

2749

Wallace



SCEGGS Darlinghurst

2004

Higher School Certificate
Trial Examination

Mathematics

This is a TRIAL PAPER only and does not necessarily reflect the content or format of the Higher School Certificate Examination for this subject.

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General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total marks – 120

- Attempt Questions 1–10
- All questions are of equal value

Total marks – 120

Attempt Questions 1–10

All questions are of equal value

Answer each question on a NEW page

Question 1 (12 marks)

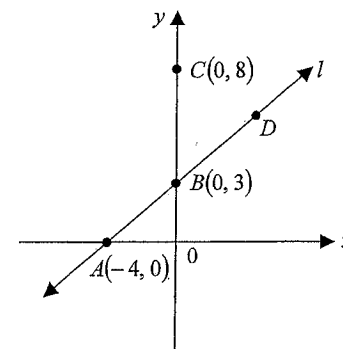
Marks

- (a) Express $\frac{5\pi}{18}$ radians in degrees. 1
- (b) The line $2x + ky - 5 = 0$ passes through the point $(3, -1)$. Find the value of k . 2
- (c) Express $\frac{6}{2 - \sqrt{3}}$ in the form $a + b\sqrt{3}$. 2
- (d) Factorise fully: 2
- $$a^2 - b^2 - 2a + 2b$$
- (e) Solve the inequality and graph your solution on a number line. 2
- $$6 - \frac{x}{3} > 7$$
- (f) If 7 apples and 2 oranges cost \$4, while 5 apples and 4 oranges cost \$4.40 find the cost of each apple and orange. 3

Question 2 (12 marks) START A NEW PAGE

Marks

The line l has intercepts at $A(-4, 0)$ and $B(0, 3)$. D is a point on line l and C has co-ordinates $(0, 8)$.



Not to scale

- (a) Show that the equation of line l is $3x - 4y + 12 = 0$. 2
- (b) Find the lengths of AB and BC . 2
- (c) What type of triangle is $\triangle ABC$? Give a clear reason. 1
- (d) If B is the midpoint of AD , find the co-ordinates of D . 2
- (e) Write down the equation of the circle through A , C and D with centre B . 1
- (f) Find the shortest distance from point C to the line l . 2
- (g) Hence, or otherwise, find the area of $\triangle ABC$. 2

Question 3 (12 marks) START A NEW PAGE

Marks

(a) Differentiate:

(i) $y = (2 - 3x)^4$

1

(ii) $f(x) = \frac{e^x}{x+1}$

2

(b) Find:

(i) $\int \cos 2x \, dx$

2

(ii) $\int_1^2 \frac{2x^2 + 1}{x^2} \, dx$

2

(c) The roots of $2x^2 + 6x - 7 = 0$ are α and β .

Find:

(i) $\alpha + \beta$

1

(ii) $\frac{1}{\alpha^2 + \beta^2}$

2

(d) Find the value of k if

2

$$\int_1^k \frac{dx}{2x-1} = \log_e 3$$

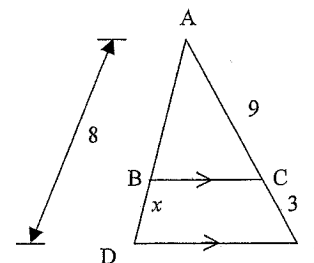
Question 4 (12 marks) START A NEW PAGE

Marks

(a) Find the equation of the normal to the curve $y = \sqrt{1-x}$ at the point $x = -3$.

3

(b)



Not to scale

2

Find the value of x , giving reasons.

(c) (i) Find the locus of the point $P(x, y)$ which moves so that its distance from point $A(4, 0)$ is always twice its distance from point $B(1, 0)$, ie $PA = 2PB$.

3

(ii) Describe this locus.

1

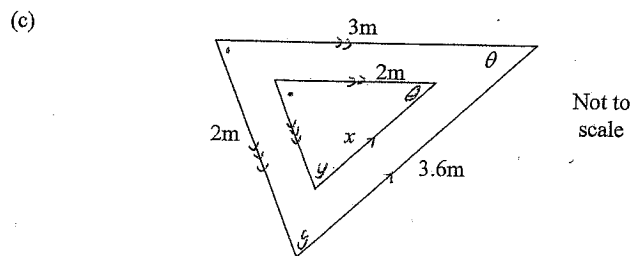
(d) Find the exact value of the area bounded by the curve $y = x^2 - 7x + 10$, the x -axis and the lines $x = 0$ and $x = 3$.

3

Question 5 (12 marks) START A NEW PAGE

Marks

- (a) Solve for x : $3^{2x} - 10 \cdot 3^x + 9 = 0$. 2
- (b) Two dates are selected at random for a school dance. One date is taken at random from those in October and another is taken from November. 1
- What is the probability that both are the twentieth of the month?



The diagram shows a small triangular garden bed surrounded by a concrete path. The sides of the path are parallel to the sides of the garden and the two triangles are similar.

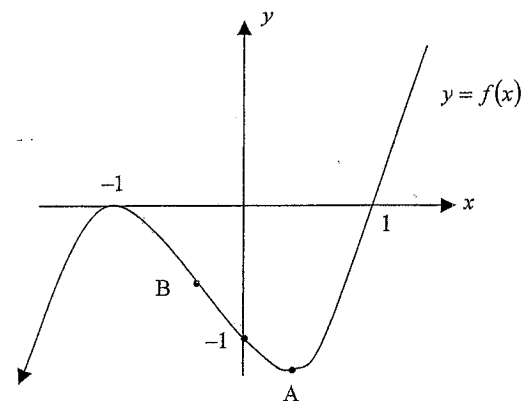
- (i) Find the length of x . 1
- (ii) Prove that the angle θ is 34° correct to the nearest degree. 1
- (iii) Hence find the area of the concrete path correct to 2 significant figures. 2

Question 5 continues on page 7

Question 5 (continued)

Marks

(d)



The curve is $y = f(x)$ where $f(x) = x^3 + x^2 - x - 1$.

It has a maximum stationary point at $(-1, 0)$ and a minimum stationary point at A. It has an inflexion point at B.

- (i) Find the co-ordinates of the point A. 2
- (ii) Show that the co-ordinates of B are $\left(-\frac{1}{3}, -\frac{16}{27}\right)$. 2
- (iii) Copy the sketch of $y = f(x)$ and, on the same diagram, sketch $y = f'(x)$. 1

Question 6 (12 marks) START A NEW PAGE

Marks

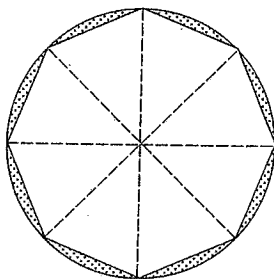
- (a) Calvin and Hobbes are making a large snowman. To collect the snow Calvin walks 1m away from the snowman, fills a bucket with snow and takes it back to the snowman. Then he walks 1.5m away, fills the bucket and returns. Each time he goes to fill the bucket he walks 0.5m further away from the snowman and returns.
- (i) How far from the snowman does Calvin go to fill the 5th bucket? 1
- (ii) In total he uses 20 buckets of snow. Find the total distance Calvin walks collecting all of the snow. 2

- (b) Consider the geometric series

$$m + 2m^2 + 4m^3 + 8m^4 \dots$$

- (i) For which values of m will the series have a limiting sum? 1
- (ii) If $m = \frac{1}{4}$ find the limiting sum. 2

- (c) 3



Not to scale

A table top is made in the shape of a regular octagon by removing the shaded segments from a circular piece of wood as shown. The radius of the original circle was 1.5m. Prove that the total area of the shaded segments is:

$$9 \frac{(\pi - 2\sqrt{2})}{4} \text{ m}^2$$

Question 6 continues on page 9

Question 6 (continued)

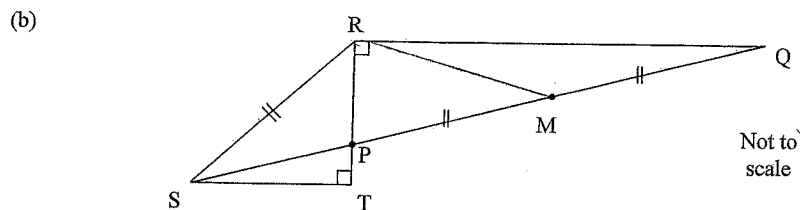
Marks

- (d) For the equation $4x^2 + (m - 4)x - m = 0$:
- (i) write an expression for the discriminant in fully factorised form. 2
- (ii) Explain why the equation has rational roots for all rational values of m . 1

Question 7 (12 marks) START A NEW PAGE

Marks

- (a) (i) Sketch the curve $y = 2 \sin x - 1$ in the domain $0 \leq x \leq 2\pi$, showing all important features. 2
- (ii) Shade the region for which $y \leq 2 \sin x - 1$ and $y \geq 0$ in this domain. 1
- (iii) Find the exact value of the area of this shaded region. 3



The triangles PRQ and RTS are right angled triangles and $PM = MQ = SR$ as shown in the diagram.

Copy the diagram onto your answer page.

- (i) Explain why $\angle PST = \angle PQR$. 1
- (ii) The point A is positioned so that PRQA is a rectangle. Label point A on your diagram. 1
- (iii) Explain why M is the midpoint of RA. 1
- (iv) Explain why $PM = RM$. 1
- (v) Hence prove that $\angle PSR = 2 \times \angle PST$. 2

Question 8 (12 marks) START A NEW PAGE

Marks

- (a) (i) Copy and complete the table given for the function $y = x + \log_e x$. Values may be calculated correct to 2 decimal places. 1

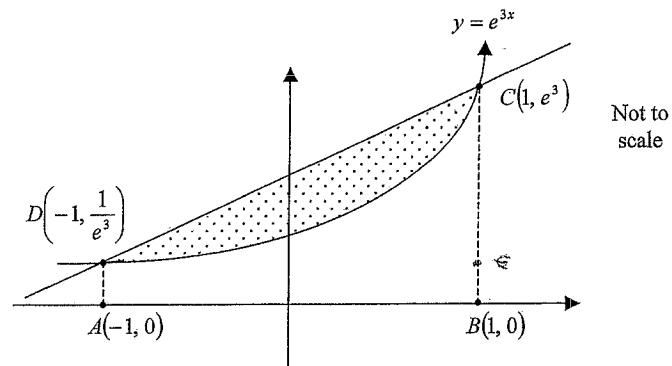
x	1	1.5	2	2.5	3
y	1				4.10

- (ii) Hence find an approximation to $\int_1^3 x + \log_e x \, dx$ using Simpson's Rule with 5 function values. 2

- (b) Find the co-ordinates of the focus of the parabola 2

$$x^2 - 8x - 12y - 8 = 0$$

- (c)



In the diagram above, a straight line has been drawn intersecting $y = e^{3x}$ at the points $C(1, e^3)$ and $D(-1, \frac{1}{e^3})$. A has co-ordinates $(-1, 0)$ and B has co-ordinates $(1, 0)$. The area between the line and the curve is shaded.

- (i) Show that the area of the trapezium ABCD is given by $\frac{e^6 + 1}{e^3}$. 1
- (ii) Hence, or otherwise, find the exact value of the shaded area. 3
- (d) If $\tan \theta = \frac{1}{\sqrt{2}}$ and $180^\circ \leq \theta \leq 270^\circ$, prove that $\frac{1}{\cot \theta - \operatorname{cosec} \theta} = \sqrt{3} - \sqrt{2}$. 3

Question 9 (12 marks) START A NEW PAGE

Marks

- (a) In a lottery, the probability of the jackpot prize being won in any draw is $\frac{1}{60}$.
- (i) What is the probability that the jackpot prize will be won in each of four consecutive draws? 1
- (ii) How many consecutive draws need to be made for there to be a greater than 98% chance that at least one jackpot prize will have been won? 3
- (b) A bowl is formed by rotating the region bounded by the curve $y = x^3$, the line $x = 3$ and the x -axis about the y -axis. 4
- Find the volume of the bowl, giving your answer correct to 3 significant figures.
- (c) On 1st January, 1994, the population of Baileyville was 25000 and the population of Brimfield Park was 12500. Fabio noticed that the population of the towns t years later could be found using the following equations:
- Baileyville: $A = 25000e^{-pt}$
Brimfield Park: $B = 12500e^{qt}$
- (i) Ten years later, on 1st January, 2004, the population of Baileyville and Brimfield Park are 20000 and 15000 respectively. Find the exact values of p and q . 2
- (ii) In what year will the populations of Baileyville and Brimfield Park be equal? 2

Marks

Question 10 (12 marks) START A NEW PAGE

- (a) Every year, Beryl deposits \$1500 into a superannuation fund that earns 4% p.a. interest, compounded annually. How much will Beryl have in her superannuation fund after 20 years? 2
- (b) The velocity of a particle in metres per second, moving in a straight line is given by the equation:

$$\dot{x} = 1 - \frac{2}{t+3}$$

The particle is initially at the origin.

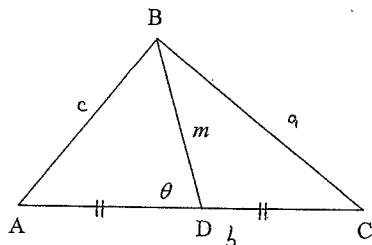
- (i) Prove that the particle is never at rest. 1
- (ii) Find an expression for the displacement after time t seconds. 2
- (iii) Find the distance travelled in the first second. 1

Question 10 continues on page 14

Question 10 (continued)

Marks

(c) ABC is a triangle and D is the midpoint of AC.



Also $BC = a$, $AC = b$, $AB = c$ and $BD = m$.

(i) Simplify $\cos(180 - \theta)$

1

(ii) Show that $\cos\theta = \frac{4m^2 + b^2 - 4c^2}{4mb}$.

2

(iii) Hence show that:

3

$$a^2 + c^2 = 2m^2 + \frac{1}{2}b^2$$

End of Paper

Trial

Question	Comm	Reas	Calc
1	-	-	-
2	e) (1)	g) (2)	
3		d) (2)	a) b) (7)
4	c) (i) (1)	b) (2)	d) (3)
5		e) (4)	d) (5)
6	a) (ii) (1)	c) (3)	
7	a) (i) (2) b) (i)(ii)(iii)(iv) (4)	b) (v) (2)	
8		d) (3)	
9		c) (ii) (2)	b) (4)
10	b) (i) 1	c) (ii)(iii) 5	b) (ii)(iii) (3)
Q. 10.		25	22

① a) $\frac{5\pi}{18} = \frac{5 \times 180}{18}$ ✓

$= 50^\circ$

b) $2 \times 3 - k - 5 = 0$ ✓

$-k + 1 = 0$

$k = 1$ ✓

c) $\frac{6}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = 12 + 6\sqrt{3}$ ✓

d) $(a-b)(a+b) - 2(a-b)$ ✓

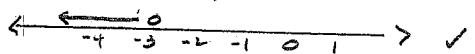
$(a-b)(a+b-2)$ ✓

e) $6 - \frac{x}{3} > 7$

$18 - x > 21$

$-x > 3$

$x < -3$ ✓



f) $7A + 2G = 4$ ✓

$5A + 4G = 4 \cdot 4$ ✓

$14A + 4G = 8$

$9A = 3 \cdot 6$

$A = 0 \cdot 4$ ✓

$2 \cdot 8 + 2G = 4$

$G = 0 \cdot 6$

∴ apples cost 40 cents
oranges cost 60 cents. ✓

② a) gradient = $\frac{3}{4}$ ✓

equation: $y - 3 = \frac{3}{4}x$

$4y - 12 = 3x$

$3x - 4y + 12 = 0$ ✓

b) $AB^2 = 4^2 + 3^2$ ✓

$AB = 5u$ ✓

$BC = 5u$ ✓

c) Isosceles since 2 sides
are equal ✓ (Comm)

d) $0 = \frac{a-4}{2}$

$a = 4$ ✓

$3 = \frac{b+0}{2}$

$b = 6$ ✓

∴ D is (4, 6)

e) $x^2 + (y-3)^2 = 25$ ✓

f) $d = \left| \frac{0 \times 3 - 4 \times 8 + 12}{\sqrt{3^2 + 4^2}} \right|$ ✓

$= \left| \frac{-32 + 12}{5} \right|$

dist = 4u. ✓

g) Area = $\frac{1}{2}bh$

$= \frac{1}{2} \times 5 \times 4$ ✓

$= 10u^2$. (Reas) ✓

Comm / 1 Reas / 2.

③ a) (i) $y' = -12(2-3x)^3$ (Calc)

well done.

(ii) $f'(x) = \frac{(x+1)e^x - e^x}{(x+1)^2}$ (Calc)
 $= \frac{xe^x}{(x+1)^2}$

use Quotient Rule and make sure it is done correctly.

b) (i) $\int \cos 2x dx = \frac{1}{2} \sin 2x + c$ (Calc)

use Standard Integral sheet.

(ii) $\int_1^2 \frac{2x^2+1}{x^2} dx = \int_1^2 (2+x^{-2}) dx$ Divide First.
 $= \left[2x - \frac{1}{x} \right]_1^2$
 $= \left(4 - \frac{1}{2} \right) - (2 - 1)$

$2\frac{1}{2}$ (Calc)

c) (i) $\alpha + \beta = -\frac{6}{2} = -3$

well done.

(ii) $\alpha\beta = -\frac{7}{2}$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$
 $= 9 + 7$
 $= 16$

This was not well known.

$\therefore \frac{1}{\alpha^2 + \beta^2} = \frac{1}{16}$

d) $\int_1^k \frac{dx}{2x-1} = \frac{1}{2} [\log_e(2x-1)]_1^k$
 $= \frac{1}{2} [\log_e(2k-1) - \log_e 1]$

log Integration needs more work.

$= \frac{1}{2} \log_e(2k-1)$

$\therefore \log_e(2k-1) = 2 \log_e 3$

$2k-1 = 9$

$k = 5$

log law: $2 \log_e 3 = \log_e 3^2$

④ a) $y = (1-x)^{\frac{1}{2}}$
 $y' = -\frac{1}{2}(1-x)^{-\frac{1}{2}}$

$= \frac{-1}{2\sqrt{1-x}}$

if $x = -3$, $y' = \frac{-1}{2\sqrt{4}} = -\frac{1}{4}$

$y = \sqrt{4} = 2$

too many forgot the $x-1$

\therefore normal: $y-2 = 4(x+3)$
 $4x - y + 14 = 0$

simplify $\sqrt{4} = 2!$

b) $\frac{x}{8-x} = \frac{3}{9}$ (lines cut off proportional intercepts on trans.)

$9x = 24 - 3x$

$12x = 24$ (Reas)

$x = 2$

c) (i) $PA^2 = (x-4)^2 + y^2$

$PB^2 = (x-1)^2 + y^2$

$PA^2 = 4PB^2$

$x^2 - 8x + 16 + y^2 = 4(x^2 - 2x + 1 + y^2)$
 $= 4x^2 - 8x + 4 + 4y^2$

many forgot to square the 2.

$\therefore 3x^2 + 3y^2 = 12$

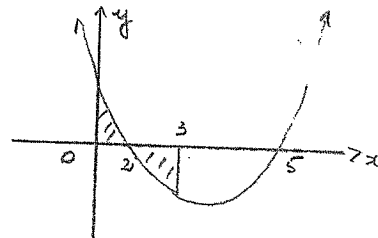
$x^2 + y^2 = 4$

\therefore circle centre (0,0) radius 2 (Comm)

had to give the centre and radius.

d) $x^2 - 7x + 10 = 0$ (Calc)

$(x-5)(x-2) = 0$



$$\begin{aligned} \text{Area}_1 &= \int_0^2 x^2 - 7x + 10 \, dx \\ &= \left[\frac{x^3}{3} - \frac{7x^2}{2} + 10x \right]_0^2 \\ &= \frac{8}{3} - 14 + 20 \\ &= 8\frac{2}{3} \text{ m}^2 \end{aligned}$$

$$\begin{aligned} \text{Area}_2 &= \left| \int_2^3 x^2 - 7x + 10 \, dx \right| \\ &= \left| \left[\frac{x^3}{3} - \frac{7x^2}{2} + 10x \right]_2^3 \right| \\ &= \left| \left(9 - \frac{63}{2} + 30 \right) - \left(\frac{8}{3} - 14 + 20 \right) \right| \\ &= 1\frac{1}{6} \text{ m}^2 \\ \therefore \text{Area required} &= 9\frac{5}{6} \text{ m}^2 \end{aligned}$$

too many didn't sketch it.

if correctly did \int_0^3 only
(one mark only).

5) a) Let $y = 3^x$
 $y^2 - 10y + 9 = 0$
 $(y-9)(y-1) = 0$
 $y = 9$ or 1
 $\therefore x = 2$ or 0

b) Prob = $\frac{1}{30} \times \frac{1}{31} = \frac{1}{930}$

c) (i) $\frac{x}{3 \cdot 6} = \frac{2}{3}$

$$x = 2.4 \text{ m}$$

(ii) $\cos \theta = \frac{3^2 + 3 \cdot 6^2 - 2^2}{2 \times 3 \times 3 \cdot 6}$
 $= 0.83148 \dots$ (read)
 $\theta = 33.7 \dots$
 $= 34^\circ$ (nearest degree)

(iii) $\text{Area}_1 = \frac{1}{2} \times 2 \times 2.4 \times \sin 34^\circ$
 $= 1.342 \dots$

$$\begin{aligned} \text{Area}_2 &= \frac{1}{2} \times 3 \times 3 \cdot 6 \times \sin 34^\circ \\ &= 3.0196 \end{aligned}$$

\therefore Area path = 1.7 m^2 (2 s.f.)

d) (i) $y = x^3 + 2x^2 - x - 1$
 $y' = 3x^2 + 2x - 1$
 $= (3x-1)(x+1)$

if $y' = 0$, $x = \frac{1}{3}$ or -1

if $x = \frac{1}{3}$, $y = \left(\frac{1}{3}\right)^3 + \left(\frac{1}{3}\right)^2 - \frac{1}{3} - 1$
 $= -\frac{15}{27}$ (calc)
 $\text{A is } \left(\frac{1}{3}, -\frac{15}{27}\right)$

(ii) $y'' = 6x + 2$

if $y'' = 0$, $x = -\frac{1}{3}$
 $y = \left(-\frac{1}{3}\right)^3 + \left(-\frac{1}{3}\right)^2 + \frac{1}{3} - 1$
 $= -\frac{16}{27}$

not well known.

$3^x = 9$ $3^x = 1$ errors here
 $\therefore x = 2$ $\therefore x = 0$ none case!
 well done.

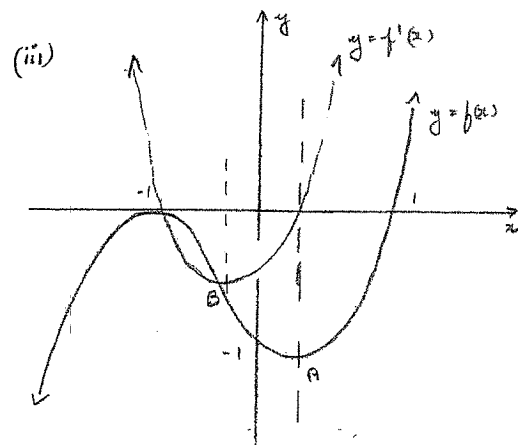
many calculation errors.
 Show all working for marks
 to be allocated.

This question was well done
 by most.

if $x < -\frac{1}{3}$, $y'' < 0$

$x > -\frac{1}{3}$, $y'' > 0$

since concavity has changed
 $(-\frac{1}{3}, -\frac{16}{27})$ is inflexion pt B.



Two sketches on the same axes.

The minimum point of $y = f'(x)$ should line up with B

b) a) (i) $T_1 = 1$, $T_2 = 1.5$, $T_3 = 2$

A.P., $a = 1$, $d = 0.5$

$$T_5 = a + 4d = 1 + 2$$

distance = 3m

(ii) $D = \frac{20 \times 2}{2} (2 + 19 \times 0.5)$

distance = 230m

b) (i) $t = 2m$

$\therefore t < 2m < 1$

$-\frac{1}{2} < m < \frac{1}{2}$ for limiting sum

(ii) $S = \frac{\frac{1}{4}}{1 - \frac{1}{2}} = \frac{1}{2}$

c) Area one triangle

$$= \frac{1}{2} \times \frac{3}{2} \times \frac{3}{2} \times \sin \frac{\pi}{4}$$

$$= \frac{9}{8\sqrt{2}} m^2$$

Area eight triangles = $\frac{9}{\sqrt{2}} m^2$

$$= \frac{9\sqrt{2}}{2} m^2$$

Area circle = $\pi \times \frac{3}{2} \times \frac{3}{2}$

$$= \frac{9\pi}{4} m^2$$

\therefore Shaded area = $\frac{9\pi}{4} - \frac{9\sqrt{2}}{2}$

$$= \frac{9}{4} (\pi - 2\sqrt{2}) m^2$$

(Reas)

must $\times 2$ (goes there and back)

not many did this correctly.

- use fractions (not decimals)

- most used 8 x area segment

- have to correct formula!

- lots of fudging.

$$\begin{aligned}
 4) (i) \Delta &= (m-4)^2 + 16m \\
 &= m^2 - 8m + 16 + 16m \\
 &= m^2 + 8m + 16 \\
 &= (m+4)^2
 \end{aligned}$$

(ii) for the roots to be rational, Δ must be a perfect square.

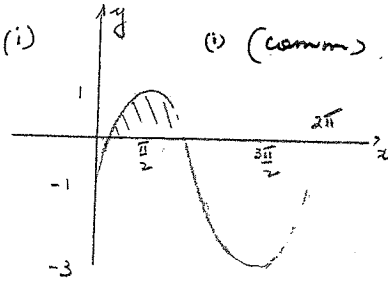
$(m+4)^2$ is a perfect square provided m is rational. (comm)

many didn't realise $c = -m$

many thought that to be rational $\Delta > 0$!

had to link $(m+4)^2$ to fact needed a perfect sq.

⑦ a) (i) (i) (comm)



$$\text{Area} = \int_a^b 2\sin x - 1 \cdot dx$$

$$\begin{aligned}
 \text{if } \sin x &= \frac{1}{2} \\
 x &= \frac{\pi}{6}, \frac{5\pi}{6}
 \end{aligned}$$

$$\begin{aligned}
 \therefore \text{Area} &= \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} 2\sin x - 1 \cdot dx \\
 &= \left[-2\cos x - x \right]_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \\
 &= -2\cos \frac{5\pi}{6} - \frac{5\pi}{6} + 2\cos \frac{\pi}{6} + \frac{\pi}{6} \\
 &= 2 \times \frac{\sqrt{3}}{2} - \frac{5\pi}{6} + 2 \times \frac{\sqrt{3}}{2} + \frac{\pi}{6} \\
 &= 2\sqrt{3} - \frac{2\pi}{3} \quad 3.
 \end{aligned}$$

Graphs very poor. Confusion with $y = \sin 2x - 1$ etc.

Must show maximum, minimum values and where curve crosses x-axis.

(ii) Very poor shading.

(iii) Integration usually correct. many students found the evaluation of $\cos \frac{5\pi}{6}$ etc difficult

b) (i) $\angle RTS = \angle TRQ$ (given) comm (i) \rightarrow (iv)

$\therefore RQ \parallel ST$ (alt. \angle 's =)

$\therefore \angle PST = \angle PQR$ (alt \angle 's = $RQ \parallel ST$)

This must be proved

(iii) M is midpoint PQ (given)

(ii) to (iv) well done.

$\therefore M$ is also midpoint RA
(diagonals of a rectangle bisect each other)

(iv) $RA = PQ$ (diagonals of a rectangle are equal)

$\therefore PM = RM$ (M is midpoint of equal intervals)

(iv) Let $\angle MRP = d$ (Reas)

$PM = RM$ (proved in iii)

$\therefore \angle MRP = \angle RPM = d$ (opp equal sides in ΔMRP)

$\therefore \angle RMP = 180^\circ - 2d$ (angle sum of $\Delta RPM = 180^\circ$)

$\therefore RS = RM$ ($RS = PM = RM$)

$\therefore \angle RMS = \angle PSR$ (opp equal sides in ΔRSM)

$\therefore \angle PSR = 180^\circ - 2d$

But $\angle SPT = \angle MPR = d$ (vert. opp \angle 's =)

$\therefore \angle PST = 180^\circ - 90^\circ - d$ (angle sum of $\Delta 180^\circ$)
 $= 90^\circ - d$

$\therefore \angle PSR = 2\angle PST$

(v) Well done to the students who succeeded in this question.

8 a) (i)

x	1	1.5	2	2.5	3
y	1	1.91	2.69	3.42	4.10

had to be to 2 d.p.

(ii) $\int_1^3 x + \log_e x \, dx$

$$= \frac{0.5}{3} [1 + 4.1 + 4(1.91 + 3.42) + 2 \times 2.69]$$

$$= 5.3$$

mostly well done.

b) $12y + 8 = x^2 - 8x$

$$12y + 8 + 16 = x^2 - 8x + 16$$

$$12(y+2) = (x-4)^2$$

vertex (4, -2)

focal length 3 units

focus (4, 1)

O.K.

Some didn't factorise

$12y+24$ and \therefore didn't

find y coord of vertex correctly.

c) (i) Area = $\frac{1}{2} \cdot AB \cdot (AD + BC)$

$$= \frac{1}{2} \times 2 \times \left(\frac{1}{e^3} + e^3 \right)$$

$$\text{Area} = \frac{e^6 + 1}{e^3} e^2$$

fine.

(ii) Area under curve

$$= \int_{-1}^1 e^{3x} \, dx$$

$$= \frac{1}{3} \left[e^{3x} \right]_{-1}^1$$

$$= \frac{1}{3} (e^3 - e^{-3}) e^2$$

$$\text{Area} = \frac{1}{e^3} + e^3 - \frac{e^5}{3} + \frac{1}{3e^3}$$

$$= \frac{2}{3} e^3 + \frac{4}{3e^3} e^2$$

too many didn't integrate correctly!

9) a) (i) $P(4 \text{ wins}) = \left(\frac{1}{60}\right)^4$

(ii) $P(\text{at least 1 win})$
 $= 1 - P(\text{no wins})$

Let no. of draws be n .

$$\therefore 1 - \left(\frac{59}{60}\right)^n > 0.98$$

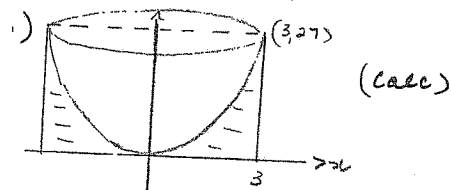
$$-\left(\frac{59}{60}\right)^n > -0.02$$

$$\left(\frac{59}{60}\right)^n < 0.02$$

$$n \log_e \frac{59}{60} < \log_e 0.02$$

$$n > \frac{\log_e 0.02}{\log_e \frac{59}{60}}$$

n is 233 draws.



$$V_1 = \pi \int_0^{27} x^2 dy$$

$$= \pi \int_0^{27} y^{\frac{2}{3}} dy$$

$$= \pi \left[\frac{3}{5} y^{\frac{5}{3}} \right]_0^{27}$$

$$= \pi \times \frac{3}{5} \times 243$$

$$= \frac{729\pi}{5}$$

$$= (458 \text{ (3 s.f.)})$$

(ii) Be careful of inequality signs.

b) Poor execution.

• Most students found the incorrect volume.

• did not find the y value (27)

• did not find x^2 correctly

• forgot π .

$$V = \pi R^2 H$$

$$= \pi \times 9 \times 27$$

$$= 243\pi$$

\therefore Volume of bowl

$$= (243 - \frac{729}{5})\pi \text{ m}^3$$

$$= \frac{48}{5}\pi \text{ m}^3$$

$$= 305 \text{ m}^3 \text{ - pt}$$

e) (i) $A = 25000e^{-10p}$

$$20000 = 25000e^{-10p}$$

$$e^{-10p} = \frac{4}{5}$$

$$p = -\frac{1}{10} \log_e \frac{4}{5}$$

$$B = 12500e^{9t}$$

$$15000 = 12500e^{9t}$$

$$\therefore e^{9t} = \frac{6}{5}$$

$$9t = \frac{1}{10} \log_e \frac{6}{5}$$

$$25000e^{-pt} = 12500e^{9t}$$

$$2e^{-pt} = e^{9t}$$

$$\log_e 2e^{-pt} = \log_e e^{9t}$$

$$\log_e 2 + \log_e e^{-pt} = \log_e e^{9t}$$

$$\log_e 2 - pt = 9t$$

$$pt + 9t = \log_e 2$$

$$t = \frac{\log_e 2}{p+9} \quad (\text{Reas})$$

$$= \frac{\log_e 2}{0.04054651}$$

$$= 17.095 \dots \text{ years}$$

201 equal populations

well done.

Difficult question.

Must take logs of both sides carefully.

10) a) First \$1500 becomes $1500(1.04)^{20}$

Second \$1500 becomes $1500(1.04)^{19}$

last \$1500 becomes $1500(1.04)$

Total:

$$1500[1.04 + 1.04^2 + \dots + 1.04^{20}]$$

$$= 1500 \times 1.04 \frac{(1.04^{20} - 1)}{1.04 - 1}$$

$$= \$46453.80 \text{ (nearest cent)}$$

b) (i) if $x=0$ at rest.

$$\therefore \frac{2}{t+3} = 1$$

$$2 = t+3 \quad (\text{Cross})$$

$$t = -1$$

Since $t \geq 0$, never at rest.

(ii) $x = t - 2 \log_e(t+3) + c$

if $x=0, t=0$

$$0 = 0 - 2 \log_e 3 + c$$

$$c = 2 \log_e 3 \quad (\text{Calc})$$

$$\therefore x = t - 2 \log_e(t+3) + 2 \log_e 3$$

iii) distance travelled

$$= 1 - 2 \log_e 4 + 2 \log_e 3$$

$$\quad \quad \quad (\text{Calc})$$

(i) $\cos(180^\circ - \theta) = -\cos \theta$

ii) $\cos \theta = \frac{BD^2 + AD^2 - AB^2}{2 \cdot BD \cdot AD}$ (Cosine Rule in $\triangle BAD$)

$$= \frac{m^2 + \frac{b^2}{4} - c^2}{2 \times m \times \frac{b}{2}} \times \frac{4}{4}$$

$$= \frac{4m^2 + b^2 - 4c^2}{4mb} \quad (\text{Reas})$$

- many thought $a=1$.
- some only did one year.

had to find $t=-1$ when $v=0$ and had to explain why this couldn't be.

had to simplify

N.B. $(\frac{b}{2})^2$ not $\frac{1}{2}b^2$ otherwise alright.

(iii) Using Cosine Rule in $\triangle BDC$
 $\cos(180^\circ - \theta) = \frac{BD^2 + DC^2 - BC^2}{2 \times BD \times DC}$

$$-\cos \theta = \frac{m^2 + \frac{b^2}{4} - a^2}{2 \times m \times \frac{b}{2}} \times \frac{4}{4}$$
$$= \frac{4m^2 + b^2 - 4a^2}{4mb}$$

$$\therefore \frac{4m^2 + b^2 - 4c^2}{4mb} = \frac{4a^2 - b^2 - 4m^2}{4mb}$$

$$\therefore 4m^2 + b^2 - 4c^2 = 4a^2 - b^2 - 4m^2$$

$$\therefore 4a^2 + 4c^2 = 8m^2 + 2b^2$$

$$a^2 + c^2 = 2m^2 + \frac{b^2}{2}$$

(RHS)

many had no idea.

nice fudging.