq \cos x \leq 1$

$$x = \frac{\pi}{3} \text{ or } x = \frac{5\pi}{3} \checkmark \quad \text{Recs-3}$$

b) i) $r^2 + h^2 = 9^2$ by Pythagoras' Thm

$$r^2 = 81 - h^2$$

$$r = \sqrt{81 - h^2} \quad \checkmark \quad \text{as } r \geq 0$$

ii) Volume of silo $= \pi r^2 h + \frac{1}{3} \pi r^2 h \checkmark$

$$= \frac{4}{3} \pi r^2 h$$

$$= \frac{4}{3} \pi (81 - h^2) h \checkmark \quad \text{Conn-2}$$

$$= 108\pi h - \frac{4}{3} \pi h^3$$

iii) Max volume occurs when $\frac{dV}{dh} = 0$

$$\frac{dV}{dh} = 108\pi - 4\pi h^2 \checkmark$$

A lot of variations

for $\sin^2 x$

Be careful to answer

$$0 \leq x \leq 2\pi.$$

Very easy mark

When asked to show don't 'fudge' to get answer.

A lot of students did not know the volume of cone formula.

Most students carried forward ii) correctly to gain this mark.

$$\therefore 108\pi - 4\pi h^2 = 0$$

$$4\pi h^2 = 108\pi$$

$$h^2 = 27$$

$$h = \pm 3\sqrt{3}$$

Since h represents a height

$$h = 3\sqrt{3} \checkmark$$

test $\frac{d^2V}{dh^2} = -8\pi h$

when $h = 3\sqrt{3}$

$$\frac{d^2V}{dh^2} = -8\pi \times 3\sqrt{3}$$

$$= -24\sqrt{3}\pi < 0 \checkmark \quad \text{Calc-3}$$

$\therefore h = 3\sqrt{3}$ gives a maximum volume.

c) i) $P(3W) = \left(\frac{1}{50}\right)^3 \checkmark$

$$= \frac{1}{125000}$$

ii) $P(\text{at least one win}) = 1 - P(\text{no win})$

\therefore find n such that

$$1 - \left(\frac{49}{50}\right)^n \geq 0.5 \checkmark$$

$$\left(\frac{49}{50}\right)^n \leq 0.5$$

$$\ln\left(\frac{49}{50}\right)^n \leq \ln(0.5)$$

$$n \ln\left(\frac{49}{50}\right) \leq \ln(0.5)$$

$$n \geq \frac{\ln(0.5)}{\ln\left(\frac{49}{50}\right)}$$

$$\geq 34.3096\dots$$

$$\therefore n = 35$$

\therefore it would require 35 draws. \checkmark

Recs-2

Be careful solving for h - careless errors being made.

Test V'' to test the h for max. volume.

Remember h is a positive value.

Well done

A lot of students didn't know how to set this up.

Very similar question given to you in the last assessment.

If you are coming out with a negative solution. "Alarm bells"

should be sounding, some mistakes being made!

Q10 a) i) $\angle AOB = \frac{2\pi}{8}$
 $= \frac{\pi}{4}$

\therefore Area of $\triangle AOB = \frac{1}{2} \times 1 \times 1 \times \sin \frac{\pi}{4}$ ✓
 $= \frac{1}{2\sqrt{2}} \text{ cm}^2$

\therefore area of the octagon $= 8 \times \frac{1}{2\sqrt{2}}$
 $= \frac{4}{\sqrt{2}}$
 $= 2\sqrt{2} \text{ cm}^2$ ✓

Most students didn't realize they needed

$A = \frac{1}{2} ab \sin C$

Many students used

$A = \frac{1}{2} r^2 (\theta - \sin \theta)$

ii) $\angle MOP = \frac{\pi}{8}$ (as $\triangle POS$ is isosceles and OM is an altitude)

$\therefore \tan \frac{\pi}{8} = \frac{PM}{OM}$
 $= \frac{PM}{1}$

$\therefore PM = \tan \frac{\pi}{8}$

\therefore Area $\triangle POS = \frac{1}{2} \times b \times h$
 $= \frac{1}{2} \times 2 \tan \frac{\pi}{8} \times 1$ ✓
 $= \tan \frac{\pi}{8}$

\therefore Area of the octagon $= 8 \tan \frac{\pi}{8}$ ✓

Most students didn't realize right angle trig was required.

iii) Area of small octagon $<$ area of circle $<$ area of large octagon

$2\sqrt{2} < \pi \times 1^2 < 8 \tan \frac{\pi}{8}$ ✓

$2\sqrt{2} < \pi < 8 \tan \frac{\pi}{8}$ Reas-5

$\sqrt{2} < \frac{\pi}{2} < 4 \tan \frac{\pi}{8}$

b) 9.12% p.a. = 0.76% p.m.

$\therefore A_1 = 400,000 + 0.76\% \times 400,000 - M$
 $= 400,000 (1.0076) - M$

$A_2 = A_1 + 0.76\% A_1 - M$
 $= A_1 (1.0076) - M$

$= [400,000 (1.0076) - M] (1.0076) - M$

$= 400,000 (1.0076)^2 - 1.0076M - M$ ✓

When the question says derive you need to show the relationship between A_1, A_2 and A_3 i.e. establish a pattern.

$F = 400,000 (1.0076)^2 - M (1 + 1.0076)$

$A_3 = A_2 + 0.76\% A_2 - M$

$= A_2 (1.0076) - M$

$= [400,000 (1.0076)^2 - 1.0076M - M] (1.0076) - M$

$= 400,000 (1.0076)^3 - 1.0076^2 M - 1.0076M - M$

$= 400,000 (1.0076)^3 - M (1 + 1.0076 + 1.0076^2)$

continuing the pattern

$\therefore A_{300} = 400,000 (1.0076)^{300} - M (1 + 1.0076 + \dots + 1.0076^{299})$ ✓
 (Comm-2)

ii) $A_{300} = 0$

$\therefore 400,000 (1.0076)^{300} - M (1 + 1.0076 + \dots + 1.0076^{299}) = 0$

AP: $a = 1$
 $r = 1.0076$
 $n = 300$

$400,000 (1.0076)^{300} - M \times \frac{(1.0076^{300} - 1)}{1.0076 - 1} = 0$ ✓

$\therefore M \frac{(1.0076^{300} - 1)}{0.0076} = 400,000 (1.0076)^{300}$

$M = 400,000 (1.0076)^{300} \times \frac{0.0076}{(1.0076^{300} - 1)}$

$= 3389.72$

\therefore monthly repayments are \$3389.72 ✓
 Reas-2

iii) find A_{120} when $M = 3389.72$

$\therefore A_{120} = 400,000 (1.0076)^{120} - 3389.72 (1 + 1.0076 + \dots + 1.0076^{119})$
 $= 400,000 (1.0076)^{120} - 3389.72 \times \frac{(1.0076^{120} - 1)}{1.0076 - 1}$

$= \$331,863$ (to nearest dollar) ✓

Comm-1

Done very well! Many students know how to manipulate the Sn formula to get the correct value of M.

Reasonably well done for those who reached it!

iv) 12% pa. = 1% pm

$$A_n = 331863(1.01)^n - 3800(1 + 1.01 + \dots + 1.01^{n-1})$$

$$= 331863(1.01)^n - 3800 \times \frac{(1.01^n - 1)}{1.01 - 1} \checkmark$$

$$= 331863(1.01)^n - 380000(1.01^n - 1)$$

now $A_n = 0$

$$\therefore 331863(1.01)^n - 380000(1.01^n - 1) = 0$$

$$331863(1.01)^n - 380000(1.01^n) + 380000 = 0$$

$$48137(1.01)^n = 380000$$

$$(1.01)^n = 7.894\dots$$

$$n = \frac{\ln(7.894\dots)}{\ln(1.01)}$$

$$= 207.64\dots \checkmark$$

It would require a further 208 months

Reas - 2

For the students who attempted this question, silly numerical errors made it difficult to arrive at the correct value.