SCEGGS Darlinghurst

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Term 1, 2003
Monday 17th March

EXTENSION 1 MATHEMATICS

Task Weighting : 10 %

General Instructions

- Time allowed - 70 minutes
- Write your name at the top of each page
- Start each question on a new page
- Attempt **all** questions.
- Marks may be deducted for careless or badly arranged work
- Approved calculators should be used
- Mathematical templates and geometrical equipment may be used.

Question 1 Reas 1 /1	12 /12
Question 2 Reas 4/4 Com 1 /1	12 /12
Question 3 ☺	12 /12
Question 4 Reas 5/5 ☺	12 /12
TOTAL	48 /48

Fantastic!

QUESTION 1 (12 marks) Marks

(a) Expand and simplify 2

$$(p+q)^2-(p-q)^2$$

(b) Factorise fully

(i) $4x^3-12x^2-x+3$ 3

(ii) $6p^2-5pq-4q^2$ 1

$-8 \quad +3$

$6p-8q \qquad 6p+3q$

$(3p-4q \qquad 2p+q$

(c) Simplify fully 4

$$\frac{x^2-9}{x^4-27x} \div \frac{x+3}{x^2+3x+9}$$

(d) (i) Expand $\left(x+\frac{1}{x}\right)^2$ 1

(ii) Given that $x+\frac{1}{x}=3$ 1R

use part (i) to evaluate $x^2+\frac{1}{x^2}$

without attempting to find the value of x.

QUESTION 2 (12 marks) START A NEW PAGE Marks

(a) Solve for x 2

$$\frac{1}{2x} - \frac{2}{3} = 1 - \frac{1}{3x}$$

(b) Solve for x 3

$$\frac{x+1}{x-1} \leq 2$$

(c) Solve simultaneously for a, b, c. 4R

$$3a - 2b - c = -8$$
$$5a + b + 3c = 23$$
$$4a + b - 5c = -18$$

(d) Katie was asked to find the values of x
for which $x \times x \leq x + x$.
To do this, she decided to solve the inequality $x^2 \leq 2x$.
She divided by x and concluded that the solution was $x \leq 2$.

Explain why Katie was incorrect in solving the inequality this way. 1C

Show the correct solution. 2

QUESTION 3 (12 marks) START A NEW PAGE Marks

(a) Given that $\tan\theta = -\frac{5}{12}$ and that θ is obtuse,

find $\sin\theta$ and $\sec\theta$. 3

(b) Solve for $-180° \le \theta \le 180°$,

$\tan\theta = -1$ 2

(c) Solve for $0° \le \theta \le 360°$,

$\sec^2\theta = 2$ 3

(d) Solve for $0° \le \theta \le 360°$,

$\sin 2\theta - 2\cos^2\theta = 0$ 4

QUESTION 4 (12 Marks) START A NEW PAGE Marks

$\cos 2\theta = \cos^2\theta - \sin^2\theta$
$= \cos^2\theta - (1-\cos^2$
$= \cos^2\theta - 1 + \cos^2$
$= 2\cos^2\theta - 1$

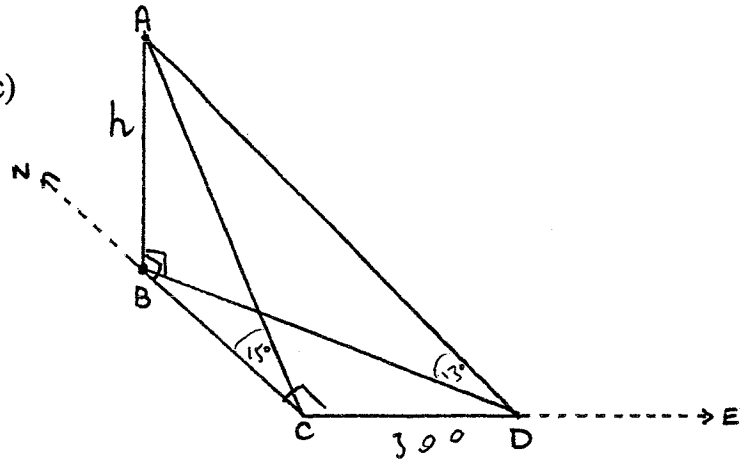
(a) Prove the identity

$$\frac{2\cos^3\theta - \cos\theta}{\sin\theta\ \cos^2\theta - \sin^3\theta} = \cot\theta$$ 3(2R)

(b) By expressing $15°$ as either $(45-30)°$ or $(60-45)°$, prove that

$$\sin 15° = \frac{\sqrt{6}-\sqrt{2}}{4}$$ 3

(c)



B, C, D are points on level ground, with D a distance of 300 metres due east of C and B due north of C.
A vertical mast AB stands at B.
At C, the angle of elevation to the top of the mast is 15° and at D, the angle of elevation of A is 13°.

(i) In ΔABC, show that $BC = h\cot 15°$ 1

(ii) Similarly, show that $BD = h\cot 13°$ 1

(iii) Show that

$$h = \frac{300}{\sqrt{\cot^2 13° - \cot^2 15°}}$$ 3R

(iv) Hence find the height of the mast to the nearest metre. 1

END OF EXAMINATION

Extension 1 Year 11

Solutions.

1) a) $(p+q)^2-(p-q)^2$

$=p^2+2pq+q^2-(p^2-2pq+q^2)$ ✓

$=p^2+2pq+q^2-p^2+2pq-q^2$

$=4pq$ ✓

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b) i) $4x^3-12x^2-x+3$

$=4x^2(x-3)-1(x-3)$ ✓

$=(x-3)(4x^2-1)$ ✓

$=(x-3)(2x-1)(2x+1)$ ✓

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ii) $6p^2-5pq-4q^2$

$3p \times -4q$

$2p \times +q$

$=(3p-4q)(2p+q)$ ✓

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c)

$$\frac{x^2-9}{x^2-27x} \div \frac{x+3}{x^2+3x+9}$$ ✓ ✓

$$=\frac{(x-3)(x+3)}{x(x^3-27)} \times \frac{x^2+3x+9}{x+3}$$

$$=\frac{(x-3)(x^2+3x+9)}{x(x-3)(x^2+3x+9)}$$ ✓

$$=\frac{1}{x}$$ ✓

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d) i) $\left(x+\frac{1}{x}\right)^2$

$=x^2+2x.\frac{1}{x}+\frac{1}{x^2}$

$=x^2+2+\frac{1}{x^2}$ ✓

ii) $x^2+\frac{1}{x^2}+2=\left(x+\frac{1}{x}\right)^2$

$x^2+\frac{1}{x^2}+2=3^2$

$x^2+\frac{1}{x^2}=9-2$

$=7$ ✓(1R)

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2) a) $\frac{1}{2x}-\frac{2}{3}=1-\frac{1}{3x}$

$\frac{3-4x}{6x}=\frac{6x-2}{6x}$

$3-4x=6x-2$ ✓

$-10x=-5$

$x=\frac{-5}{-10}$

$x=\frac{1}{2}$ ✓

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b) $\frac{x+1}{x-1} \leq 2$

undefined for $x=1$ ✓

$\frac{x+1}{x-1} \times (x-1)^2 \leq 2(x-1)^2$

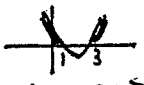
$(x+1)(x-1) \leq 2(x^2-2x+1)$

$x^2-1 \leq 2x^2-4x+2$

$0 \leq x^2-4x+3$

$x^2-4x+3 \geq 0$ ✓

$(x-3)(x-1) \geq 0$



$x \leq 1, x \geq 3$ ✓

Solution

$x<1, x>3$

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c)

$3a-2b-c=-8$ ①

$5a+b+3c=23$ ②

$4a+b-5c=-18$ ③

②×2 $10a+2b+6c=46$ ④

③×2 $8a+2b-10c=-36$ ⑤

$3a-2b-c=-8$ ①

④−⑤ $2a+16c=82$ ⑥

⑤+① $11a-11c=-44$ ⑦ ✓

⑥×11 $22a+176c=902$ ⑧

⑦×2 $22a-22c=-88$ ⑨

⑧−⑨ $198c=990$

$c=5$ ✓

Substitute into ⑧

$22a+880=902$

$22a=22$

$a=1$ ✓

Substitute into ②

$5+b+15=23$

$b+20=23$

$b=3$ ✓

Solution

$a=1$

$b=3$

$c=5$

(4R)

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d) It is incorrect to divide both sides by x.

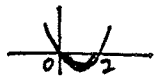
Part of the solution is lost this way.

She needs to factorize the quadratic and then solve the inequality. ✓(1C)

$x^2 \leq 2x$

$x^2-2x \leq 0$

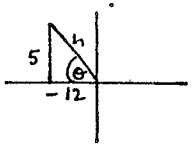
$x(x-2) \leq 0$ ✓



$0 \leq x \leq 2$ ✓

a) $\tan\theta = -5/12$, θ is obtuse

θ lies in quadrant 2



By pythagoras

$h^2 = 5^2 + 12^2$

$= 25 + 144$

$= 169$

$h = 13$

$\sin\theta = \frac{5}{13}$ ✓

$\sec\theta = \frac{1}{\cos\theta}$

$= \frac{1}{-\frac{12}{13}}$ ✓✓

$= -\frac{13}{12}$

b) $\tan\theta = -1$

θ lies in Quadrants 2 and 4

Acute angle

$\tan\theta = 1$

$\theta = 45°$

$\therefore \theta = (180-45)°, (360-45)°$

$= 135°, 315°$

for $-180° \le \theta \le 180°$

$= 135°, -45°$

✓ ✓

c) $\sec^2\theta = 2$

$\sec\theta = \pm\sqrt{2}$

$\sec\theta = \sqrt{2}$	$\sec\theta = -\sqrt{2}$
$\frac{1}{\cos\theta} = \sqrt{2}$	$\frac{1}{\cos\theta} = -\sqrt{2}$
$\cos\theta = \frac{1}{\sqrt{2}}$ ✓	$\cos\theta = -\frac{1}{\sqrt{2}}$
Quad 1 & 4	Quad 2 and 3
$\theta = 45°, (360-45)°$	$\theta = (180-45)°, (180+45)°$
$= 45°, 315°$ ✓	$= 135°, 225°$ ✓

d) $\sin 2\theta - 2\cos^2\theta = 0$

$2\sin\theta\cos\theta - 2\cos^2\theta = 0$ ✓

$2\cos\theta(\sin\theta - \cos\theta) = 0$ ✓

$2\cos\theta = 0$	$\sin\theta - \cos\theta = 0$
$\cos\theta = 0$	$\sin\theta = \cos\theta$
$\theta = 90°, 270°$	$\frac{\sin\theta}{\cos\theta} = 1$
	$\tan\theta = 1$
	quad 1 and 3
	$\theta = 45°, (180+45)°$
	$= 45°, 225°$

✓ ✓

a)

$LHS = \frac{2\cos^3\theta - \cos\theta}{\sin\theta\cos^2\theta - \sin^3\theta}$

$= \frac{\cos\theta(2\cos^2\theta - 1)}{\sin\theta(\cos^2\theta - \sin^2\theta)}$ ✓

$= \frac{\cos\theta(2\cos^2\theta - 1)}{\sin\theta(\cos^2\theta - (1-\cos^2\theta))}$ ✓

$= \frac{\cos\theta(2\cos^2\theta - 1)}{\sin\theta(2\cos^2\theta - 1)}$ (2R) ✓

$= \frac{\cos\theta}{\sin\theta}$

$= \cot\theta$

$= RHS$

OR

$\frac{\cos\theta(2\cos^2\theta - 1)}{\sin\theta(\cos^2\theta - \sin^2\theta)}$

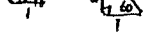
$= \frac{\cos\theta \cdot \cos 2\theta}{\sin\theta \cdot \cos 2\theta}$

$= \frac{\cos\theta}{\sin\theta}$

$= \cot\theta$

$= RHS$

b) $\sin 15°$



$= \sin(45° - 30°)$

$= \sin 45°\cos 30° - \cos 45°\sin 30°$ ✓

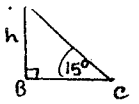
$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$

$= \frac{\sqrt{3} - 1}{2\sqrt{2}}$ ✓

$= \frac{\sqrt{3} - 1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{\sqrt{6} - \sqrt{2}}{4}$ ✓

i) In $\triangle ABC$

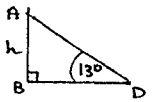


$\tan 15° = \frac{h}{BC}$

$BC = \frac{h}{\tan 15°}$

$BC = h\cot 15°$ ✓

ii) In $\triangle ABD$

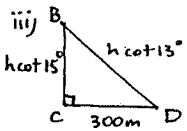


$\tan 13° = \frac{h}{BD}$

$BD = \frac{h}{\tan 13°}$

$BD = h\cot 13°$ ✓

iii)



By Pythagoras ✓

$h^2\cot^2 13° = 300^2 + h^2\cot^2 15°$

$h^2(\cot^2 13° - \cot^2 15°) = 300^2$ ✓

$h^2 = \frac{300^2}{(\cot^2 13° - \cot^2 15°)}$

$h = \sqrt{\frac{300^2}{\cot^2 13° - \cot^2 15°}}$

$= \frac{300}{\sqrt{\cot^2 13° - \cot^2 15°}}$ ✓

(3R)

iv) $h \doteq 136.455...$

$h \doteq 136$ m (to nearest metre) ✓