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Centre Number

Student Number

SCEGGS Darlinghurst

2008

Preliminary Course
Semester 1 Examination

Mathematics Extension 1

Outcomes Assessed: P1, P3, P4, P5 and PE3
Task Weighting: 20%

General Instructions

- Time allowed – 1 hour
- This paper has **four** questions
- Write your Student Number at the top of each page
- Attempt **all** questions
- Write using blue or black pen
- Answer all questions on the pad paper provided
- Draw all diagrams using a pencil and ruler
- Begin each question on a new page
- Marks will be deducted for careless or badly arranged work
- Approved scientific calculators and mathematical templates may be used

Total marks – 40

- Attempt Questions 1 – 4

Question	Reasoning	Communication	Marks
1	/2		/10
2	/2	/4	/10
3	/3	/3	/10
4	/3		/10
TOTAL	/10	/7	/40

Marks

Question 1 (10 marks)

(a) It is given that $\dfrac{1}{A} = \dfrac{1}{B} + \dfrac{1}{C}$ **2**

Evaluate B when $A = 3.6$ and $C = 4.7$, answering correct to 1 decimal place.

(b) If $f(x) = 2x - x^2$, find:

(i) $f(-2)$ **1**

(ii) $f(x-2)$ **2**

(c) Evaluate $\dfrac{\cos 315° + \sin 240°}{\tan 120°}$. Answer in simplest rationalised form. **3**

(d) Simplify $\dfrac{3x^3}{x-2} + \dfrac{24}{2-x}$ **2**

- Start a new page

Marks

Question 2 (10 marks)

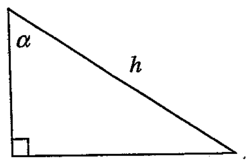
(a) If $f(x) = \dfrac{1}{x+1} + 1$,

(i) state the domain of $f(x)$. **1**

(ii) sketch $y = f(x)$, showing all important features. **3**

(b) Solve $3\sin\theta = 2\cos\theta$ for $-180° \le \theta \le 180°$. **3**
Answer correct to the nearest minute.

(c) (i) **2**



Prove that the area of this triangle is $\frac{1}{2}h^2 \sin\alpha \;\cos\alpha$.

(ii) If it is given that the area is 25cm^2 and $\alpha = 37°$, calculate the length of h correct to 2 significant figures. **1**

- Start a new page

Marks

Question 3 (10 marks)

(a) Sketch the function $y = f(x)$ defined by: **3**

$$f(x)\begin{cases} = -x-2 & \text{for } x < -2 \\ = \sqrt{4-x^2} & \text{for } -2 \le x \le 2 \\ = x-2 & \text{for } x > 2 \end{cases}$$

(b) (i) Solve $\dfrac{3}{2x-1} \le 2$ **3**

(ii) Graph your solution on a number plane. **1**

(c) Prove that: **3**

$$\cot A\,(\operatorname{cosec} A - \cot A) = \frac{\cos A}{1+\cos A}$$

- Start a new page

Marks

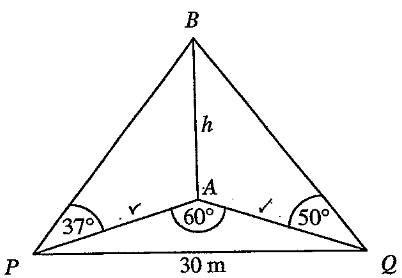
Question 4 (10 marks)

(a) (i) Sketch the curves $y=(x+3)^2$ and $y=(x+1)(x+4)$ on the same set of axes. **1**

(ii) Prove that they intersect at one point only, stating the co-ordinates of this point. **2**

(iii) Hence, or otherwise, solve $(x+3)^2>(x+1)(x+4)$. **1**

(b) A vertical tower AB of height h metres is standing on horizontal ground. Two points, P and Q are on ground level, 30 m apart. The angles of elevation of the top of the tower from P and Q are $37°$ and $50°$ respectively. It is given that $\angle PAQ = 60°$.



(i) Prove $AP = \dfrac{h}{\tan 37°}$. **1**

(ii) Find a similar expression for AQ. **1**

(iii) Prove that: **2**

$$h^2\left[\frac{1}{\tan^2 37°}+\frac{1}{\tan^2 50°}-\frac{1}{\tan 37° \tan 50°}\right] = 900$$

(iv) Hence find the height of the tower correct to the nearest metre. **2**

End of paper

Semester 1 Preliminary Extension 1 2008 Solutions.

1) a) $\dfrac{1}{3.6} = \dfrac{1}{B} + \dfrac{1}{4.7}$

$\dfrac{1}{B} = \dfrac{1}{3.6} - \dfrac{1}{4.7}$

$B = 15.4$ (1 dp) ✓✓

b) i) $f(-2) = 2\times -2 - (-2)^2$
$= -4 - 4$
$= -8$ ✓

(ii) $f(x-2) = 2(x-2) - (x-2)^2$
$= 2x - 4 - (x^2 - 4x + 4)$
$= 2x - 4 - x^2 + 4x - 4$
$= -x^2 + 6x - 8$

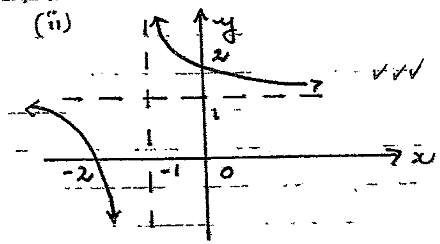
c) $\dfrac{\frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2}}{-\sqrt{3}} = \dfrac{\frac{2-\sqrt{6}}{2\sqrt{2}}}{-\sqrt{3}}$
$= \dfrac{\sqrt{6}-2}{2\sqrt{6}} \times \dfrac{\sqrt{6}}{\sqrt{6}}$
$= \dfrac{6-2\sqrt{6}}{12}$
$= \dfrac{3-\sqrt{6}}{6}$

d) $\dfrac{3x^3}{x-2} + \dfrac{24}{2-x} = \dfrac{3x^3-24}{x-2}$
$= \dfrac{3(x^3-8)}{x-2}$
$= \dfrac{3(x-2)(x^2+2x+4)}{x-2}$
$= 3(x^2+2x+4)$

✓✓ Reas.

2) a) (i) all real x except $x=-1$ ✓ com

(ii)



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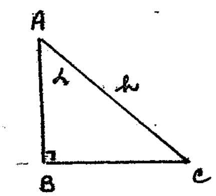
b) $\tan\theta = \dfrac{2}{3}$

Acute angle $= 33°41'$
1st, 3rd quadrants
$\theta = 33°41',\ 213°41'$ $\quad 0 \le \theta \le 360°$
$\therefore \theta = 33°41',\ -146°19'$ $\quad -180° \le \theta \le 180°$

c) (i)



Area of triangle $= \dfrac{1}{2} AB \times BC$

$\sin\alpha = \dfrac{BC}{h}$

$\therefore BC = h\sin\alpha$

$\cos\alpha = \dfrac{AB}{h}$

$\therefore AB = h\cos\alpha$

$\therefore$ Area $= \dfrac{1}{2} h\sin\alpha\, h\cos\alpha$ Reas ✓✓

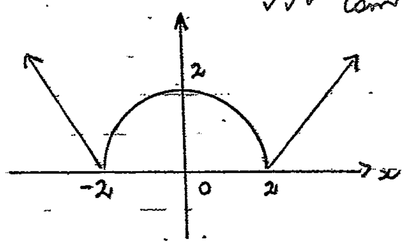
$= \dfrac{1}{2} h^2 \sin\alpha\cos\alpha$

(ii) $25 = \dfrac{1}{2} h^2 \sin 37° \cos 37°$.

$h^2 = 50 \div (\sin 37° \cos 37°)$
$h^2 = 104.0299..$
$h = 10$ (2 s.f.)

✓✓✓ com

) a)



b) (i) $3 \quad \frac{(2x-1)^2}{2x-1} \le 2(2x-1)^2$

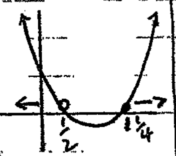
$3(2x-1) \le 2(4x^2-4x+1)$

$6x-3 \le 8x^2-8x+2$

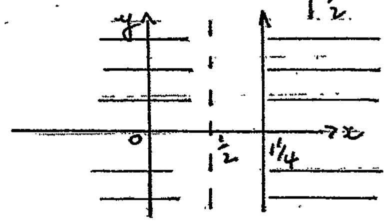
$8x^2-14x^2+5 \geq 0$

$(2x-1)(4x-5) \geq 0$

$x < \frac{1}{2}, \quad x \geq 1\frac{1}{4}$



i)



c) LHS $= \cot A(\operatorname{cosec} A - \cot A)$

$= \frac{\cos A}{\sin A}\left(\frac{1}{\sin A} - \frac{\cos A}{\sin A}\right)$

$= \cos A \; \frac{(1-\cos A)}{\sin^2 A}$

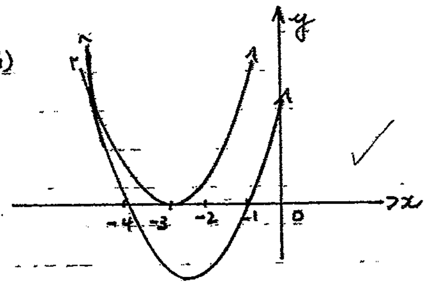
$= \cos A \; \frac{(1-\cos A)}{1-\cos^2 A}$

$= \cos A \; \frac{(1-\cos A)}{(1+\cos A)(1-\cos A)}$

$= \frac{\cos A}{1+\cos A}$ ✓✓✓

= R.H.S. Reas

(4) a) (i)



✓

(ii) $(x+3)^2 = (x+1)(x+4)$

$x^2+6x+9 = x^2+5x+4$

$x = -5$ ✓

$y = (-5+3)^2$

$= 4$

there is only one simultaneous solution

i.e. there is only one point of intersection $(-5, 4)$ ✓

(iii) $x > -5$ ✓

b) (i) $\tan 37° = \frac{h}{AP}$ ✓

$\therefore AP = \frac{h}{\tan 37°}$

(ii) $AQ = \frac{h}{\tan 50°}$ ✓ Reas

(iii) By Cosine Rule:

✓ CNE

$30^2 = \frac{h^2}{\tan^2 37°} + \frac{h^2}{\tan^2 50°} - \frac{2 \times h^2 \cos 60°}{\tan 37° \tan 50°}$

$900 = h^2\left[\frac{1}{\tan^2 37°} + \frac{1}{\tan^2 50°} - \frac{1}{\tan 37° \tan 5}\right.$ ✓✓ Reas

(iv) $h^2 = 900 \div 1.35161\ldots$ ✓

$h = 25.804\ldots$ ✓

$h = 26\text{m}$ (nearest metre)

height is 26m (nearest metre)