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Centre Number

Student Number

SCEGGS Darlinghurst

2009

Preliminary Course
Semester 2 Examination

Mathematics Extension 1

Outcomes Assessed: P1-P8, PE1, PE2, PE3 and PE6
Task Weighting: 40%

General Instructions

- Reading time – 5 minutes
- Time allowed – 1½ hours
- Write using black or blue pen
- Write your Student Number at the top of this page
- This paper has **four** questions
- Attempt all questions on the pad paper provided
- Answer **all** questions and show all necessary working
- **Start each question on a new page**
- Marks will be deducted for careless or badly arranged work
- Mathematical templates, geometrical equipment and scientific calculators may be used
- **Do not** attach all questions together in one bundle

Total marks – 60

- Attempt Questions 1 – 4

Question	Comm	Reason	Calc	TOTAL
1		/4	/2	/15
2	/3	/4		/16
3		/5		/13
4		/5	/2	/16
TOTAL	/3	/16	/4	/60

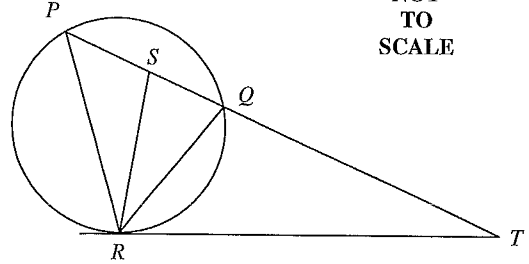
Marks

Question 1 (15 marks)

(a) The polynomial $x^3 + 2x^2 + 1$ is divided by $x + 3$. Calculate the remainder. **2**

(b) The interval AB, where A is $(4, 5)$ and B is $(19, -5)$ is divided internally in the ratio $2:3$ by the point $P(x, y)$. Find the values of x and y. **2**

(c)



NOT TO SCALE

In the diagram ΔPQR is drawn in a circle. The tangent to the circle at R meets PQ produced to T. S is a point on PQ such that RS bisects $\angle QRP$.

Copy the diagram.

(i) Explain why $\angle TRQ = \angle RPT$. **1**

(ii) Show that $TR = TS$, giving clear reasons. **3**

(d) A monic polynomial $P(x)$ of degree 4 has one root equal to 2 and a double root equal to -1. **2**

If the polynomial passes through the point $(0, 6)$ find the equation of $P(x)$.

Question 1 continues on the next page

Question 1 (continued)

Marks

(e) Let $f(x) = 3x^2 - 2x$. **2**

Use the definition $f'(a) = \lim_{h \to 0} \dfrac{f(a+h) - f(a)}{h}$

to find the derivative by first principles of $f(x)$ at the point $x = a$.

(f) By making an appropriate substitution, or otherwise, solve for x. **3**

$$2\left(x + \frac{1}{x}\right)^2 + \left(x + \frac{1}{x}\right) - 15 = 0$$

- Start a new page

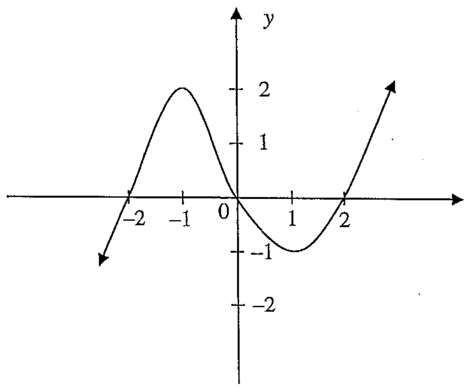
Marks

Question 2 (16 marks)

(a) Solve for θ, $0° \le \theta \le 360°$ **3**

$$\sin 2\theta = \sin \theta$$

(b) The curve $y = f(x)$ is shown below.



On the separate answer page attached, draw the graph of:

(i) $y = |f(x)|$ **1**

(ii) $y = f(x+2)$ **1**

(iii) $y = f(x) - 1$ **1**

Question 2 continues on the next page

Question 2 (continued)

Marks

(c) The polynomial $P(x)=2x^3+3x^2+kx-2$ has roots α, β and γ.

(i) Find the value of $\alpha+\beta+\gamma$. **1**

(ii) Find the value of $\alpha\beta\gamma$. **1**

(iii) If one root is the reciprocal of the other, find the third root and hence find the value of k. **2**

(d) (i) Express $\sin x+\cos x$ in the form $R\cos(x-\alpha)$ where $R>0$ and $0°<\alpha<90°$. **2**

(ii) Hence solve for $0°\le x\le 360°$ **2**

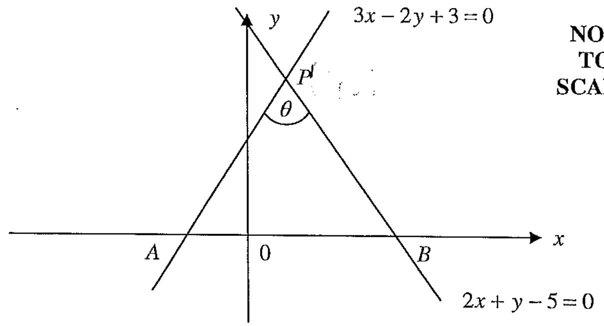
$$\sin x+\cos x=1$$

(iii) Find all possible solutions of $\sin x+\cos x=1$. **2**

Question 3 (13 marks)

Marks

(a)



The diagram shows the lines $3x-2y+3=0$ and $2x+y-5=0$ intersecting at point P. The lines cut the x and y axes at A and B respectively.

(i) Find the co-ordinates of P. **2**

(ii) Find the size of the acute angle, θ, between the lines $3x-2y+3=0$ and $2x+y-5=0$. Answer correct to the nearest minute. **3**

(b) (i) By using the substitution $t=\tan\frac{x}{2}$, show that **2**

$\sin x-3\cos x-2$

can be written as $\dfrac{t^2+2t-5}{1+t^2}$.

(ii) Hence, find correct to the nearest minute **3**

the values of x, $0°\le x\le 360°$ for which

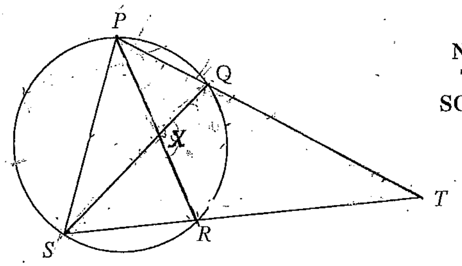
$\sin x-3\cos x-2=0$.

Question 3 continues on the next page

Marks

Question 3 (continued)

(c)



NOT TO SCALE

The points P, Q, R, S are placed on a circle of radius r such that PR and QS meet at X.
The lines PQ and SR are produced to meet at T, and $QXRT$ is a cyclic quadrilateral.

Copy or trace this diagram onto your answer page.

(i) Find the size of $\angle SQT$, giving reasons for your answer. **2**

(ii) Find an expression for the length of PS in terms of r. **1**

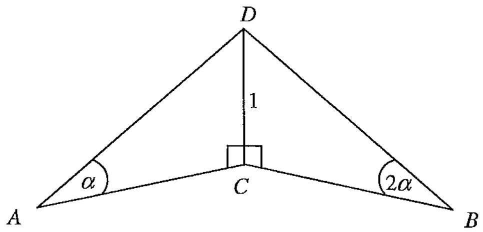
Marks

Question 4 (16 marks)

(a) Solve for x. **3**

$$\frac{3}{2x-1} \geq x$$

(b)



In the diagram CD is a vertical flagpole of height 1 metre which stands on horizontal ground.
The angles of elevation of the top D of the flagpole from points A and B on the ground are α and 2α respectively.

(i) Show that $BC = \dfrac{1-\tan^2\alpha}{2\tan\alpha}$. **1**

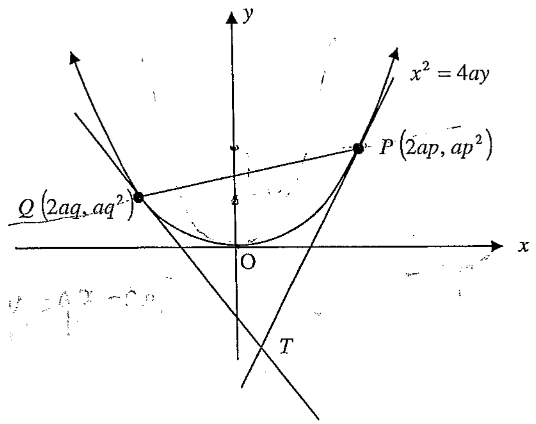
(ii) Show that $AC - BC = BD$. **3**

Question 4 continues on the next page

Marks

Question 4 (continued)

(c) $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ are two points on the parabola $x^2 = 4ay$.
The tangents at P and Q intersect at T.



(i) Show that the tangent at P is given by $y = px - ap^2$. **2**

(ii) Show that the tangents intersect at the point $T\ (a(p+q), apq)$. **2**

(iii) Show that the chord PQ has equation $y = \frac{1}{2}(p+q)x - apq$. **2**

(iv) The line PQ is a tangent to the parabola $x^2 = 2ay$. **2**

Show that $(p+q)^2 = 8pq$.

(v) Show that the Cartesian equation of the locus of T is a parabola. **1**

End of paper

Year 11 Extension 1 Mathematics

Preliminary Semester 2 Exam 2009

Comments

Question 1

a) $P(x) = x^3 + 2x^2 + 1$

Remainder

$P(-3) = (-3)^3 + 2(-3)^2 + 1$

$= -27 + 18 + 1$

$= -8$

Using the Remainder Theorem is the best way to do this. $P(a) = R$. If you use long division you must make a clear conclusion.

b) $A(4,5)$ $B(19,-5)$

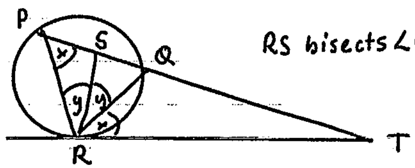
$2:3$ Internal division

$P = \left(\dfrac{3\times4 + 2\times19}{2+3}, \dfrac{3\times5 + 2\times-5}{2+3} \right)$

$= \left(\dfrac{50}{5}, \dfrac{5}{5} \right)$

$= (10, 1)$

This part was well done.

c) 

RS bisects $\angle QRP$

Drawing a clear $\frac{1}{3}$ to $\frac{1}{2}$ page diagram will help you.

i) $\angle TRQ = \angle TPR$

(Angle between a tangent and a chord is equal to the angle in the alternate segment.)

Make sure you know the correct wording. The key words are essential.

ii) Let $\angle TRQ = \angle TPR = x$

Let $\angle QRS = \angle SRP = y$ (equal angles since RS bisects $\angle QRP$)

$\angle TSR = \angle SPR + \angle SRP$ (exterior angle of a triangle is equal to sum of opposite interior angles)

$= x + y$

$\angle SRT = x + y$

$\therefore$ $TR = TS$ (sides opposite equal angles are equal.)

(Reas 4)

Question 1 continued.

d) $P(x) = (x-2)(x+1)^2(x+k)$

monic coefficient of x^4 is one

must put extra factor because of degree 4

passes through $(0,6)$

$6 = 2\times(-1)^2 \times k$

$-2k = 6$

$k = -3$

This was an easy question if you started with the correct factors. Your polynomial must have degree 4.

e) $f(x) = 3x^2 - 2x$

$f(a) = 3a^2 - 2a$

$f(a+h) = 3(a+h)^2 - 2(a+h)$

$= 3(a^2 + 2ah + h^2) - 2a - 2h$

$= 3a^2 + 6ah + 3h^2 - 2a - 2h$

$f'(a) = \lim_{h\to0} \dfrac{f(a+h) - f(a)}{h}$

$= \lim_{h\to0} \dfrac{3a^2 + 6ah + 3h^2 - 2a - 2h - (3a^2 - 2a)}{h}$

$= \lim_{h\to0} \dfrac{6ah + 3h^2 - 2h}{h}$

$= \lim_{h\to0} \dfrac{h(6a + 3h - 2)}{h}$

$= \lim_{h\to0} (6a + 3h - 2)$

$= 6a - 2$

(Calc 2)

A variation on first principles.

The question asks for $f'(a)$ not $f'(x)$ for the final answer.

Because the formula was given, this wasn't really as hard as some students thought!

It is a 2U question.

Question 1 (continued)

f) $2\left(x+\frac{1}{x}\right)^2 + \left(x+\frac{1}{x}\right) - 15 = 0$

Let $m = x + \frac{1}{x}$

$2m^2 + m - 15 = 0$

$(2m - 5)(m + 3) = 0$

$m = \frac{5}{2}$

$x + \frac{1}{x} = \frac{5}{2}$

$x^2 + 1 = \frac{5}{2}x$

$2x^2 + 2 = 5x$

$2x^2 - 5x + 2 = 0$

$(2x - 1)(x - 2) = 0$

$x = \frac{1}{2}, 2$

$m = -3$

$x + \frac{1}{x} = -3$

$x^2 + 1 = -3x$

$x^2 + 3x + 1 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-3 \pm \sqrt{9 - 4.1.1}}{2}$

$= \frac{-3 \pm \sqrt{5}}{2}$

← This is a quadratic equation. Use a substitution to reduce it.

← Very basic work here. You should recognise these as quadratic equations and be able to rearrange them.

Decimal answers are not required. Don't waste time finding them.

Question 2

a) $\sin 2x = \sin x$

$2\sin x \cos x - \sin x = 0$

$\sin x(2\cos x - 1) = 0$

$\sin x = 0$

$x = 0°, 180°, 360°$

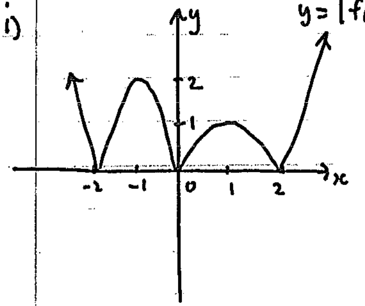
$\cos x = \frac{1}{2}$

quad 1 & 4

$x = 60°, 300°$

Many divided this line by $\sin x$, doing this eliminates one of the resulting equations. You must factorise then solve each equation. Also remember that 360° is in the domain

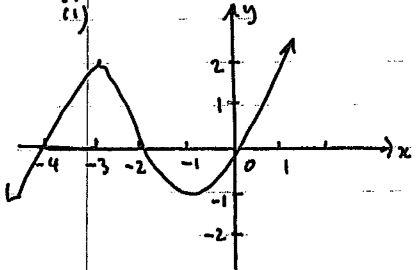
b) i)



$y = |f(x)|$

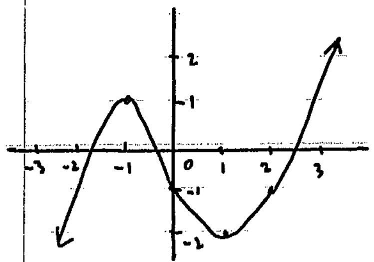
These were well done

ii)



$y = f(x+2)$

Shift left 2 units.



$y = f(x) - 1$

Shift down 1 unit

If your curve is shifted down then the intercepts will also change

Question 2 (continued)

c) $P(x) = 2x^3 + 3x^2 + kx - 2$

i) $\alpha+\beta+\gamma = -\frac{b}{a}$

$= -\frac{3}{2}$

ii) $\alpha\beta\gamma = -\frac{d}{a}$

$= \frac{2}{2}$

$= 1$

Parts (i) and (ii) were well done.

iii) One root is reciprocal $\Rightarrow$ Roots are $\alpha, \frac{1}{\alpha}, \gamma$

from part ii) R 2

$\alpha\beta\gamma = 1$

$\alpha \cdot \frac{1}{\alpha}\gamma = 1$

$\gamma = 1$ ✓

Read the question carefully it says one root is the reciprocal of the other, not the negative reciprocal

Since this is a root of the polynomial $P(1)=0$

$2\times(1)^3 + 3\times(1)^2 + k\times 1 - 2 = 0$

$2 + 3 + k - 2 = 0$

$k + 3 = 0$

$k = -3$ ✓

Question 2 (continued)

d) i)

$\sin x + \cos x = R\cos(x-\alpha)$

$= R(\cos x\cos\alpha + \sin x\sin\alpha)$

$= R\cos x\cos\alpha + R\sin x\sin\alpha$

Part (i) well done

Match the coefficients

$R\cos\alpha = 1$ (1)

$R\sin\alpha = 1$ (2)

Find R

$R^2 = 1^2 + 1^2$

$R = \sqrt{1+1}$

$= \sqrt{2}$ ✓

Find α

(2)/(1)

$\tan\alpha = 1$

$\alpha = 45^\circ$ ✓

$\therefore \sin x + \cos x = \sqrt{2}\cos(x-45^\circ)$

ii) $\sin x + \cos x = 1$

$\sqrt{2}\cos(x-45^\circ) = 1$

$\cos(x-45^\circ) = \frac{1}{\sqrt{2}}$

quad 1 & 4

Check domain

$0^\circ \le x \le 360^\circ$

$-45^\circ \le x - 45^\circ \le 315^\circ$

Remember to check the domain, often a solution is left out if this is not done

$\therefore x - 45^\circ = -45^\circ, 45^\circ, 360^\circ - 45^\circ$

quad 4, quad 1, quad 4

$x = 0^\circ, 90^\circ, 360^\circ$ ✓✓

iii) all solutions $\rightarrow$ general solution

R 2

$\cos(x-45^\circ) = \frac{1}{\sqrt{2}}$

$x - 45^\circ = 360^\circ n \pm \cos^{-1}\frac{1}{\sqrt{2}}$

$x = 360^\circ n \pm 45^\circ + 45^\circ$

$\therefore x = 360^\circ n$ or $x = 360^\circ n + 90^\circ$

where $n \in \mathbb{Z}$ ✓

Not well done, all possible solutions means general solution

Two methods

- learn the rule
- look for a pattern

Another Method

$\cos(x - 45°) = \frac{1}{\sqrt{2}}$ quadrants 1 or 4

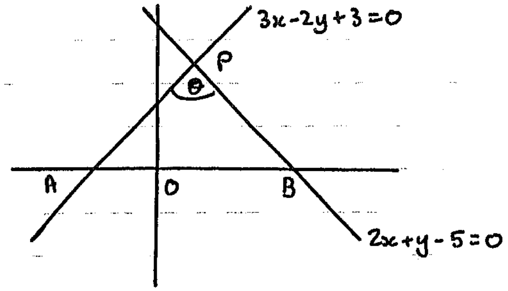
in Q1: $x - 45° = 45°, 360° + 45°, 2\times 360° + 45° \ldots$
$x = 90°, 360° + 90°, 2\times 360° + 90°$

Q4: $x - 45° = 360 - 45°, 2\times 360 - 45°, \ldots$
$x = 360°, 2\times 360°, \ldots$

$\therefore\ x = 360°n + 90°$ or $x = 360°n$

Question 3

a)



i) $3x - 2y + 3 = 0$ ①
$2x + y - 5 = 0$ ②

②×2 $4x + 2y - 10 = 0$ ③
$3x - 2y + 3 = 0$ ①

Add ①&③ $7x - 7 = 0$
$7x = 7$
$x = 1$

subst. into ① $3 - 2y + 3 = 0$
$2y = 6$
$y = 3$ $\therefore P(1,3)$

This part was well done. Very basic work.

ii) $3x - 2y + 3 = 0$ $2x + y - 5 = 0$
$m_1 = -\frac{3}{-2}$ $m_2 = -\frac{2}{1}$
$= \frac{3}{2}$ $= -2$

$\boxed{\begin{array}{l} ax + by + c = 0 \\ \text{To find gradient without making } y \text{ the subject} \\ m = -\frac{a}{b} \end{array}}$

Acute angle $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$
$= \left|\frac{\frac{3}{2} - -2}{1 + \frac{3}{2}\times -2}\right|$
$= \left|\frac{3\frac{1}{2}}{-2}\right|$
$= \left|-\frac{7}{4}\right|$
$\tan\theta = \frac{7}{4}$
$\theta = 60°15'$

You must learn the formula correctly. A standard Ext① question. It's disappointing to see incorrect formulas!

Question 3 (continued)

b) i) $t = \tan\frac{x}{2}$

$\sin x = \frac{2t}{1+t^2}$ $\cos x = \frac{1-t^2}{1+t^2}$

$\sin x - 3\cos x - 2$

$= \frac{2t}{1+t^2} - 3\left(\frac{1-t^2}{1+t^2}\right) - 2$ ✓

$= \frac{2t - 3 + 3t^2 - 2(1+t^2)}{1+t^2}$

$= \frac{2t - 3 + 3t^2 - 2 - 2t^2}{1+t^2}$

$= \frac{t^2 + 2t - 5}{1+t^2}$ ✓

This part was well done by students who knew their substitutions.

ii) $\sin x - 3\cos x - 2 = 0$

$\frac{t^2 + 2t - 5}{1+t^2} = 0$

$t^2 + 2t - 5 = 0$

$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-2 \pm \sqrt{4 - 4\times1\times-5}}{2}$

$= \frac{-2 \pm \sqrt{24}}{2}$ ✓

$t = \frac{-2+\sqrt{24}}{2}$

$\tan\frac{x}{2} = 1.449\ldots$

(Quad 1 & 3 but Quad 3 angle will be too big when doubled)

$\frac{x}{2} = 55°24'$

$x = 110°48'$ ✓

$t = \frac{-2-\sqrt{24}}{2}$

$\tan\frac{x}{2} = -3.449\ldots$

(Quad 2 and 4 but Quad 4 angle will be too big.)

$\frac{x}{2} = (180° - 73°50')$

$\frac{x}{2} = 106°10'$

$x = 212°20'$ ✓

Solving the quadratic equation was fine but many students had difficulty with the calculator work solving for the angle x.
That's all work so keep practising.

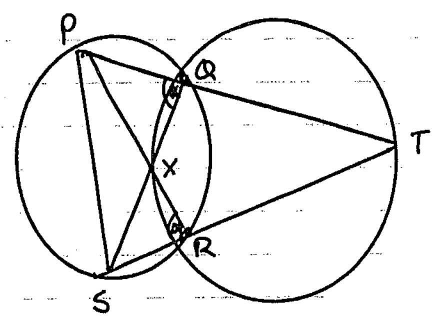
Check domain

$0° \leq x \leq 360°$

$0° \leq \frac{x}{2} \leq 180°$

So look for answers in quadrants 1 & 2 only.

Question 3 (continued)



Reas 3

Very impressive if you got this out. Circle geometry is time consuming but worthwhile! ALWAYS Draw a large clear diagram before you start your proof.

i) Let $\angle PQS = \angle PRS = x$
(angles in the same segment are equal)

$\angle XQT = 180° - \angle PQS = 180° - x$

$\angle TRP = 180° - \angle PRS = 180° - x$

(Angle sum of a straight line is 180°) ✓

$\angle XQT = \angle TRP$

Since QXRT is a cyclic quadrilateral

$\angle XQT + \angle TRX = 180°$

$\therefore 2\angle XQT = 180°$ ✓

$\angle XQT = 90°$

$\therefore \angle SQT = 90°$

ii) Since $\angle SQT = 90°$

$\angle SQP = 90°$

(Angle sum straight line is 180°)

$\therefore$ PS is a diameter of the circle

(Angle subtended by the diameter is 90° at the circumference)

Since r is the radius of the circle

$PS = 2r$ ✓

As the question just stated find an expression, many students got this mark without reasons.

Question 4

a) $\frac{3}{2x-1} \geq x$

undefined when $x = \frac{1}{2}$ ✓

Not well done

multiply both sides by $(2x-1)^2$

$\frac{3}{2x-1} \times (2x-1)^2 \geq x(2x-1)^2$

$3(2x-1) \geq x(2x-1)^2$

$3(2x-1) - x(2x-1)^2 \geq 0$

don't expand.
factorise it instead!

most students chose to expand at this point although this can be done it takes a lot more time and effort
Please factorise, it is much more efficient and you will be less likely to make a mistake

$(2x-1)\left[3 - x(2x-1)\right] \geq 0$

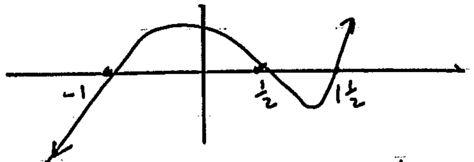
$(2x-1)(3 - 2x^2 + x) \geq 0$

↑ factorise out −1 and then reverse signs

$(2x-1) \cdot -1 (2x^2 - x - 3) \geq 0$

$(2x-1)(2x^2 - x - 3) \leq 0$

$(2x-1)(2x-3)(x+1) \leq 0$ ✓



$x \leq -1 \;,\; -\frac{1}{2} \leq x \leq 1\frac{1}{2}$

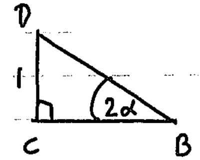
but $x \neq \frac{1}{2}$ ∴ The solution is

$x \leq -1, \; -\frac{1}{2} < x \leq 1\frac{1}{2}$ ✓

make sure you answer the question and incorporate any undefined values

Question 4 (continued)

b)



Part (i) was well done

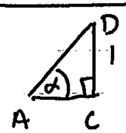
$\tan 2\alpha = \frac{1}{BC}$

$BC = \frac{1}{\tan 2\alpha}$

$= \frac{1}{\frac{2\tan\alpha}{1-\tan^2\alpha}}$ ✓

$= \frac{1-\tan^2\alpha}{2\tan\alpha}$

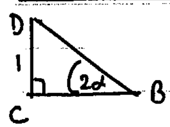
ii)



$\tan\alpha = \frac{1}{AC}$

$AC = \frac{1}{\tan\alpha}$

R
2



$\sin 2\alpha = \frac{1}{BD}$

$BD = \frac{1}{\sin 2\alpha}$

✓

This was a tricky question.
You needed to establish an expression for AC and BD in terms of α then set the question up as a trig identity proof.
Problems arose when students wrote an expression for BD using pythagoras - although this can be done it makes the question very difficult.

Question 4 (continued)

Prove that $AC - BC = BD$

$$AC - BC = \frac{1}{\tan\alpha} - \left(\frac{1-\tan^2\alpha}{2\tan\alpha}\right)$$

$$= \frac{2 - 1 + \tan^2\alpha}{2\tan\alpha}$$

$$= \frac{1+\tan^2\alpha}{2\tan\alpha}$$

you needed to use $1+\tan^2 x = \sec^2 x$

$$= \frac{\sec^2\alpha}{2\tan\alpha}$$

$$= \frac{\frac{1}{\cos^2\alpha}}{2\frac{\sin\alpha}{\cos\alpha}}$$

$$= \frac{1}{\cos^2\alpha} \times \frac{\cos\alpha}{2\sin\alpha}$$

$$= \frac{1}{2\sin\alpha\cos\alpha}$$

$$= \frac{1}{\sin 2\alpha}$$

$$= BD$$

Question 4 (continued)

i) $x^2 = 4ay$ — Calc (2)

$$y = \frac{x^2}{4a}$$

$$y' = \frac{2x}{4a}$$

$$= \frac{x}{2a}$$

At $P(2ap, ap^2)$

$$m_T = \frac{2ap}{2a}$$

$$= p$$

Eqn tangent

$$y - y_1 = m(x - x_1)$$

$$y - ap^2 = p(x - 2ap)$$

$$y - ap^2 = px - 2ap^2$$

$$y = px - ap^2$$

Parts (i) to (iii) were quite good
* Important note: if a question says show then you must show working eg. don't just write $m_T = p$ you must differentiate and substitute.

ii) Similarly tangent at Q

$$y = qx - aq^2 \quad (2)$$

$$y = px - ap^2 \quad (1)$$

Solve simultaneously

$$px - ap^2 = qx - aq^2$$

$$px - qx = ap^2 - aq^2$$

$$x(p-q) = a(p^2 - q^2)$$

$$x = \frac{a(p-q)(p+q)}{p-q}$$

$$x = a(p+q)$$

Subst. into (1) $\quad y = ap(p+q) - ap^2$

$$= ap^2 + apq - ap^2$$

$$= apq$$

$T(a(p+q), apq)$

iii) $P(2ap, ap^2)$ $Q(2aq, aq^2)$

Gradient PQ

$$m_{PQ} = \frac{aq^2 - ap^2}{2aq - 2ap}$$
$$= \frac{a(q^2-p^2)}{2a(q-p)}$$
$$= \frac{a(q-p)(q+p)}{2a(q-p)}$$
$$= \frac{p+q}{2}$$ ✓

Equation of chord PQ

$$y - y_1 = m(x - x_1)$$
$$y - ap^2 = \frac{(p+q)}{2}(x - 2ap)$$
$$y - ap^2 = \tfrac{1}{2}(p+q)x - \tfrac{1}{2}(p+q) \times 2ap$$
$$y - ap^2 = \tfrac{1}{2}(p+q)x - ap^2 - apq$$
$$y = \tfrac{1}{2}(p+q)x - apq$$ ✓

Alternate Method

Chord of contact: PQ is a chord of contact from the external point $T(a(p+q), apq)$

$$xx_0 = 2a(y + y_0)$$
$$xa(p+q) = 2a(y + apq)$$
$$x(p+q) = 2y + 2apq$$
$$2y = x(p+q) + 2apq$$
$$y = \tfrac{1}{2}(p+q)x + apq$$

Question 4 (continued)

(iv) R ②

$$x^2 = 2ay \quad ①$$
$$y = \tfrac{1}{2}(p+q)x - apq \quad ②$$

Solve simultaneously

$$x^2 = 2a\left(\tfrac{1}{2}(p+q)x - apq\right)$$
$$x^2 = a(p+q)x - 2a^2pq$$
$$x^2 - a(p+q)x + 2a^2pq = 0$$

This is a quadratic equation. Since the line PQ is a tangent to the parabola there is only one solution

$\therefore$ Using the discriminant $\Delta = 0$

$$\Delta = b^2 - 4ac$$
$$= (-a(p+q))^2 - 4.1.2a^2pq$$
$$= a^2(p+q)^2 - 8a^2pq$$
$$= a^2\left[(p+q)^2 - 8pq\right]$$

Since $\Delta = 0$

$$a^2\left[(p+q)^2 - 8pq\right] = 0$$
$$(p+q)^2 - 8pq = 0$$
$$(p+q)^2 = 8pq$$

This was a tricky question: we have the line PQ and the parabola $x^2 = 2ay$. Start by solving these simultaneously

** This is essential to solve this question*

Question 4 (continued)

v) $T(a(p+q), apq)$

$x = a(p+q) \Rightarrow p+q = \frac{x}{a}$

$y = apq \Rightarrow pq = \frac{y}{a}$

Using part iv)

$$(p+q)^2 = 8pq$$

$$\left(\frac{x}{a}\right)^2 = 8\frac{y}{a}$$

$$\frac{x^2}{a^2} = \frac{8y}{a}$$

$$x^2 = \frac{8y}{a} \times a^2$$

$$x^2 = 8ay$$ ✓

The locus of T is a parabola.

You need to eliminate p&q. Use your x and y coordinates to get an expression for p+q and for pq

Substitute these expressions into the equation from part (v)