SCEGGS Darlinghurst

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Term 2, 2003
Monday 23rd June

EXTENSION 1 MATHEMATICS

Preliminary Course Assessment Task 3

Weighting: 15%

General Instructions

- Total Time allowed - 60 minutes.
- Attempt all questions
- All questions are of equal value
- Show ALL working
- Start each question on a NEW PAGE
- Marks may be deducted for careless or badly arranged work.
- Approved scientific calculators and mathematical templates can be used.

		S	R
Question 1 :	10 /10	3	3
Question 2 :	8 /10		2
Question 3 :	10 /10	5	1
Question 4:	10 / 10	2	5
TOTAL	38 / 40	10	11
	95 %		

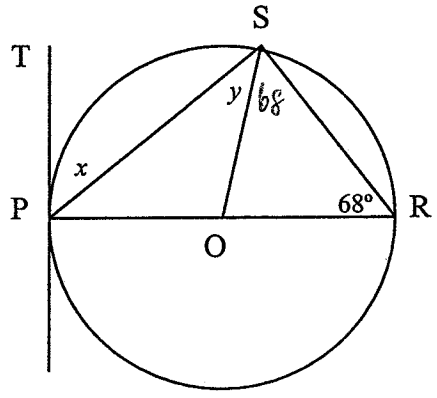
Question 1 (10 Marks) **START A NEW PAGE**

(a) Find the coordinates of the point P which internally divides the interval joining $A(-2,3)$ and $B(10,11)$ in the ratio 3 : 1. 2

(b) Evaluate: 2

$$\lim_{x\to 2}\frac{x^2-5x+6}{x-2}$$

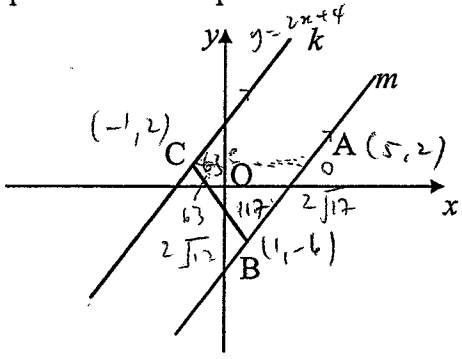
(c) P, S and R are three points on the circumference of a circle with centre O. PT is a tangent to the circle at point P. Find the value of x and y, giving reasons. 3



(d) Find the obtuse angle between the lines $2x+3y-7=0$ and $y=5x-2$ to the nearest degree. 3

Question 2 (10 Marks) **START A NEW PAGE**

The line k passes through $C(-1, 2)$ and has equation $y = 2x + 4$. The point B has coordinates $(1, -6)$. The line m is parallel to k and point A lies on line m.



NOT TO SCALE

Copy the diagram, showing all information.

(a) Find the length of BC in simplest surd form. **1**

(b) Calculate the acute angle line k makes with the x axis to the nearest degree. **2**

(c) Show that the equation of m is given by $y = 2x - 8$. **2**

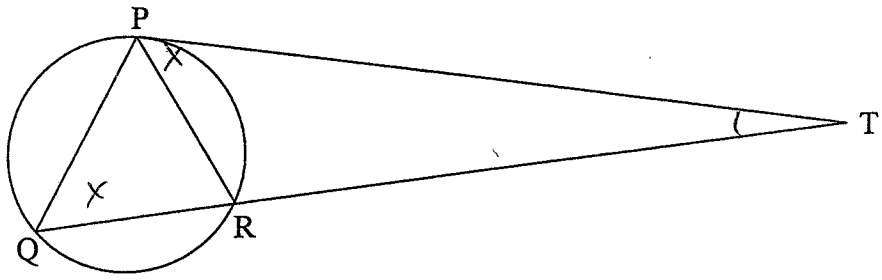
(d) Find the size of $\angle ABC$. **2**

(e) Find the coordinates of point A if AC has a gradient of 0. **1**

(f) Find the area of ΔABC. **2**

Question 3 (10 Marks) **START A NEW PAGE**

(a) PT is a tangent to the circle, and QT is a secant intersecting the circle at Q and R.



Copy or trace the diagram

(i) Prove that $\Delta PRT \,|||\, \Delta QPT$. **3**

(ii) Hence show $PT^2 = QT \times RT$. **2**

(b) Evaluate: **2**

$$\lim_{x\to\infty} \frac{5x^2+3x^3}{5x^3+1}$$

(c) (i) Sketch the curve $y = \dfrac{2}{x^2-4}$ showing all important features. **2**

(ii) Hence solve the inequality $\dfrac{2}{x^2-4} \le -\dfrac{1}{2}$ graphically. **1**

Question 4 (10 Marks) **START A NEW PAGE**

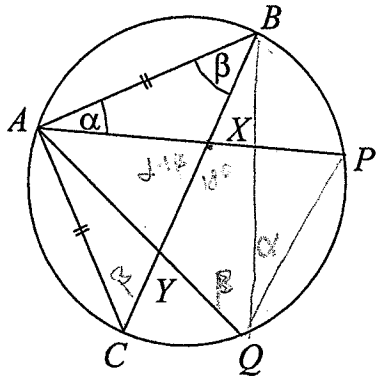
(a) (i) Give the equation of the two vertical asymptotes of the function: **2**

$$y = \frac{2(x-5)}{(x-2)(x-8)}$$

(ii) Show that $\frac{1}{x-2} + \frac{1}{x-8} = \frac{2(x-5)}{(x-2)(x-8)}$. **1**

(iii) Hence sketch $y = \frac{1}{x-2} + \frac{1}{x-8}$ showing all important features. **2**

(b) Let $ABPQC$ be a circle such that $AB = AC$, AP meets BC at X, and AQ meets BC at Y, as in the diagram. Let $\angle BAP = \alpha$ and $\angle ABC = \beta$.



Copy the diagram

(i) State why $\angle AXC = \alpha + \beta$. **1**

(ii) Prove that $\angle BQP = \alpha$. **1**

(iii) Prove that $\angle BQA = \beta$. **2**

(iv) Prove that $PQYX$ is a cyclic quadrilateral. **1**

- END OF ASSESSMENT -

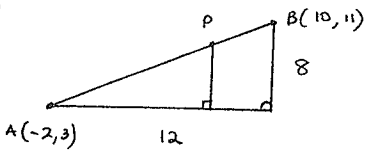
EXTENSION 1 MATHS

ASSESSMENT TASK 3 — /40

Yr 11, 2003

QUESTION 1: (10 marks) C/3 R/3

(a)



$P\left(-2+\frac{3}{4}\times 12,\ 3+\frac{3}{4}\times 8\right)$ ✓✓

$= P(7, 9)$

OR by formula:

$m:n = 3:1$

$x = \frac{3\times 10 + 1\times -2}{4}$ $\quad y = \frac{3\times 11 + 1\times 3}{4}$ ✓✓

$= \frac{28}{4}$ $\quad = \frac{36}{4}$

$\therefore P(7, 9)$

(b) $\lim_{x\to 2} \frac{x^2-5x+6}{x-2}$

$= \lim_{x\to 2} \frac{(x-3)(x-2)}{x-2}$ ✓

$= -1$ ✓

(c) $x = 68^\circ$ ($\angle$ between chord + tangent = $\angle$ in alt segment). ✓

$\angle TPO = 90^\circ$ ($\angle$ between tangent + radii)

$\therefore \angle SPO = 22^\circ$ ✓

$OP = OS$ (radii of same circle)

$\therefore \angle SPO = \angle PSO$ ($\angle$ opp = sides in isos Δ =)

$\therefore y = 22^\circ$ ✓

Com 3 (right $\angle$ ✓ isos Δ ✓ 22 ✓)

(d) $y = 5x - 2$

$m_1 = 5$

$2x + 3y - 7 = 0$

$3y = -2x + 7$

$y = -\frac{2}{3}x + \frac{7}{3}$

$\therefore m_2 = -\frac{2}{3}$ ✓

$\therefore \tan\theta = \left|\frac{5 - -\frac{2}{3}}{1 + 5\times -\frac{2}{3}}\right|$

$= \left|-\frac{17}{7}\right|$

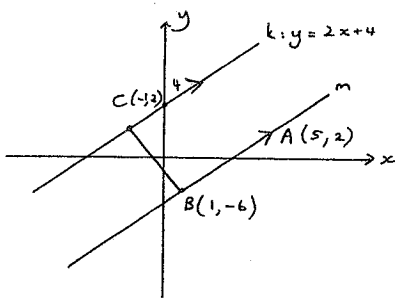
$\therefore \theta = 67^\circ 37'$ ✓

$\therefore$ Obtuse angle $= 112^\circ 23'$

$\doteq 112^\circ$ (nearest degree) ✓

Reas 3

QUESTION 2: (10 marks) R/4 (R/2)



(a) $BC = \sqrt{(1--1)^2 + (-6-2)^2}$

$= \sqrt{4+64}$

$= 2\sqrt{17}$ ✓ (must be simplified)

(b) $m = 2$ ✓

$\therefore \tan\theta = 2$

$\theta \doteq 63^\circ$ (nearest degree) ✓

(c) $m \parallel k$ $\therefore$ grad $= 2$ ✓

$y + 6 = 2(x-1)$

$y = 2x - 8$ ✓

(d) $m_{BC} = \frac{2--6}{-1-1} = -4$ ✓

$\tan\theta = \left|\frac{-4-2}{1+-4\times 2}\right|$

$= \frac{6}{7}$ Reason/2

$\therefore \theta = 40^\circ 36'$ ✓

(e) Grad $= 0 \Rightarrow$ AC is horizontal

$\therefore y = 2$

$\therefore 2 = 2x - 8$

$\therefore x = 5$ $\therefore A(5, 2)$ ✓

(f) $AC = 6$ ✓

$\perp$ height $= 8$

$\therefore$ Area $= \frac{1}{2}\times 6\times 8$

$= 24\ u^2$. ✓ Reas 2

OR $AB = \sqrt{16+64}$

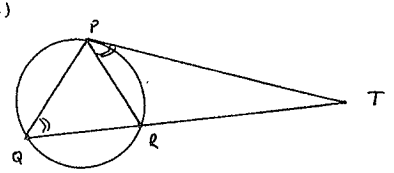
$= 4\sqrt{5}$ ✓

Area $= \frac{1}{2}\times 2\sqrt{17}\times 4\sqrt{5}\times \sin 40^\circ 36'$

$= 24\ u^2$ ✓

QUESTION 3: (10 marks) C 1/5 R 1/5 H 4

(a)



(i) $\angle PTR$ is common ✓

$\angle TPR = \angle PQR$ ($\angle$ between tangent + chord = $\angle$ in alt segment) ✓

$\therefore \Delta PRT \,|||\, \Delta QPT$ (equiangular) ✓

Com 1/3

(ii) $\therefore \frac{PT}{RT} = \frac{QT}{PT}$ (sides in same ratio in $|||$ s) ✓

$\therefore PT^2 = QT \times RT$ ✓

(b) $\lim_{x \to \infty} \frac{5x^2 + 3x^3}{5x^3 + 1}$

$= \lim_{x \to \infty} \frac{\frac{5}{x^3} + 3}{5 + \frac{1}{x^3}}$ ✓ ($\div$ by x^3)

$= \frac{3}{5}$ ✓

(c)(i) $y = \frac{2}{x^2 - 4}$

Asymptotes: $x^2 - 4 = 0$

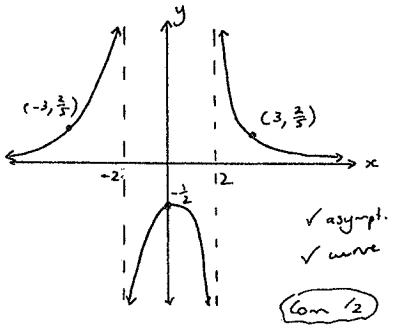
$\therefore x = \pm 2$.

Intercepts: when $x = 0$, $y = -\frac{1}{2}$

no x intercepts

Limits: As $x \to \infty$, $y \to 0^+$

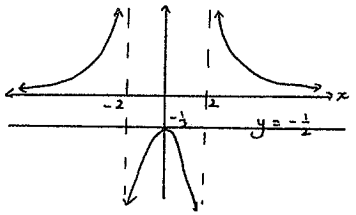
As $x \to -\infty$, $y \to 0^+$



✓ asympt.
✓ curve

Com 1/2

(ii) $\frac{2}{x^2 - 4} \leq -\frac{1}{2}$



$\therefore -2 < x < 2$ ✓

Reas 1

QUESTION 4: (10 marks) C 1/2 R 1/5 H 4

(a) (i) $y = \frac{2(x-5)}{(x-2)(x-8)}$

Vertical Asymptotes:

$x = 2$ ✓

$x = 8$ ✓

(ii) LHS $= \frac{1}{x-2} + \frac{1}{x-8}$

$= \frac{x-8+x-2}{(x-2)(x-8)}$

$= \frac{2x-10}{(x-2)(x-8)}$ ✓

$= \frac{2(x-5)}{(x-2)(x-8)}$

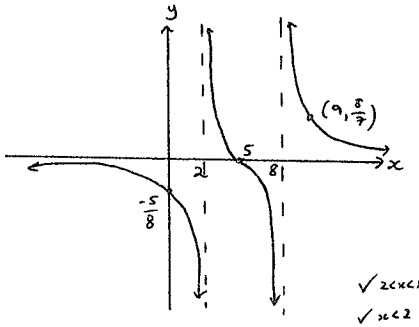
(iii) Asymptotes: $x = 2$, $x = 8$

Intercepts: when $x = 0$, $y = -\frac{5}{8}$

when $y = 0$, $x = 5$

Limits: As $x \to \infty$, $y \to 0^+$

$x \to -\infty$, $y \to 0^-$

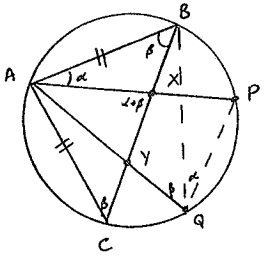


✓ $2 < x < 5$

✓ $x < 2$, $x > 8$

Com 1/2

(b)



(i) $\angle AXC = \alpha + \beta$ (exterior $\angle$ of Δ = Σ two interior opp $\angle$s) ✓

(ii) $\angle BQP = \alpha$ ($\angle$ at circum. standing on arc BP are =) ✓

(iii) $\angle BCA = \beta$ ($\angle$ opp = sides in isos ΔABC are =) ✓

$\therefore \angle AQP = \beta$ ($\angle$ at circum. standing on arc AB are =) ✓

(iv) $\angle PXY = 180 - (\alpha + \beta)$ ($\angle \Sigma$ of str. line = 180°)

$\angle PQY = \alpha + \beta$.

$\therefore \angle PXY + \angle PQY = 180 - (\alpha+\beta) + \alpha + \beta$

$= 180$ ✓

$\therefore$ Since opp $\angle$ are supplementary,

PQYX is a cyclic quadrilateral

Reas 1/5