SCEGGS Darlinghurst

Name: .....

Teacher: .....

Preliminary Assessment Task 3
Wednesday, 1st June 2005

Mathematics

Task Weighting 15%

General Instructions

- Time allowed – 50 minutes
- Write using black or blue pen
- Write your name and your teacher's name at the top of each page
- Attempt **all** questions
- Show all necessary working
- Marks will be deducted for careless or badly arranged work
- Approved calculators should be used
- Mathematical templates, geometrical equipment and scientific calculators may be used
- Begin each question on a new page.

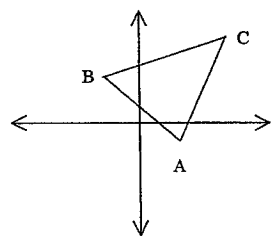
Question	Com.	Reas.	Marks Awarded
1	/2	/4	/12
2	/6	/2	/12
3		/2	/8
Total	**/8**	**/8**	**/32**

Question 1 (12 marks)

- Answer on the pad paper provided
- Write **your name** and **your teacher's name** at the start of each question
- Start each question on a new page
- Clearly label each question

Marks

(a) The three points $A(2,-1)$, $B(-2,3)$ and $C(4,5)$ form a triangle.



NOT TO SCALE

(i) Find the coordinates of P, the midpoint of BC. **1**

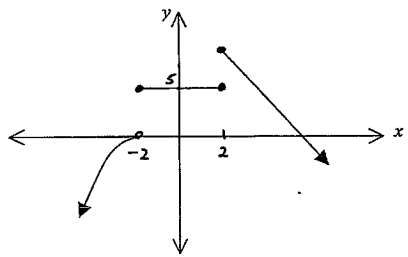
(ii) Show that Q, the midpoint of AB lies on the y axis. **1**

(iii) Find the equation of PQ. **2**

(iv) Show that $PQ \parallel AC$ **2**

(v) Show that AC is twice the length of PQ. **2**

(b) Consider the following graph: **2**



Does this graph represent a function? Explain your answer.

(c) State the domain and range of $y = \sqrt{16-x^2}$. **2**

Question 2 (12 marks)

- **START A NEW PAGE**

Marks

(a) (i) Given $f(x) = x^3 - x$, find $f(2)$ **1**

(ii) Show that $f(x) = x^3 - x$ is an odd function. **2**

(b) Sketch the following graphs, showing all important features.
Use at least *one third* of a page for each sketch.

(i) $y = |2x - 1|$ **2**

(ii) $y = 5^{-x} + 1$ **2**

(iii) $y = \dfrac{-3}{x - 6}$ **2**

(c) Shade the region defined by $y \leq -x^2 + 4x - 3$ and $y > x - 3$ **3**

Question 3 begins on the next page ...

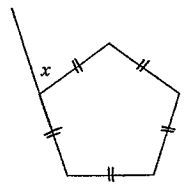
Question 3 (8 marks)

- **START A NEW PAGE**

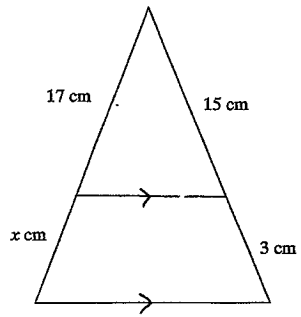
Marks

(a) Find the exact value of x, giving reasons. **2**

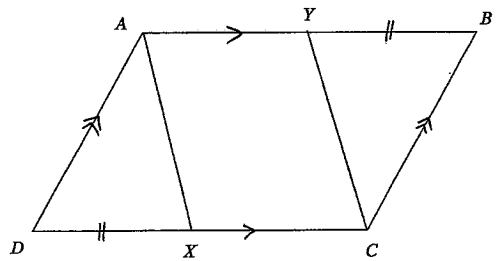
(i)



(ii)



(b) $ABCD$ is a parallelogram and $BY = DX$.



(i) Explain why $\angle ADX = \angle CBY$. **1**

(ii) Show that $\Delta ADX \equiv \Delta CBY$. **3**

(iii) Hence prove that $AX \parallel YC$. **2**

End of Assessment

Year 11 : Mathematics

Assessment Task 3

SOLUTIONS

QUESTION 1 : (12 marks) Com ½ Reas ¼

(a) (i) $P(1,4)$ ✓

(ii) $Q(0,1)$ which lies on the y axis ✓

(iii) $m_{PQ} = \frac{4-1}{1-0} = 3$ ✓

$y - y_1 = m(x - x_1)$

$y - 4 = 3(x-1)$

$y = 3x + 1$ ✓

(iv) $m_{AC} = \frac{5--1}{4-2} = 3$ ✓

∴ Since $m_{AC} = m_{PQ}$,

$AC \parallel PQ$. ✓ (Reas ½)

(v) $AC = \sqrt{(4-2)^2 + (5--1)^2} = \sqrt{40} = 2\sqrt{10}$ ✓

$PQ = \sqrt{(1-0)^2 + (4-1)^2} = \sqrt{10}$ ✓

∴ $AC = 2 \times PQ$. (Reas ½)

(b) No, it is not a function.

At $x = 2$, there are Two function values (y values). ✓✓ (Com)

(c) $y = \sqrt{16 - x^2}$

semicircle, radius 4, centre $(0,0)$

Domain : $-4 \le x \le 4$ ✓

Range : $0 \le y \le 4$ ✓

QUESTION 2: (12 marks) Com 1/6 Reas ½

(a) (i) $f(x) = x^3 - x$

$f(2) = 8 - 2 = 6$ ✓

(ii) $f(-x) = (-x)^3 - (-x)$

$= -x^3 + x$ ✓

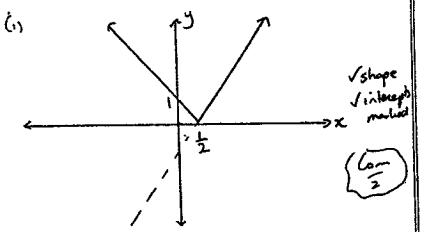
$-f(-x) = x^3 - x$

$= f(x)$

∴ Since $f(x) = -f(-x)$, ✓ (Reas ½)

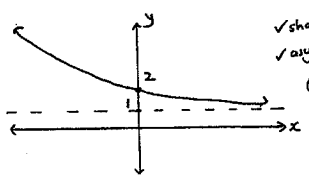
$f(x)$ is odd.

(b) (i)



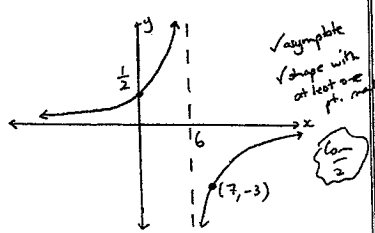
✓shape ✓intercepts marked (Com ½)

(ii)



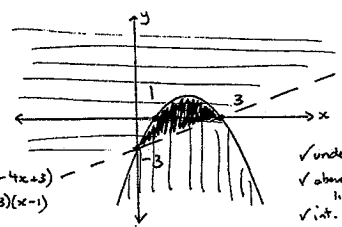
✓shape ✓asymptote (Com ½)

(iii)



✓asymptote ✓shape with at least one pt. marked. (Com ½)

(c)



$y \le -(x^2 - 4x + 3)$ ✓

$y \le -(x-3)(x-1)$

✓under para. ✓above dotted line ✓int. indicated

QUESTION 3: (8 marks) Reas ½

(a) (i) Ext. angle sum $= 360°$

$\therefore x = \frac{360}{5} = 72°$ ✓

(ii) $\frac{17}{x} = \frac{15}{3}$ ✓

$\therefore x = \frac{17}{5} = 3\frac{3}{5}$ (ratio of intercepts on parallel lines is equal).

(b) (i) $\angle ADX = \angle CBY$ because opposite angles in a parallelogram are equal. ✓

(ii) In ΔADX and ΔCBY,

$\angle ADX = \angle CBY$ (proven above) ✓

$DX = BY$ (given)

$AD = BC$ (opp. sides in a parallelogram are equal) ✓

$\therefore \Delta ADX \equiv \Delta CBY$ (SAS) ✓

(iii) $\angle BYC = \angle AXD$ (corresponding angles in $\equiv \Delta$ are $=$).

$\angle BYC = \angle YCX$ (alt. $\angle$ s are $=$ as $AB \parallel CD$) ✓

$\therefore \angle AXD = \angle YCX$

∴ Since corresponding angles are equal, $AX \parallel YC$. ✓ (Reas ½)