Student Number: ______________

SCEGGS Darlinghurst

YR 11/12

HSC Assessment #1

Wednesday 22nd November, 2006

Mathematics

General Instructions

- Time allowed – 60 minutes
- Weighting 15%
- This paper has **four** questions
- Attempt **all** questions and show all necessary working
- Marks may be deducted for careless or badly arranged work
- Write using blue or black pen, **diagrams in pencil**
- Write your student number at the top of each page
- Start each question on a new page
- Approved calculators, mathematical templates and geometrical instruments may be used

Question	Communication	Reasoning	Calculus	Marks
1	/2	/1	/2	/10
2	/3	/2	/5	/10
3	/3		/5	/10
4		/4	/3	/10
TOTAL	/8	/7	/15	/40

QUESTION 1: (10 marks)

(a) Sketch the locus of the point $P(x,y)$ that moves so that it is always 2 units to the right of the y-axis. Hence write down its equation. 2

(b) By using the substitution $u = x^3$, solve the equation 2

$$x^6 + 7x^3 - 8 = 0$$

(c) Find the values of P, Q and R, given that 3

$$3x^2 - 5x + 7 \equiv P(x-1)^2 + Q(x-1) + R$$

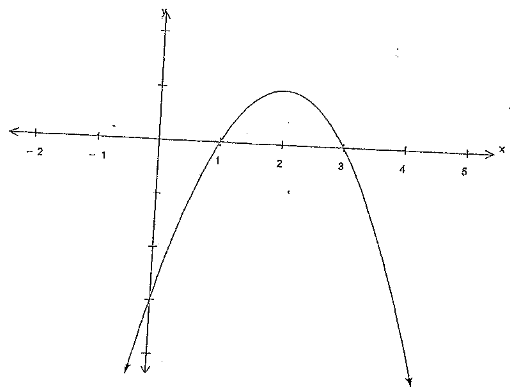
(d) The tangent to the curve $y = x^2 - px + 8$ has a gradient of 5 when $x = -2$. Find the value of p. 2

(e) For the function $y = 2x^8 + 1$ it is known that both $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} = 0$ when $x = 0$. 1

What type of stationary point occurs at $x = 0$? Explain your answer clearly.

QUESTION 2: (10 marks) START A NEW PAGE

(a) The diagram shows a curve $y = f(x)$.
Copy or trace this diagram onto your answer page.
Either on the same axes or directly underneath your copy, draw a possible graph of $y = f'(x)$. 1



(b) Loretta was asked to find the equation of the normal to the curve $y = \sqrt[3]{2x-3}$ at the point (2,1)

She wrote down that the derivative of the function was $\frac{dy}{dx} = \frac{1}{3}(2x-3)^{\frac{-2}{3}}$

and then she used it to calculate that the gradient of the tangent was equal to $\frac{1}{3}$.

(i) Did Loretta differentiate the function correctly? Explain your answer clearly. 1

(ii) Using Loretta's working, or otherwise, find the equation of the normal to the given curve at (2,1). 2

QUESTION 2 continues on the next page.

QUESTION 2 continued.

(c) The points A and B are (1,2) and (-3,-1) respectively.

The point $P(x,y)$ moves so that $\angle APB$ is a right angle.

(i) Show that the locus of P has the equation 2

$$x^2 + y^2 + 2x - y - 5 = 0$$

(ii) Hence give an accurate geometrical description of the locus of P. 2

(d) Find the values of x for which the curve $y = 5 + 4x^3 - x^4$ is concave up. 2

QUESTION 3: (10 marks) START A NEW PAGE

Consider the function $f(x) = 5x^3(x-2)$

(i) Find the coordinates of the turning points of $y = f(x)$ and determine their nature. 3

(ii) Find the coordinates of any points of inflexion. 2

(iii) Using at least half a page, sketch the graph of $y = f(x)$ in the domain $-1 \le x \le 3$. 3

Your sketch must clearly show the turning points, the points of inflexion and the points where the curve meets the x-axis.

(iv) State the absolute minimum and absolute maximum values of $f(x)$ in the domain $-1 \le x \le 3$? 2

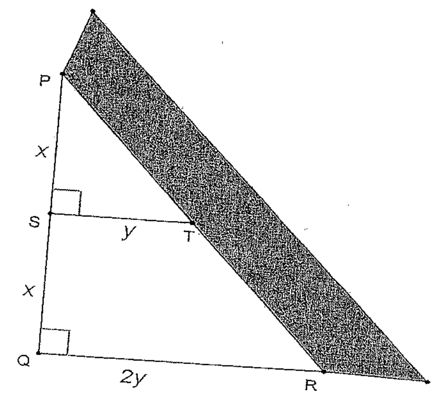
QUESTION 4: (10 marks) START A NEW PAGE

(a) By expressing the parabola $x = y^2 - 8y + 4$ in the form $(y-k)^2 = 4a(x-h)$,

find: (i) the coordinates of the vertex, 2

(ii) the coordinates of the focus. 2

(b) A new animal safari enclosure is to be built at Zooland Safari Park. The enclosure is to be triangular in shape with a total area of 2000 m^2. To keep the meerkats separate from the hyenas, the Zooland surveyor sets up the field with fences at PQ, QR and ST as shown in the diagram. The side PR will be a large canal full of water so no fence is required along that side. The point S is the midpoint of PQ and QR is twice the length of ST.
Let $PS = x$ metres and $ST = y$ metres.



(i) Show that the total length of fencing, L, required for the enclosure is given by $L = 2x + \dfrac{3000}{x}$ 2

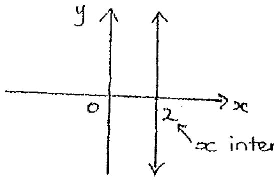
(ii) Hence find the exact value of x that will minimise the length of fencing required for the enclosure. 3

(iii) Show that the minimum length of fencing required for the enclosure is exactly $40\sqrt{15}$ metres. 1

End of paper

Test #1 22-11-06

ESTION ONE:



x intercept must be labelled!

e locus has equation $x = 2$.

* Any old vertical line to the right was not sufficient – it had to pass through 2 on the x axes
* Too many people did not use a ruler or label the axes!

Comm 2

$x^6 + 7x^3 - 8 = 0$

Let $u = x^3$

$u^2 + 7u - 8 = 0$

$(u+8)(u-1) = 0$

$u = -8$ $\quad$ $u = 1$

$x^3 = -8$ $\quad$ $x^3 = 1$

$x = -2$ $\quad$ $x = 1$

* Many said $\sqrt[3]{-8}$ did not exist, & several others said $\sqrt[3]{8} = \pm 2$. Be careful to check your answers, & use your calculator if you need to

${}^2 - 5x + 7 \equiv P(x-1)^2 + Q(x-1) + R$

$= P(x^2 - 2x + 1) + Qx - Q + R$

$= Px^2 - 2Px + P + Qx - Q + R$

$= Px^2 - (2P - Q)x + (P - Q + R)$

watch signs!

Most got 2 or 3 out of 3 for this question

← Several mistakes with -ve signs & brackets

tch the coefficients.

$P = 3$ ①

$Q = 5$ ②

$R = 7$ ③

Subst. ① into ②

$6 - Q = 5$

$-Q = -1$

$Q = 1$

ubst. into ③

$3 - 1 + R = 7$

$2 + R = 7$

$R = 5$.

Solution

$P = 3$

$Q = 1$

$R = 5$

d) $y = x^2 - px + 8$

$y' = 2x - p$

At $x = -2$, $y' = 5$

$\therefore 5 = -4 - p$

$9 = -p$

$\therefore p = -9$

* Several tried to find the equation of a tangent – read the question & use the given information to answer it!

Calc 2

e) $y = 2x^8 + 1$

$y' = 16x^7$

$y'' = 42x^6$

When $x = 0$ $\quad$ $y' = 0$ and $y'' = 0$

$\therefore$ There is a possible horizontal point of inflexion at $x = 0$

Method A

Test slope

$y' = 16x^7$

x	-1	0	1
y'	-16	0	16
	$-$	0	$+$
	\	—	/

$\therefore$ There is a minimum turning point at $x = 1$

Method B

Test concavity

$y'' = 42x^6$

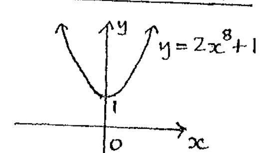
x	-1	0	1
y''	42	0	42
	$+$ up	0	$+$ up

The concavity does not change.

$\therefore$ the point cannot be a horizontal POI.

It is a minimum TP.

Alternative Method



By drawing a sketch of $y = 2x^8 + 1$, it can be seen that there is a minimum T.P at $x = 0$

* Only 6 people answered this question correctly.

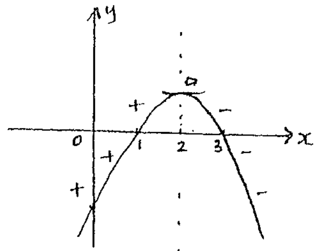
* It is very rare for information to be given in a question that is not needed – if you said that there was a horizontal POI, you didn't use the information '$y = 2x^8 + 1$', such things should make you think twice about whether you've answered the question correctly.

* You never ever need to write a half page essay to get 1 mark – don't waste time like that, especially in Question 1.

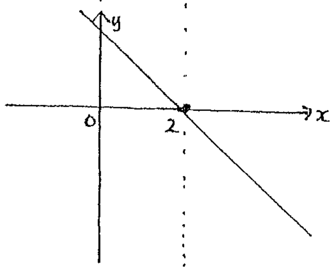
Reas 1

QUESTION 2.

a)



$y=f(x)$



$y=f'(x)$

Axes and Graphs should be labelled.

If numbers are labelled, It is very important that you indicate the numerical position where the stationary point occurs.

Calc. 1

b) i) $y = \sqrt[3]{2x-3}$

$= (2x-3)^{1/3}$

$\frac{dy}{dx} = \frac{1}{3}(2x-3)^{-2/3} \times 2$

$= \frac{2}{3}(2x-3)^{-2/3}$

Loretta did not differentiate the function correctly because when she used the function of a function rule she forgot to multiply by the derivative of the inside of the bracket.

Learn & practise your index rules.

Try Ex 1·11 p20
Ex 1·12 p23
Ex 8·4 p330
Ex 8·5 p333

Comm 1

ii) Gradient tangent at $x=2$

$m_1 = \frac{2}{3}(4-3)^{-2/3}$

$= \frac{2}{3} \times 1$

$= 2/3$

Gradient normal at $x=2$

$m_2 = -3/2$

when $x=2$

$y = (4-3)^{1/3}$

$= 1^{1/3}$

$= 1$ $(2,1)$

Equation of normal

$y - y_1 = m(x-x_1)$

$y - 1 = -\frac{3}{2}(x-2)$

$2y - 2 = -3x + 6$

$3x + 2y - 8 = 0$

Calc 2

Using Loretta's incorrect answer.

Gradient tangent

$m_1 = \frac{1}{3}$

Gradient normal

$m_2 = -3$

Point $x=2$, $y=1$ $(2,1)$

Equation of normal

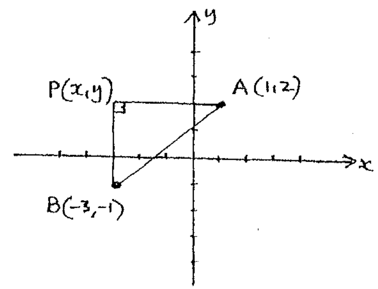
$y - y_1 = m(x-x_1)$

$y - 1 = -3(x-2)$

$y - 1 = -3x + 6$

$3x + y - 7 = 0$

c) i)



$\angle APB = 90^\circ$

$AP \perp PB$

$\therefore m_{PA} \times m_{PB} = -1$

$\frac{y-2}{x-1} \times \frac{y+1}{x+3} = -1$

$(y-2)(y+1) = -1(x-1)(x+3)$

$y^2 - 2y + y - 2 = -1(x^2 + 3x - x - 3)$

$y^2 - y - 2 = -1(x^2 + 2x - 3)$

$y^2 - y - 2 = -x^2 - 2x + 3$

$x^2 + y^2 + 2x - y - 5 = 0$

Reas 2

ii) $x^2 + 2x + y^2 - y = 5$

complete the squares

$x^2 + 2x \underline{+1} + y^2 - y \underline{+ 1/4} = 5 \underline{+1} \underline{+1/4}$

$(x+1)^2 + (y-\frac{1}{2})^2 = 6\frac{1}{4}$

$(x+1)^2 + (y-\frac{1}{2})^2 = \frac{25}{4}$

The locus is a circle with Centre $(-1, \frac{1}{2})$ and radius $\sqrt{\frac{25}{4}} = \frac{5}{2}$

Comm

$y = 5 + 4x^3 - x^4$

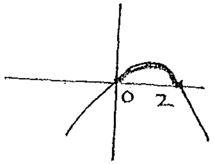
$y' = 12x^2 - 4x^3$

$y'' = 24x - 12x^2$

Concave upwards $y'' > 0$

$24x - 12x^2 > 0$

$12x(2 - x) > 0$



$0 < x < 2$

Draw the graph with more care. It is concave down.

Calc 2

QUESTION 6:

$f(x) = 5x^3(x-2)$

$= 5x^4 - 10x^3$

i) $f'(x) = 20x^3 - 30x^2$

$f''(x) = 60x^2 - 60x$

Stationary points $f'(x) = 0$

$20x^3 - 30x^2 = 0$

$10x^2(2x - 3) = 0$

$10x^2 = 0$ $\quad\quad$ $2x - 3 = 0$

$x = 0$ $\quad\quad$ $x = 3/2$

when $x = 0$

$y = 0 - 0$

$= 0$ $\quad\quad$ $(0, 0)$

when $x = 3/2$

$y = 5 \times (\frac{3}{2})^4 - 10 \times (\frac{3}{2})^3$

$= -8\frac{7}{16}$ $\quad\quad$ $(1\frac{1}{2}, -8\frac{7}{16})$

No penalty for incorrect y value in part (i)

Test nature using $y'' = 60x^2 - 60x$

when $x = 0$

$\left.\begin{array}{l} y' = 0 \\ y'' = 0 \end{array}\right\}$ possible horizontal POI at $(0,0)$

Stating this fact is not sufficient. You must give a clear conclusion by using either of the methods shown.

$y' \to$ tests slope

$y'' \to$ tests concavity

Method A:

Test slope

$y' = 20x^3 - 30x^2$

x	-1	0	1
y'	-50	0	-10

DO ~~concavity~~ test like this!

$\therefore$ Horizontal point of inflexion at $(0,0)$

→ SHOW VALUES

Method B:

Check concavity

$y'' = 60x^2 - 60x$

x	-1	0	$\frac{1}{2}$
y''	120	0	-15
	up	0	down

Since concavity changes there is a horiz. POI at $(0,0)$

Please do not abbreviate horizontal point of inflexion to just HPOI when $x = \frac{3}{2}$

$$y'' = 60 \times \tfrac{3}{2}\left(\tfrac{3}{2} - 1\right)$$
$$= 90 \times \tfrac{1}{2}$$
$$= 45$$

$y'' > 0$

$\therefore$ concave up

$\therefore$ A minimum turning point at $\left(1\tfrac{1}{2}, -8\tfrac{7}{16}\right)$

You must give a clear conclusion that it is a mini. T.P. The mark will not be awarded if there's no conclusion.

Calc 3

ii) Points of inflexion $y'' = 0$

$$60x(x-1) = 0$$
$$x = 0 \qquad x = 1$$

Horizontal POI at $(0,0)$ already found above.

when $x = 1$
$y = 5(1-2)$
$= -5$ $\quad (1, -5)$

Check concavity change at $(1, -5)$

$$y'' = 60x^2 - 60x$$

x	$\frac{1}{2}$	1	$1\frac{1}{2}$
y''	-15	0	45
	$-$ down	0	$+$ up.

Since concavity changes there is a POI at $(1, -5)$

You must show the checking concavity step at $(1, -5)$

You need to have your wits about you here because the other horiz. POI is at $(0,0)$ so you can't go that far past it.

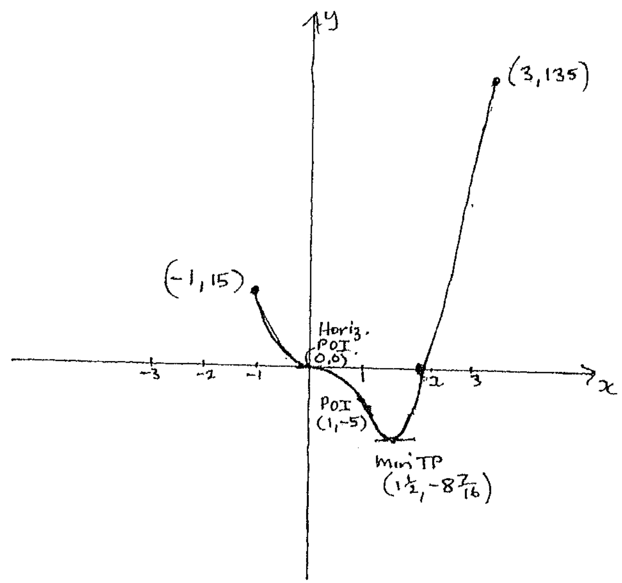
Calc 2

Endpoints $\quad y = 5x^3(x-2)$

When $x = -1$
$y = 5 \times -1^3 \times (-1-2)$
$= 5 \times -1 \times -3$
$= 15$

when $x = 3$
$y = 5 \times 3^3 \times (3-2)$
$= 5 \times 27 \times 1$
$= 135$

Horizontal POI at	$(0,0)$	
Minimum TP at	$\left(1\tfrac{1}{2}, -8\tfrac{7}{16}\right)$	
POI at	$(1,-5)$	
x-intercepts	$(0,0)$	$(2,0)$



✓ endpoints

✓ stationary points and points of inflexion ALL x and y values correct. Any calculation errors made in part (i) are penalised here.

✓ shape of graph correct showing concavity changes clearly

Comm 3

iv) Absolute maximum $= 135$ ✓

Absolute Minimum $= -8\tfrac{7}{16}$ ✓

State only the y values for this part. You don't state the coordinates of the point as a whole.

QUESTION 4 :

i) $x = y^2 - 8y + 4$

$y^2 - 8y = x - 4$

Complete the square

$y^2 - 8y \underline{+16} = x - 4 \underline{+16}$

$(y-4)^2 = x + 12$

$(y-4)^2 = 4 \cdot \frac{1}{4}(x+12)$ ✓

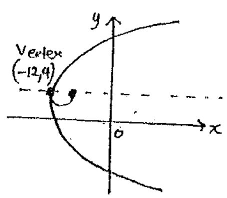
Vertex $(-12, 4)$ ✓

* A few mistakes when completing the square watch your + & −

* Make sure you get your x & y values in the correct order ie (x, y)

focal length $a = \frac{1}{4}$

Focus $(-11\frac{3}{4}, 4)$ ✓ ✓



one mark for focus a distance of $\frac{1}{4}$ away from vertex in any direction

* Please do a quick sketch this will help to show you where the focus should be

Reas 2

* lots of parabolas around the wrong way.

) Area $A = \frac{1}{2}bh$

$2000 = \frac{1}{2} \times 2y \times 2x$

$2xy = 2000$

$y = \frac{2000}{2x}$ ✓

Length of fencing, L.

$L = 2x + 3y$

$= 2x + 3 \times \frac{2000}{2x}$ ✓

$= 2x + \frac{3000}{x}$

* When a question says SHOW you must explain what formula your working has come from.

$2xy = 2000$ does not show that you have used the area formula correctly

Reas 2

ii) $L = 2x + 3000x^{-1}$

$L' = 2 - 3000x^{-2}$

$L'' = 6000x^{-3}$

* Differentiation was well done.

Stationary values $L' = 0$

$2 - \frac{3000}{x^2} = 0$ ✓

$2 = \frac{3000}{x^2}$

$2x^2 = 3000$

$x^2 = 1500$

$x = \pm\sqrt{1500}$

$x = \pm 10\sqrt{15}$

x is positive since it is a length

$\therefore \quad x = 10\sqrt{15}$ ✓

you must state that $x = 10\sqrt{15}$ only, as it is a length and $\therefore > 0$.

Test max/min.

when $x = 10\sqrt{15}$

$L'' = \frac{6000}{(10\sqrt{15})^3}$

$L'' > 0$

$\therefore$ Concave up ✓

$\therefore$ Minimum occurs at $x = 10\sqrt{15}$

* MUST Test for max/min

Calc 3

iii) when $x = 10\sqrt{15}$

$L = 2 \times 10\sqrt{15} + \frac{3000}{10\sqrt{15}}$

$= 20\sqrt{15} + \frac{300}{\sqrt{15}}$

$= 20\sqrt{15} + \frac{300}{\sqrt{15}} \times \frac{\sqrt{15}}{\sqrt{15}}$ ✓

$= 20\sqrt{15} + 20\sqrt{15}$

$= 40\sqrt{15}$ m.

* Again if asked to show do not skip steps

$L = 20\sqrt{15} + \frac{3000}{10\sqrt{15}}$

$= 20\sqrt{15} + 20\sqrt{15}$

$= 40\sqrt{15}$

Is NOT acceptable this does not show you can rationalise.