Centre Number

Student Number

SCEGGS Darlinghurst

2008

Preliminary Course
Assessment Task 2

Mathematics Extension 1

Outcomes Assessed: PE1, PE2, PE6
Task Weighting: 20%

General Instructions

- Time allowed – 1 hour
- This paper has **four** questions
- Write your Student Number and Centre Number at the top of each page
- Attempt **all** questions
- Write using blue or black pen
- Answer all questions on the pad paper provided
- Draw all diagrams using a pencil and ruler
- Begin each question on a new page
- Marks will be deducted for careless or badly arranged work
- Approved scientific calculators and mathematical templates may be used

Total marks – 42

- Attempt Questions 1 – 4

Question	Reasoning	Communication	Marks
1			10
2			12
3			11
4			9
TOTAL			42

Marks

Question 1 (10 marks)

(a) Find the acute angle between the lines $x + 2y + 1 = 0$ and $y = 3x + 2$ to the nearest minute. 2

(b) When asked to solve the equation $2\sin x - \cos x = 1$, $0^\circ \le x \le 180^\circ$,

Alex correctly used the t-method and wrote the following:

$$2\left(\frac{2t}{1+t^2}\right) - \left(\frac{1-t^2}{1+t^2}\right) = 1$$

He correctly solved this equation to obtain the answer

$$t = \frac{1}{2}$$

He continued his solution with

$$\tan\frac{x}{2} = \frac{1}{2}$$

$$\frac{x}{2} = 27^\circ$$

$$x = 54^\circ$$

Comment on Alex' solution. 2.

Question 1 continues on next page.

Question 1 (continued)

(c) A and B have coordinates (1, -4) and (-3,-2) respectively.

The point P divides the interval AB **externally** in the ratio of 5:3.

Find the coordinates of P. 3

(d) Express $\cos 3\theta$ in terms of $\cos\theta$. 3

Question 2 (12 marks) – Start a new page.

(a) If $Sin\,A = \frac{2}{3}$ and $\tan B = \frac{2}{3}$ and A and B are acute angles,

(i) Find the exact value of $\cos A$ and $\cos B$ 2

(ii) Show that the exact value of $\cos(A+B) = \frac{3\sqrt{5}-4}{3\sqrt{13}}$. 2

(b) The angle between the lines $y = mx + 2$ and $x = 2y$ is 45°. Find the exact value(s) of m . 2

Question 2 continues on next page

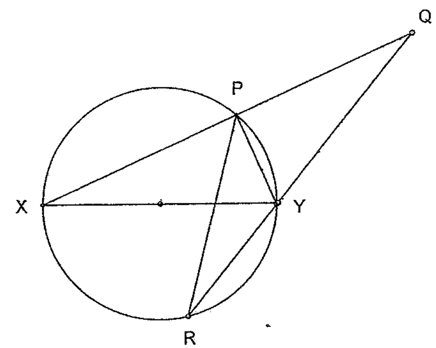
Question 2 continued.

(c) XY is the diameter of the circle XPYR. XPQ lie on a straight line as do RYQ. 3

PR, XY and PY are joined as shown in the diagram.

Copy the diagram onto your solution sheet.

Given that $\angle PXY = 35^\circ$ *and* $\angle PQY = 25^\circ$, find the size of $\angle YPR$ giving reasons for your answer.



Not to Scale

(d) Find the exact value of $\sec 22.5^\circ \operatorname{cosec} 22.5^\circ$ 3

Question 3 (11 marks) – Start a new page

(a) Find the general solution for $\cos\theta = \frac{1}{\sqrt{2}}$. 2

(b) Solve $\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x = 1$ for $0^\circ \le x \le 360^\circ$ by expressing it in the form of $R\sin(x + \alpha)$. 3

(c) Solve $4\sin^2\theta - 13\sin\theta + 3 = 0$ *for* $0^\circ \le \theta \le 360^\circ$ to the nearest degree. 3

(d) Prove that $\frac{\sin 2\theta}{\sin\theta} - \frac{\cos 2\theta}{\cos\theta} = \sec\theta$ 3

Question 4 (9 marks) – Start a new page

(a) Barbara considers this question: 3

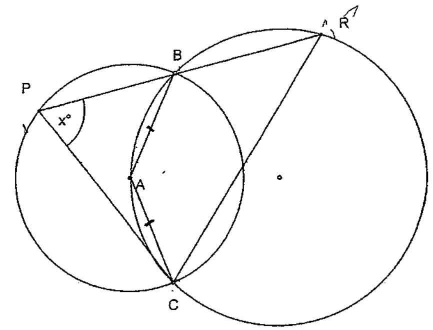
Point A is the centre of the circle BCP.

Point A lies on the circumference of circle BAC.

Circles BCP and BAC intersect at B and C as shown.

The points P, B and R are collinear.

Prove that RP = RC.



Barbara commences her proof as follows:

AIM: Prove that RP = RC

CONSTRUCTION: Join AB and AC

PROOF: let $\angle CPB = x^\circ$

Copy the first three lines of Barbara's proof into your solution book and then complete it.

Question 4 continues on next page.

Question 4 continued.

(b) The interval joining A (1,3) and B (4,9) is cut by the straight line $3x - y - 2 = 0$ at the point P. In what ratio does P divide AB? 3

(c) Solve $\sin 2x = \tan x$ for $0^\circ \leq x \leq 360^\circ$ 3

End of Examination

Note Title 19/06/2008

Preliminary Ass 2 June 2008
SOLUTIONS

1 (a) $y = -\frac{1}{2}x - \frac{1}{2}$ $\quad y = 3x+2$

$m_1 = -\frac{1}{2}$ $\quad m_2 = 3$

$\tan\theta = \left|\frac{3-(-\frac{1}{2})}{1+3(-\frac{1}{2})}\right| = \left|\frac{\frac{7}{2}}{-\frac{1}{2}}\right| = |-7| = 7$ ✓

(b) $\tan\frac{x}{2}$ is not defined when $x = 180°, 540°$, etc ✓

∴ Alex must check $x = 180°$ manually to see whether it is a solution.

Sub $x = \pi$: $2\sin 180°\cos 180° = 2(0) - (-1)$
$= 1$
$=$ RHS

∴ Alex has omitted the solution $x = 180°$ ✓

(c) Ratio $m:n = 5:-3$ ✓

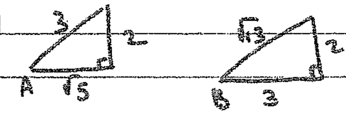
$x = \frac{5(3)-3(1)}{2}$ ✓ $\quad y = \frac{5(-2)-3(-4)}{2}$ ✓

$P = (-9, 1)$

(d) $\cos 3\theta = \cos(2\theta + \theta)$ ✓
$= \cos 2\theta\cos\theta - \sin 2\theta\sin\theta$
$= (2\cos^2\theta - 1)\cos\theta - 2\sin\theta\cos\theta\sin\theta$ ✓
$= 2\cos^3\theta - \cos\theta - 2\sin^2\theta\cos\theta$
$= 2\cos^3\theta - \cos\theta - 2(1-\cos^2\theta)\cos\theta$
$= 2\cos^3\theta - \cos\theta - 2\cos\theta + 2\cos^3\theta$
$= 4\cos^3\theta - 3\cos\theta$ ✓

Question 2

(a) (i)



$\cos A = \frac{\sqrt{5}}{3}$ ✓

$\cos B = \frac{3}{\sqrt{13}}$ ✓

(ii) $\cos(A+B) = \cos A\cos B - \sin A\sin B$ ✓
$= \frac{\sqrt{5}}{3}\cdot\frac{3}{\sqrt{13}} - \frac{2}{3}\cdot\frac{2}{\sqrt{13}}$
$= \frac{3\sqrt{5}-4}{3\sqrt{13}}$ ✓

(b) $y = mx+2$ $\quad y = x/2$
$m_1 = m$ $\quad m_2 = \frac{1}{2}$ $\quad \theta = 45$

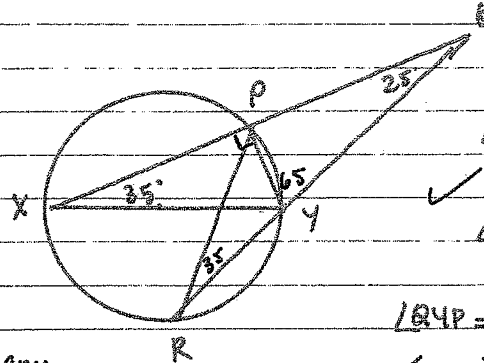
$\tan 45° = \left|\frac{m-\frac{1}{2}}{1+\frac{1}{2}m}\right| = 1$

∴ $\left|m - \frac{1}{2}\right| = \left|1 + \frac{1}{2}m\right|$

$m - \frac{1}{2} = 1 + \frac{m}{2}$
$\frac{m}{2} = \frac{3}{2}$
$m = 3$ ✓

$m - \frac{1}{2} = -(1 + \frac{1}{2}m)$
$\frac{3m}{2} = -1 + \frac{1}{2}$
$m = -\frac{1}{3}$ ✓

(c)



$\angle XPY = 90°$ (angle in a semi circle) ✓

$\angle YPQ = 90°$ (a straight angle has 180°)

$\angle QYP = 180 - (90+25)$
$= 65$ (angle sum of Δ is 180°) ✓

1 mark for any 2 correctly reasoned deductions

$\angle PRY = 35°$ (angles on the same arc in the same segment are equal)

∴ $\angle YPR = 65 - 35$
$= 30°$ ✓ (exterior angle of Δ = sum of two interior opposite angles)

(d) $\sec 22.5° \operatorname{cosec} 22.5° = \frac{1}{\cos 22.5} \cdot \frac{1}{\sin 22.5}$ ✓

$= \frac{1}{\sin(2 \times 22.5)}$ ✓ $= \frac{1}{\sin 45} = \frac{1}{1/\sqrt{2}}$ ✓

$= \sqrt{2}$

Question 3

(a) $\theta = 360°n \pm 45°$ ✓ ✓

(b) $R\sin(x+\alpha) = R\sin x\cos\alpha + R\cos x\sin\alpha$
$= \frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x$

∴ $R\cos\alpha = \frac{1}{2}$ $\quad R\sin\alpha = \frac{\sqrt{3}}{2}$

$R^2 = \frac{1}{4} + \frac{3}{4} = 1$ $\quad \tan\alpha = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$

$R = 1$ ✓ $\quad \alpha = 60°$ ✓

∴ $\sin(x+60) = 1$
$x + 60 = 90°$ for $0 \le x \le 360°$
∴ $x = 30°$ ✓

(c) $4\sin^2\theta - 13\sin\theta + 3 = 0$ $\quad 0° \le \theta \le 360°$
let $\sin\theta = A$
$4A^2 - 13A + 3 = 0$
$(4A - 1)(A - 3) = 0$ ✓
$A = \frac{1}{4}$ or 3

$\sin\theta = \frac{1}{4}$ or $\sin\theta = 3$ ✓
No Solution

$\theta = 14°, 166°$ ✓

(d) $LHS = \frac{2\sin\theta\cos\theta}{\sin\theta} - \frac{(2\cos^2\theta - 1)}{\cos\theta}$ ✓
$= 2\cos\theta - 2\cos\theta + \frac{1}{\cos\theta}$ ✓
$= \sec\theta$ ✓ $= RHS$

Question 4

Aim: Prove that $RP = RC$

Construction: Join AB and AC

Proof: Let $\angle CPB = x$

✓ $\angle BAC = 2x$ (angle at the centre is double the angle at the circumference on the same arc)

✓ $\angle BRC = (180 - 2x)$ (opposite angles in cyclic quad ABRC are supplementary)

✓ $\therefore \angle RCP = 180 - (x + 180 - 2x)$ (angle sum of Δ is $180°$)

$= x$

$= \angle CPR$

$\therefore \Delta PRC$ is isosceles

$\therefore RP = RC$ (sides opposite equal angles are equal)

(b) Slope of line joining A to B $= \frac{6}{3} = 2$

$\therefore$ Eqn of AB $\Rightarrow y - 3 = 2(x-1)$

$y = 2x + 1$ ✓

To find point of intersection solve simultaneously

$y = 2x + 1$ and $3x - y - 2 = 0$

$3x - (2x+1) - 2 = 0$

$3x - 2x - 1 - 2 = 0$

$x = 3$

$y = 7$

$P(3,7)$ ✓

$\therefore P(3,7)$ divides line joining $A(1,3)$ and $B(4,9)$ in the ratio of $m:n$

ie $\frac{m(4)+n(1)}{m+n} = 3$ and $\frac{m(9)+n(3)}{m+n} = 7$

$4m + n = 3m + 3n$

$m = 2n \Rightarrow \frac{m}{n} = 2$

$\therefore m:n = \frac{m}{n} = \frac{2}{1}$

✓ $\therefore$ P divides AB in the ratio of $2:1$

(c) $\sin 2x = \tan x$

$2\sin x \cos x = \frac{\sin x}{\cos x}$ ✓

$2\sin x \cos^2 x = \sin x$

$2\sin x \cos^2 x - \sin x = 0$ ✓

$\sin x(2\cos^2 x - 1) = 0$

$\sin x = 0$ $\quad$ $2\cos^2 x - 1 = 0$

$x = 0°, 180°, 360°$ $\quad$ $\cos^2 x = \frac{1}{2}$

$\cos x = \pm\frac{1}{\sqrt{2}}$ ✓

$= 45°, 135°, 225°, 315°$