SCEGGS Darlinghurst

2007

Preliminary Course

Assessment Task 3
Wednesday 6 June 2007

Mathematics

Task Weighting: 15%
Outcomes Assessed: P2, P3 and P4

General Instructions

- Time allowed – 55 minutes
- You are allowed to bring in one A4 handwritten page of notes
- Write your Student Number at the top of each page
- Begin each question on a new page
- Attempt all questions and show all necessary working
- Answer all questions on the pad paper provided
- Marks will be deducted for careless or badly arranged work
- Mathematical templates, geometrical equipment and scientific calculators may be used

Total marks – 36

- Attempt Questions 1 – 3

Question	Reasoning	Communication	Marks
1	/2	/7	/12
2	/3	/2	/12
3	/5	/1	/12
TOTAL	/10	/10	/36

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Question 1 (12 marks)

(a) Given $f(x) = 2 - x^2$

   (i) $f(3)$ **1**

   (ii) $f(1) + f(4)$ **1**

(b) Find the exact value of $\operatorname{cosec} 225°$.   **1**

(c) Consider the following graph.   **2**
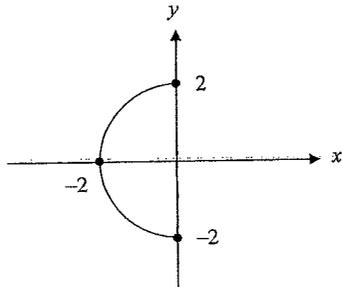
   Does it represent a function? Explain your answer.

(d) Given that $\sin\theta = \dfrac{3}{8}$ and $\tan\theta < 0$, find the exact value of $\cos\theta$.   **2**

(e) Sketch the curve $y = 3\sin 2x$ for $0° \le x \le 360°$.   **2**

(f) (i) Explain what is meant by an asymptote.   **1**

   (ii) Sketch the curve $y = \dfrac{2}{x-1}$ showing all important features.   **2**

- Start a new page

Question 2 (12 marks)

(a) Show that the function $f(x) = x^2 - 1$ is even.   **2**

(b) (i) Sketch the curve $y = \sqrt{16 - x^2}$ **1**

   (ii) State the domain and range of the function.   **2**

(c) Solve for $0° \le x \le 360°$ **2**

$$2\sin x = -1$$

(d)
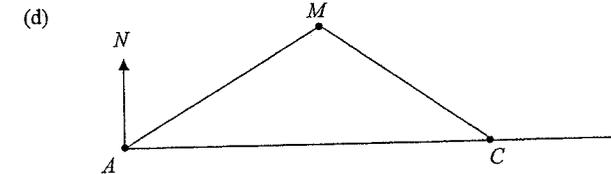
Adriana and Charlotte are bushwalking in the Royal National Park. Charlotte (C) is due east of Adriana (A).

From their positions they see a mountain (M). M is 38 km from A and 20 km from C.

The bearing of the mountain (M) from Charlotte (C) is 325°.

   (i) Copy the diagram, show all the given information.   **1**

   (ii) Find the size of $\angle ACM$.   **1**

   (iii) What is the bearing of the mountain (M) from Adriana (A)? (Answer to the nearest degree.)   **3**

Marks

Question 3 (12 Marks)

(a) The curve $y = \sin x$ is shown for $0° \le x \le 360°$.
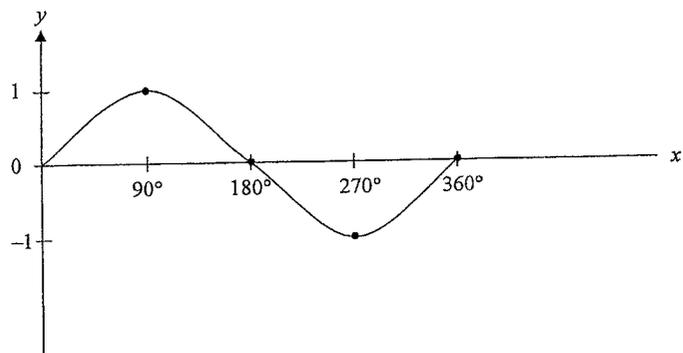
(i) Copy the diagram and on the same set of axes, draw the graph of $y = \cos x$.

1

(ii) By solving the equation $\sin x = \cos x$, find the exact values of x where the two curves intersect.

2

(b) A function is defined by $f(x) = |\, 2x + 3\,|$.
Find the values of x for which $f(x) = 10$.

2

(c) A function is given by:

$$f(x) = \begin{cases} x^2 & x \le -1 \\ x + 2 & -1 < x < 2 \\ 4 & x \ge 2 \end{cases}$$

2

Evaluate $f(0) - 2f(3)$.

Question 3 continues on the next page

Marks

Question 3 (continued)

(d) Determine the point(s) of intersection for the functions:

$$y = 2x - 3$$
$$y = x^2 - 6x + 12$$

2

(e) Prove that:

$$\frac{\cos\theta}{1 + \sin\theta} - \frac{\cos\theta}{1 - \sin\theta} = -2\tan\theta$$

3

End of paper

question 1.   Prelim +155 ... ... | Comments

a) $f(x) = 2 - x^2$

i) $f(3) = 2 - (3)^2$
$= 2 - 9$
$= -7$ ✓

ii) $f(1) + f(4)$
$= (2-1^2) + (2-4^2)$
$= 1 + (-14)$
$= -13$ ✓

part a) was well done.
Just some silly algebraic mistakes. Be careful.

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b)
$\dfrac{1}{\sin 225°} = \dfrac{1}{\sin(180+45)°}$

$= \dfrac{1}{-\sin 45°}$

$= \dfrac{1}{-\frac{1}{\sqrt{2}}}$

✓ $= -\sqrt{2}$

$\sin \theta$ is *negative* in quadrant 3.

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c) No. Using the vertical line test, it cuts the curve more than once. It does not have one to one correspondence. ✓

(Comm 1)
This was done quite well but please make sure you use the right words.

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d)

3 8 ✓
  θ
$-\sqrt{55}$
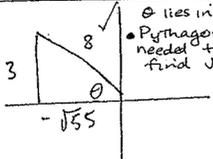
• θ lies in Quadrant.
• Pythagoras rule needed to find $\sqrt{55}$ [Reas 2]

$\cos\theta = -\dfrac{\sqrt{55}}{8}$ ✓

θ is located in Quadrant 2 so cos θ has to be negative.
Although most people used Pythagoras' rule correctly, the final sign is very important.

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i)

√ amplitude
√ period

[6m2]

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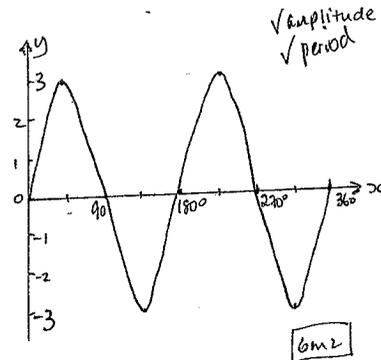
i) i) A line that a graph approaches but never touches   [6m1]

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ii)

(2,1) ✓
(0,-2)
✓
[COM 2]

Note: This value at x=1 must be very clearly seen.
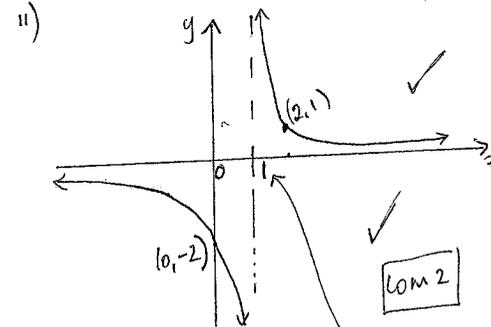
one mark correct asymptote at $x=1$

one mark y intercept clearly shown at $(0,-2)$

To save time, learn how to sketch curves like this without plotting points.

Q2

a) $f(x) = -x^2 - 1$

$f(-x) = -(-x)^2 - 1$

$= -x^2 - 1$

$f(x) = f(-x)$

∴ even function

Some careless mistakes made when setting out this Q. Remember to put the substitution inside the brackets + be careful of y° signs

b) i)

-4   4   →x

[correct]

Well done.
Remember to use a ruler and a compass.

ii) $-4 \leq x \leq 4$ ✓

$0 \leq y \leq 4$ ✓
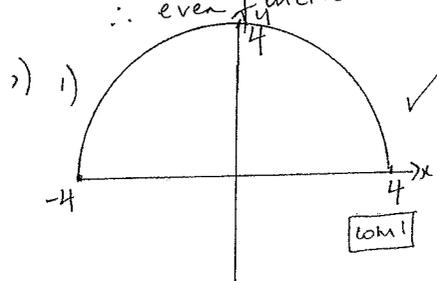
c) $2 \sin x = -1$

$\sin x = -\frac{1}{2}$

Q 3 + 4 $\theta = 30°$

$180 + \theta = 210°$ ✓

$360° - \theta = 330°$ ✓

Sin is -ve in 2 Quads, 3 + 4 ∴ 2 solutions

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1)

i)

Again, your diagrams need to be neat and easily read. Draw them big. Make sure to show the bearing.

ii) $\angle ACm = 325° - 270°$

$= 55°$
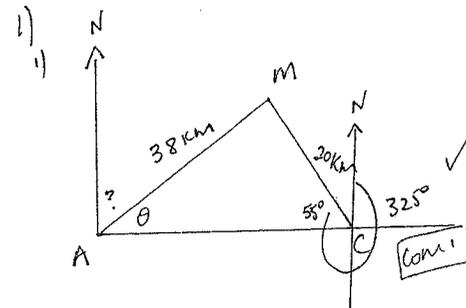
iii) $\dfrac{20}{\sin \theta} = \dfrac{38}{\sin 55°}$ ✓

$\sin \theta = \dfrac{20 \sin 55°}{38}$

$\theta = \sin^{-1}\left(\dfrac{20 \sin 55°}{38}\right)$

$= 26°$ ✓

∴ Bearing of m from A

$= 90 - 26°$

$= 064°T$ ✓ [Reas 3]

This part was poorly answered. Some students didn't even attempt this part, losing valuable marks.

Not forgot to find the bearing after finding the angle!

Remember a bearing is written $064°T$.

Q3

1)

$y = \cos x$

$y = \sin x$

Com1

) $\sin x = \cos x$ Reas 2

$\dfrac{\sin x}{\cos x} = 1$ $45°$ ✓

$\tan x = 1$ $225°$ ✓

) $f(x) = |2x+3|$

$10 = 2x+3$ $10 = -(2x+3)$

$7 = 2x$ $10 = -2x-3$

$\dfrac{7}{2} = x$ ✓ $13 = -2x$

$-\dfrac{13}{2} = x$ ✓

) $f(0) = x+2$ $2f(3)$

$= 0+2$ $2(4)$

$= 2$ ✓ $= 8$ ✓

$2 - 8 = -6$

) $2x - 3 = x^2 - 6x + 12$

$x^2 - 8x + 15 = 0$

$(x-5)(x-3) = 0$

$x = 5$ $x = 3$

$y = 2x - 3$ $y = 2x - 3$

$= 2(5) - 3$ $= 2(3) - 3$

$= 7$ $= 3$

$(5,7)$ ✓ $(3,3)$ ✓

) LHS $\dfrac{\cos\theta}{1+\sin\theta} - \dfrac{\cos\theta}{1-\sin\theta}$

$= \dfrac{\cos\theta(1-\sin\theta) - \cos\theta(1+\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}$ ✓

$= \dfrac{\cos\theta - \cos\theta\sin\theta - \cos\theta - \cos\theta\sin\theta}{\cos^2\theta}$ ✓

$= \dfrac{-2\cos\theta\sin\theta}{\cos^2\theta}$

$= -\dfrac{2\sin\theta}{\cdots\theta}$ ✓   Reas 3