SCEGGS Darlinghurst

Student Number: .....

2010
Preliminary Course
Common Test 3

Mathematics Extension 1

Task Weighting: 20%

Outcomes Assessed: PE2, PE3 & PE6

General Instructions

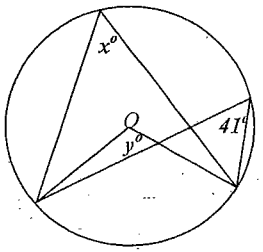
- Time allowed – 50 minutes
- **Start each question on a new page.**
- Attempt **all** questions and show all necessary working.
- Write your student number at the top of each page.
- Marks can be deducted for careless or badly arranged work
- Mathematical templates, geometrical equipment and scientific calculators may be used.

Question	Reasoning	Communication	Total
1			/13
2			/11
3			/11
Total			/35

Marks

Question 1 (13 Marks)

a) 3



Not To Scale

O is the centre of a circle

Find x and y giving reasons

b) If $6x^2 - 11 \equiv A(x+2)^2 + Bx + C$, find the values of A, B and C. 3

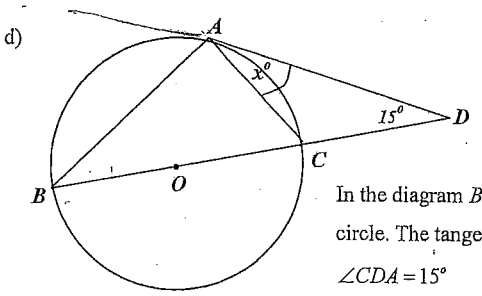
c) NSW numberplates for motor vehicles consist of 2 letters, followed by 2 numbers, followed by a further 2 letters.

e.g. **BC 01 RC**

i) How many different number plates can be made? 2

ii) Scott purchases a new car with NSW number plates as described above. What is the probability that his initials SL appear together, in that order, in the number plates? 2

d) 3



In the diagram BC is a diameter of the circle. The tangent at A meets BC at D

$\angle CDA = 15^o$

Find the value of x giving reasons.

Start A New Page **Marks**

Question 2 (11 Marks)

a) Find the value of k in the equation:

$$(k-1)x^2+(2k+1)x-2=0$$

so that it has:

i) one root the reciprocal of the other **2**

ii) equal roots **2**

b) In the final of Masterchef there were 13 men and 11 women. For a team challenge Gary the judge wants to randomly choose a group of 5 contestants for a team.

i) How many different teams of 5 are possible? **1**

ii) How many different teams of 5 could Gary choose if there were more women than men in the team. **3**

c) At a dinner party, Chris and Laurie have 10 guests. They are seated around a circular table. **3**
How many seating arrangements are there if Chris and Laurie do not sit together?

Start A New Page **Marks**

Question 3 (11 Marks)

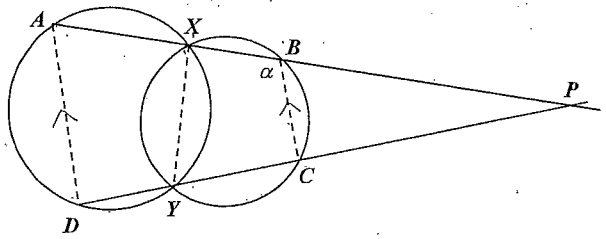
a) Find the values of b if the line $7x-y-3=0$ is a tangent to the parabola $y=2x^2+bx-1$. **3**

b) Let each different arrangement of all the letters of QUOLANGATTA be called a word.

i) How many words are possible? **1**

ii) If one of these words is chosen at random, what is the probability that all the vowels are together? **2**

c) Two circles intersect at the points X and Y. The straight lines AXB and DYC intersect at P, as shown:
let $\angle CBX=\alpha$



i) Prove that $AD \parallel BC$ **3**

ii) If $PB=5$, $BX=3$ and $PC=4$ find YC. **2**

END OF THE PAPER

Year 11 Extension 1 Mathematics Ass Tst 3 2010 – Solutions

Q1 a) $x = 41°$ (angles on the same arc are equal)
$y = 82°$ (angle at the centre is twice the angle at the circumference lying on the same arc.)

Comm 3 ✓ for x and y ✓✓ for both reasons

Some confusion with students. Look carefully – clearly identify angles.

b) $6x^2 - 11 \equiv A(x+2)^2 + Bx + C$
$\equiv Ax^2 + 4Ax + 4A + Bx + C$
$\equiv Ax^2 + (4A+B)x + 4A + C$ ✓

$\therefore A = 6$

$4A + B = 0$ $\quad 4A + C = -11$

$4 \times 6 + B = 0$ $\quad 4 \times 6 + C = -11$

$B = -24$ $\quad C = -35$

$\therefore A = 6$ ✓ $B = -24$ $C = -35$ ✓

well done – a few students with algebraic errors.

c) i) No of different numberplates $= (26)^4 \times 10^2$ ✓
$= 45\,697\,600$ ✓

well done

ii) No of numberplates with SL $= (26)^2 \times 10^2 \times 2$ ✓
But this will lead to double counting of S L N N S L
$1 \times 1 \times 10 \times 10 \times 1 \times 1$

Reas 4 $\therefore P(SL) = \dfrac{26^2 \times 10^2 \times 2 - 10^2}{(26)^4 \times 10^2}$

$= 0.00295\ldots = 0.003$ (3 dp)

Very tricky to handle the adjustment for double counting.

d) $\angle ABC = x$ (angle at the tangent equals angle in alternate segment) ✓

$\angle BAC = 90°$ (angle in the semi-circle) ✓

Comm 3 $\therefore x + x + 90° + 15° = 180°$ (angle sum of a triangle)

$2x + 105 = 180°$

$2x = 75°$

$x = 37\frac{1}{2}°$ ✓

Some students failed to clearly state the reasons.

Q2 a) i) $\alpha, \frac{1}{\alpha}$

$\alpha \cdot \frac{1}{\alpha} = \frac{c}{a}$

$1 = \frac{-2}{k-1}$ ✓

$k - 1 = -2$

$k = -1$ ✓

Students must realise that if the roots are reciprocal, their product = 1.

ii) $\Delta = 0$ when roots are equal

$\therefore (2k+1)^2 - 4 \times (k-1) \times -2 = 0$ ✓

$4k^2 + 4k + 1 + 8k - 8 = 0$

$4k^2 + 12k - 7 = 0$

$(2k-1)(2k+7) = 0$

$k = \frac{1}{2}$ $\quad k = -\frac{7}{2}$ ✓

The rules must be well known – solutions that did not use the Δ were much more time consuming.

b) i) ${}^{24}C_5 = 42504$ ✓

ii) 3 women 2 men $= {}^{11}C_3 \times {}^{13}C_2 = 12\,870$ ✓

4 women 1 man $= {}^{11}C_4 \times {}^{13}C_1 = 4290$ ✓

5 women 0 men $= {}^{11}C_5 \times {}^{13}C_0 = 462$

Reas 4 17622 ✓

Some students forgot this option.

c) 12 people

No of arrangements $= 11! - 10! \times 2$ ✓ ✓

$= 32\,659\,200$ ✓

Reas 3 (or $1 \times 9 \times 10!$)

Well done

Q3 a) Find the point of intersection:

$7x - y - 3 = 0$ ..... ①

$y = 2x^2 + 5x - 1$ .... ②

$y = 7x - 3$ .... ③

sub ② & ③

$2x^2 + 5x - 1 = 7x - 3$ ✓

$2x^2 + (5-7)x + 2 = 0$

∴ for a tangent → one point of intersection

∴ $\Delta = 0$ ✓

$(5-7)^2 - 4 \times 2 \times 2 = 0$

$5^2 - 14b + 49 - 16 = 0$

$5^2 - 14b + 33 = 0$

Reas 3 $(5-11)(5-3) = 0$

$5 = 3 \quad 5 = 11$ ✓

b) i) $\frac{11!}{2!3!2!} = 1\,663\,200$ ✓

ii) $\frac{7!}{2!} \times \frac{5!}{2!3!}$ ✓

$= 25\,200$

$P(\text{all vowels}) = \frac{25200}{1665200}$

Reas 3 $= \frac{1}{66}$ ✓

Remember to find the probability!

c) $\angle CBX = \alpha$

$\angle XYC = 180 - \alpha$ (opp. angles in a cyclic quad. are supplementary) ✓

$\angle DAY = 180 - \alpha$ (ext. angle to cyclic quad equals opp. interior angle) ✓

$\therefore \angle CBX + \angle DAX = 180°$

Reas 3 ∴ BC // DA as cointerior angles are supplementary ✓

ii) $PB \times PX = PC \times PY$ (The product of the intercepts of 2 intersecting chords are equal) ✓

ie $5 \times 8 = 4(4+x)$

$40 = 16 + 4x$

$24 = 4x$

$x = 6$

$\therefore CY = 6$ ✓

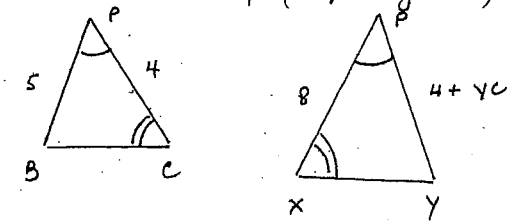
a) Alternative method:

In the triangles PBC and PXY,

$\angle P$ is common.

$\angle PCB = \angle PXY$ (exterior angle of a cyclic quadrilateral equals the interior opposite angle)

$\therefore \Delta PBC \,|||\, \Delta PXY$ (equiangular)



Be very careful in choosing the proportional sides.

$\therefore \frac{4+YC}{5} = \frac{8}{4}$ ratios of sides of similar triangles are equal

$\therefore 5 \times 8 = 4(4+YC)$

Then as above.

b) Alternative method.

Construct a tangent PT to circle XBCY

then $PT^2 = PX \times PB$

and $PT^2 = PY \times PC$

$\therefore PX \times XB = PY \times PC$

Then as above.