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SCEGGS Darlinghurst

Preliminary Assessment Task 4
Friday, 15 August, 2003

Preliminary Mathematics

Task Weighting 15%

General Instructions

- Time allowed – 50 minutes
- Write your name at the top of each page
- Start each question on a new page
- Attempt **all** questions
- Show all working
- Marks may be deducted for careless or badly arranged work
- Approved calculators should be used
- Mathematical templates, geometrical equipment and scientific calculators may be used.

	Marks
Question 1	
Communication 1 / 2	6 / 9
Question 2	
Communication 3 / 3	7 / 9
Question 3	
Reasoning 3 / 3	8 / 9
Question 4	
Reasoning 6 / 7	5 / 9
TOTAL C 4 R 9	26 / 36

Question 1 (9 Marks)

Marks

- (a) If α and β are the roots of the equation $2x^2 - 7x - 5 = 0$, find the value of
- i) $\alpha + \beta$ 1
 - ii) $\alpha\beta$ 1
 - iii) $\alpha^2 + \alpha\beta + \beta^2$ 2
- (b) Kerry said that the roots of $10 + 4x - 3x^2 = 0$ are real, unequal and rational. Do you agree with Kerry? Justify your answer with appropriate calculations. 2
- (c) Find the coordinates of the focus of the parabola $x^2 = -8(y-2)$ 1
- (d) Sketch the parabola $y = -3x^2 + 6x - 7$, showing all important features. 2

Question 2 (9 marks)

START A NEW PAGE

- (a) Solve for x : $2x^2 + 7x > 30$ 3
- (b) A point, P, moves according to the equation $x^2 + 2x + y^2 - 6y + 7 = 0$. Describe the locus of P. 3
- (c) Solve for x : $36^x - 4(6^x) - 12 = 0$ 3

Question 3 (9 marks)**START A NEW PAGE****Marks**

- (a) A point $P(x, y)$ moves so that its distance from $A(3, 2)$ is always twice its distance from $B(-1, 6)$. Find the equation of the path of P . 3
- (b) Express $2x^2 - 7x - 4$ in the form $A(x+2)^2 + B(x+2) + C$. 3
- (c) Is the line, $3x + 4y - 15 = 0$, a tangent to the circle $(x-3)^2 + (y+6)^2 = 25$? 3
Justify your answer with appropriate calculations.

Question 4 (9 marks)**START A NEW PAGE**

- (a) A point moves so that it is always the same distance from the point $(4, -1)$ as it is from the line $y = -5$. Find the equation of the locus. 2
- (b) Find the value of m in the quadratic, $5x^2 - 2mx + 10 = 0$ if it has no real roots. 3
- (c) The roots of $2x^2 - 2kx + k + 7 = 0$ are consecutive integers. Find the value of k . 4

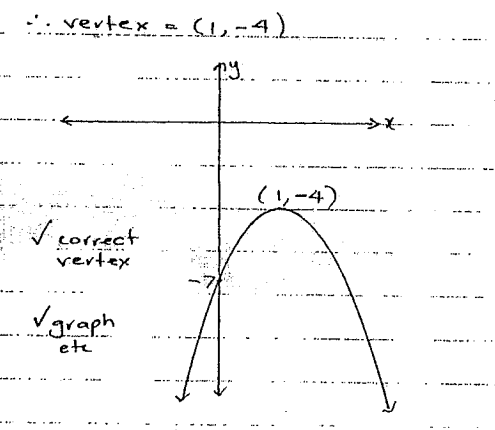
END OF ASSESSMENT

Yr. 11 Assessment 4 Solutions

a) $2x^2 - 7x - 5 = 0$
 i) $\alpha + \beta = -b/a$
 $= 7/2$ ✓
 ii) $\alpha\beta = c/a$
 $= -5/2$ ✓
 iii) $\alpha^2 + \alpha\beta + \beta^2$
 $= (\alpha + \beta)^2 - \alpha\beta$
 $= (7/2)^2 - (-5/2)$ ✓
 $= 14\frac{3}{4}$ ✓

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
 $x = \frac{-(-7) \pm \sqrt{49 - 4(2)(-5)}}{2(2)}$
 $x = \frac{7 \pm \sqrt{121}}{4}$
 $x = \frac{7 \pm 11}{4}$
 $x = 2$ or $x = -1$
 \therefore no solution \therefore no x-intercept
 axis of sym = $-\frac{b}{2a} = \frac{7}{4}$
 $x = \frac{-b}{2a} = \frac{7}{4}$
 $x = 1, y = -3(1)^2 + 6(1) - 7 = -4$

b) $10 + 4x - 3x^2 = 0$
 $\Delta = b^2 - 4ac$
 $= 16 - 4(-3)(10)$
 $= 16 + 120$
 $= 136$ ✓
 \therefore No 1 do not agree with Kerry. The roots are real and unequal but they are irrational not rational. ✓



c) $x^2 = -8(y-2)$
 vertex = $(0, 2)$
 focal length = 2
 \therefore Focus = $(0, 0)$ ✓

Marking
 1. CB ✓
 2. KB ✓
 3. MF ✓
 4. SB ✓

d) $y = -3x^2 + 6x - 7$
 $y_{int} = -7$
 $x_{int} : -3x^2 + 6x - 7 = 0$
 $x = \frac{-6 \pm \sqrt{36 - 4(-3)(-7)}}{2(-3)}$
 $x = \frac{-6 \pm \sqrt{36 - 84}}{-6}$

2a) $2x^2 + 7x > 30$
 $2x^2 + 7x - 30 > 0$
 $(2x-5)(x+6) > 0$ ✓
 $x > 2\frac{1}{2}$ and $x < -6$ ✓

b) $x^2 + 2x + y^2 - 6y + 7 = 0$
 $x^2 + 2x + 1 + y^2 - 6y + 9 = -7 + 1 + 9$
 $(x+1)^2 + (y-3)^2 = 3$ ✓
 \therefore The locus is a circle with the centre at $(-1, 3)$ and a radius of $\sqrt{3}$ units. ✓
 (must be a description)

c) $36^x - 4(6^x) - 12 = 0$
 Let $m = 6^x$
 $m^2 - 4m - 12 = 0$
 $(m-6)(m+2) = 0$
 $m = 6$ or -2 ✓
 $\therefore 6^x = 6$ or $6^x = -2$
 $x = 1$ no solⁿ ✓
 $\therefore x = 1$ ✓

3a) $PA = 2PB$ ✓
 $\sqrt{(x-3)^2 + (y-2)^2} = 2\sqrt{(x+1)^2 + (y-6)^2}$
 $x^2 - 6x + 9 + y^2 - 4y + 4 = 4[x^2 + 2x + 1 + y^2 - 12y + 36]$
 $x^2 - 6x + y^2 - 4y + 13 = 4x^2 + 8x + 4y^2 - 48y + 136$
 $3x^2 + 14x + 3y^2 - 44y + 246 = 0$ ✓

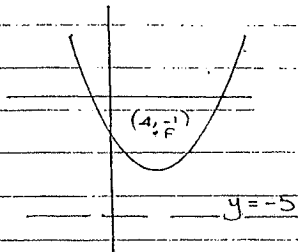
b) $2x^2 - 7x - 4 \equiv A(x+2)^2 + B(x+2) + C$
 $\equiv A(x^2 + 4x + 4) + Bx + 2B + C$
 $\equiv Ax^2 + (4A+B)x + (4A+2B+C)$
 $\therefore A = 2$
 $4A+B = -7$
 $8+B = -7$
 $B = -15$
 $4A+2B+C = -4$
 $8-30+C = -4$
 $C = 18$

$\therefore 2x^2 - 7x - 4 \equiv 2(x+2)^2 - 15(x+2) + 18$ ✓
 c) centre = $(3, -6)$ radius = 5
 perp. dist. from centre to line
 $d = \frac{|3(3) + 4(-6) - 15|}{\sqrt{3^2 + 4^2}}$ ✓
 $= \frac{|-30|}{5}$
 $= 6$ ✓

As this distance is greater than the radius the line is not a tangent to the circle. they do not touch at all. ✓

4a) Focus = (4, -1)

directrix: $y = -5$



\therefore vertex = (4, -3) ✓

$a = 2$

$\therefore (x-4)^2 = 8(y+3)$ ✓

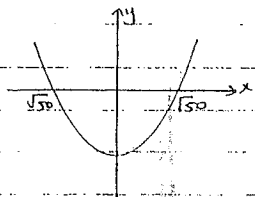
b) $5x^2 - 2mx + 10 = 0$

if no real roots $\Delta < 0$ ✓_R

$b^2 - 4ac = (-2m)^2 - 4(5)(10)$
 $= 4m^2 - 200$ ✓_R

$4m^2 - 200 < 0$

$m^2 - 50 < 0$



$-\sqrt{50} < m < \sqrt{50}$ ✓_R

$-5\sqrt{2} < m < 5\sqrt{2}$

c) $2x^2 - 2kx + (k+7) = 0$

Let roots be α and $\alpha+1$

$\alpha + \alpha + 1 = \frac{-b}{a} = \frac{+2k}{2}$

$2\alpha + 1 = k$ ①

$\alpha(\alpha+1) = \frac{c}{a} = \frac{k+7}{2}$ ✓_R
 both equations

$\alpha^2 + \alpha = \frac{k+7}{2}$ ②

Sub ① into ②

$\alpha^2 + \alpha = \frac{2\alpha + 1 + 7}{2}$ ✓_R
 method

$2\alpha^2 + 2\alpha = 2\alpha + 8$

$2\alpha^2 = 8$

$\alpha^2 = 4$

$\alpha = \pm 2$ ✓_R

$\therefore k = 2 \times 2 + 1$ or $2 \times -2 + 1$

$k = 5$ or -3 ✓_R