

Name: \_\_\_\_\_



**SCEGGS Darlinghurst**

**Preliminary Assessment Task 2**

**May Examination, 2001**

# *Mathematics*

**Outcomes to be assessed:**

- P2:** provides reasoning to support conclusions that are appropriate to the context
- P3:** performs routine arithmetic and algebraic manipulation involving surds, simple rational expressions and trigonometric identities
- P4:** chooses and applies appropriate arithmetic, algebraic, graphical, trigonometric and geometric techniques

**TIME ALLOWED: 1½ HOURS**

**DIRECTIONS TO CANDIDATES:**

- Attempt all questions
- ALL QUESTIONS ARE OF EQUAL VALUE
- START EACH QUESTION ON A NEW PAGE
- Write your answers on the paper provided.
- Write your name and your teacher's name on each page.
- Approved scientific calculators should be used.
- All necessary working should be shown. Marks may be deducted for careless or badly arranged work.
- Mathematical templates and geometrical instruments may be used.

QUESTION 1 (15 Marks)

Marks

- a) Evaluate  $\sqrt{\frac{(2.63)^3}{1 - 0.6 \times 0.3}}$  to 2 decimal places 1
- b) Subtract  $2x - 3$  from  $6x^2 - 3x + 8$  1
- c) If  $x = -3$ , find the value of  $-2x^2$  1
- d) Evaluate  $\frac{2.65 \times 10^7}{3.4 \times 10^{-3}}$   
expressing your answer in scientific notation correct to 2 significant figures. 2
- e) Simplify:  $3(3x + 2) - 2(3x + 1)$  2
- f) Expand and simplify:  $(6 - 2\sqrt{3})(5\sqrt{3} - 2)$  2
- g) Show that  $x = 5$  is not the only solution of  $x^2 = 5x$  2
- h) Simplify:  $\frac{3x}{(x+1)^2} - \frac{5}{x+1}$  3
- i) A student solved a problem in the following way: 1
- $$\begin{aligned} & |6 \times (-3)| + |-2|^3 \\ & = 18 - 8 \\ & = 10 \end{aligned}$$

Explain the mistake she made in arriving at this answer.

QUESTION 2 (15 Marks) Start a new page

Marks

- a) Express  $0.4\dot{2}$  as a fraction in simplest form. 2
- b) Solve simultaneously: 2
- $$2x - y = 15$$
- $$2y - x = 0$$
- c) An agent charges 6% commission on the first \$200000 of the value of a property sold and 2% on the remaining value. What commission does he receive for selling a house for \$650000? 2
- d) Make  $h$  the subject of the formula: 2
- $$d = 5\sqrt{\frac{h}{2}}$$
- e) Ben made a mistake in solving this problem. Identify the line in which the mistake was made and correct the solution: 3
- Line (i)  $\frac{x}{2} + \frac{x-2}{(x+3)} = \frac{x(x+3) + 2(x-2)}{2(x+3)}$
- Line (ii)  $= \frac{x + 2(x-2)}{2}$
- Line (iii)  $= \frac{x + 2x - 4}{2}$
- Line (iv)  $= \frac{3x - 4}{2}$
- f) Explain how you know that  $5x^2 - x + 10 = 0$  has no real solutions. (Show all necessary working.) 2
- g) If  $3 = m^{0.67}$ , find the value of  $m^{2.01}$  2

QUESTION 3 (15 Marks) Start a new page.

Marks

- a) Factorise fully:

$$6x^2y^2 - 6$$

2

- b) Find to the nearest integer the value of

2

$$M^4 \text{ if } M = \sqrt{a - 2bc} \text{ where } a = 300.6, b = 2.4 \text{ and } c = -5$$

- c) Simplify:

4

$$\frac{x^3 + 27}{9x + 18} \times \frac{5x^2 - 20}{x^2 + 3x + 9}$$

- d) A decimal approximation to  $\frac{5}{4\sqrt{7}}$  is 0.47.

2

When David was asked to find a decimal approximation for  $\frac{5\sqrt{7}}{28}$ , he immediately

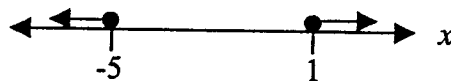
wrote down 0.47. Explain why  $\frac{5}{4\sqrt{7}}$  and  $\frac{5\sqrt{7}}{28}$  must both equal 0.47.

- e) Solve  $\frac{3x}{5} = 4 - \frac{x+3}{3}$

3

- f) Write an absolute value question which has this solution when graphed:

2



**QUESTION 4** (15 Marks) Start a new page

**Marks**

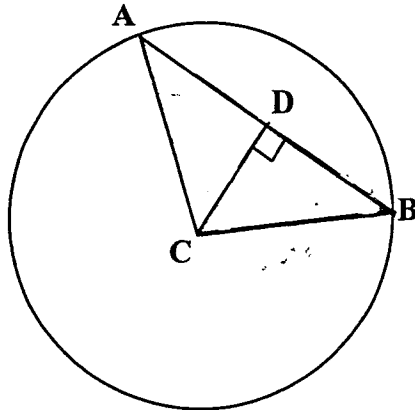
- a) Solve for  $x$ ,  $0 \leq x \leq 180^\circ$ :

1

$$\cot 60^\circ = \tan x^\circ$$

- b) Find the length of AB in this diagram to 2 decimal places:

2



radius = 20cm  
 $\angle DCB = 42^\circ$

C is the centre of the circle.

- c) Find all possible values of  $\theta$  to the nearest minute where  $0^\circ \leq \theta \leq 360^\circ$ .

(i)  $\sin \theta = 0.9897$

2

(ii)  $2 \sin^2 \theta - 1 = 0$

3

- d) Show that  $\cos^2 45 + \sin 60 = \frac{1 + \sqrt{3}}{2}$

2

- e) A ship sails 70km from a harbour on a bearing of  $030^\circ$  and then 40km on a bearing of  $120^\circ$ .

- (i) Draw a diagram showing all this information.

2

- (ii) Find the ship's distance from its starting point, correct to 1 decimal place.

1

- (iii) What bearing would a rescue boat leaving from the same harbour need to sail to meet the ship?

2

**END OF PAPER**

QUESTION 1

a)  $4.71$  ✓

b)  $6x^2 - 3x + 8 - (2x - 3)$   
 $= 6x^2 - 3x + 8 - 2x + 3$   
 $= 6x^2 - 5x + 11$  ✓

c)  $-2x^2$   
 $= -2 \times 3^2$   
 $= -2 \times 9$   
 $= -18$  ✓

d)  $7.8 \times 10^9$  ✓✓

e)  $3(3x+2) - 2(3x+1)$   
 $= 9x + 6 - 6x - 2$   
 $= 3x + 4$  ✓✓

f)  $(6 - 2\sqrt{3})(5\sqrt{3} - 2)$   
 $= (6 - 2\sqrt{3})(5\sqrt{3})$   
 $= (6 - 2\sqrt{3}) \cdot 5$   
 $= 30 - 10\sqrt{3}$  ✓✓

OR misprint on paper  
Question should have been  
 $(6 - 2\sqrt{3})(5\sqrt{3} - 2)$

$$= 30\sqrt{3} - 12 - 10 \times 3 + 4\sqrt{3}$$
$$= 34\sqrt{3} - 12 - 30$$
$$= 34\sqrt{3} - 42$$

g)  $x^2 = 5x$   
 $x^2 - 5x = 0$   
 $x(x - 5) = 0$   
 $x = 0, x = 5$   
 $\therefore x = 0$  is also a solution ✓✓ (R)

h)  $\frac{3x}{(x+1)^2} - \frac{5}{x+1}$   
 $= \frac{3x - 5(x+1)}{(x+1)^2}$   
 $= \frac{3x - 5x - 5}{(x+1)^2}$   
 $= \frac{-2x - 5}{(x+1)^2}$  ✓✓✓

i) Correct solution is  
 $|6x - 3| + |-2|^3$   
 $= |18| + |-8|$   
 $= 18 + 8$   
 $= 26$  ✓ (C)

She forgot to take the absolute value of the  $-2^3$ . It should be positive.

QUESTION 2

a) Let  $x = 0.4\dot{2}$   
 $x = 0.424242\dots$   
 $100x = 42.424242\dots$   
Subtract  
 $99x = 42$   
 $x = \frac{42}{99}$   
 $x = \frac{14}{33}$  ✓✓

b)  $2x - y = 15$  ①  
 $2y - x = 0$  ②  $x = 2y$   
Sub ② into ①  
 $2(2y) - y = 15$   
 $4y - y = 15$   
 $3y = 15$   
 $y = 5$   
Sub. into ②  
 $x = 2 \times 5$   
 $= 10$

$\therefore x = 10$   
 $y = 5$  ✓✓

c) Commission  
 $= 6\% \times 200000 + 2\% \times 450000$   
 $= 12000 + 9000$   
 $= \$ 21000$  ✓✓

d)  $d = 5\sqrt{\frac{h}{2}}$   
 $\frac{d}{5} = \sqrt{\frac{h}{2}}$   
Square both sides  
 $\frac{d^2}{25} = \frac{h}{2}$   
 $h = \frac{2d^2}{25}$  ✓✓

e) A cancelling mistake has been made in line (i). You cannot cancel unless it is a common factor. ✓

Correct solution  
 $\frac{x}{2} + \frac{x-2}{x+3} = \frac{x(x+3) + 2(x-2)}{2(x+3)}$   
 $= \frac{x^2 + 3x + 2x - 4}{2(x+3)}$   
 $= \frac{x^2 + 5x - 4}{2(x+3)}$  ✓ (R)

f)  $5x^2 - x + 10 = 0$   
 $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$   
 $= \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \cdot 5 \cdot 10}}{2 \times 5}$   
 $= \frac{1 \pm \sqrt{1 - 200}}{10}$   
 $= \frac{1 \pm \sqrt{-199}}{10}$  ✓

This equation has no real solutions since you can't take the square root of a negative number. ✓ (C)

g)  $m^{0.67} = 3$   
 $m^{2.01} = m^{0.67 \times 3}$   
 $= (m^{0.67})^3$   
 $= 3^3$   
 $= 27$  ✓ (C)

QUESTION 3:

a)  $6x^2y^2 - 6$

$= 6(x^2y^2 - 1)$  ✓

$= 6(xy-1)(xy+1)$  ✓

b)  $M^4$

$= \sqrt{300 \cdot 6 - 2 \times 2.4x - 5}$

$= 105365.16$  ✓✓

$= 105365$

(to nearest integer)

c)  $\frac{x^3+27}{9x+18} \times \frac{5x^2-20}{x^2-3x+9}$  ✓✓✓

$\frac{(x+3)(x^2-3x+9) \times 5(x-2)(x+2)}{9(x+2) \times (x^2-3x+9)}$

$= \frac{5(x-2)(x+3)}{9}$  ✓

d) When you rationalise

$\frac{5}{4\sqrt{7}}$  you get  $\frac{5\sqrt{7}}{28}$  ✓

Rationalising does not change the number so the decimal approximation is the same. ✓

✓ (C)

e)  $\frac{3x}{5} = 4 - \frac{x+3}{3}$  ✓✓

multiply all terms by 15

$9x = 60 - 5(x+3)$  ✓

$9x = 60 - 5x - 15$

$14x = 45$  ✓

$x = \frac{45}{14}$  ✓

$x = 3\frac{3}{14}$  ✓

f)  $|x+2| > 3$  ✓✓  
(must be  $>$ )

Easiest way is to find middle value at -2

✓ (R)

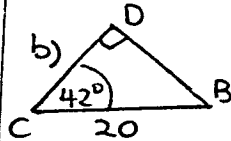
QUESTION 4:

a) Using complement rule

$\tan(90-\theta) = \cot \theta$

$\cot 60^\circ = \tan(90-60)^\circ$   
 $= \tan 30^\circ$

$\therefore x = 30$  ✓



$\sin 42^\circ = \frac{DB}{20}$

$DB = 20 \cdot \sin 42^\circ$   
 $= 13.38 \dots$  ✓

$\therefore AB = 2 \times DB$   
 $= 26.77$  (2dp.)

✓ (R)

c) i)  $\sin \theta = 0.9897$   
 $\sin \theta$  is positive in Quadrants ① and ②

$\theta = 81^\circ 46'$ ,  $(180 - 81^\circ 46')$   
 $= 81^\circ 46'$ ,  $98^\circ 14'$

ii)  $2 \sin^2 \theta - 1 = 0$

$2 \sin^2 \theta = 1$

$\sin^2 \theta = \frac{1}{2}$

$\sin \theta = \pm \frac{1}{\sqrt{2}}$  ✓

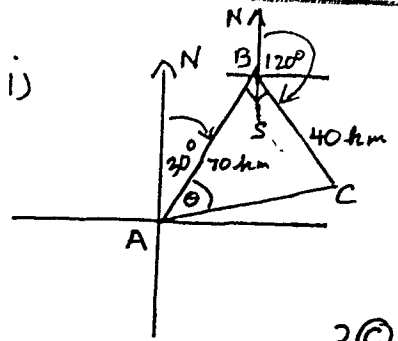
$\sin \theta = \frac{1}{\sqrt{2}}$   
Quads ① & ②

$\sin \theta = -\frac{1}{\sqrt{2}}$   
Quads ③ & ④

$\theta = 45^\circ, (180-45)^\circ$  |  $\theta = (180+45)^\circ, (360-45)^\circ$   
 $= 45^\circ, 135^\circ$  |  $= 225^\circ, 315^\circ$

$\theta = 45^\circ, 135^\circ, 225^\circ, 315^\circ$  ✓✓

e) i)



2 (C) diagram

$\angle NAB = \angle ABS = 30^\circ$  (alt angles)  
 $\angle CBS = 180 - 120 = 60^\circ$  (straight)

ii)  $\angle ABC = 30^\circ + 60^\circ$   
 $= 90^\circ$

$\therefore \triangle ABC$  is right angled.

By Pythagoras

$AC^2 = AB^2 + BC^2$   
 $= 70^2 + 40^2$   
 $= 6500$

$AC = \sqrt{6500}$   
 $= 80.6$  km ✓

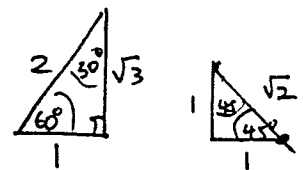
iii) Find  $\angle BAC$

$\tan \theta = \frac{40}{70}$

$\theta = 29^\circ 45'$

$\therefore$  Bearing  $= 30^\circ + 29^\circ 45'$   
 $= 59^\circ 45'$  ✓✓

d)



$\cos^2 45^\circ + \sin^2 60^\circ$

$= \left(\frac{1}{\sqrt{2}}\right)^2 + \frac{\sqrt{3}}{2}$  ✓

$= \frac{1}{2} + \frac{\sqrt{3}}{2}$  ✓

$= \frac{1+\sqrt{3}}{2}$