Name: ________________

SCEGGS Darlinghurst

2005
HIGHER SCHOOL CERTIFICATE
Assessment Task
Tuesday, 29 March

Extension 1 Mathematics

Task Weighting: 25%

General Instructions

- Time allowed - 75 minutes
- Answer on the pad paper provided
- Write your name at the top of each page
- Start each question on a new page
- Attempt **all** questions
- Marks may be deducted for careless or badly arranged work
- Mathematical templates and geometrical equipment may be used
- Approved scientific calculators should be used
- A table of standard integrals is provided

	Com	Calc	Reas	Marks
Question 1				/12
Question 2				/10
Question 3				/10
Question 4				/10
TOTAL				/42

Question 1 (12 marks) **Marks**

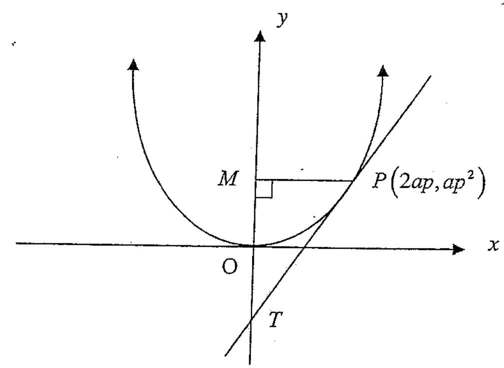
(a) The polynomial equation 2

$$P(x)=x^3-2x^2-2x$$

has a solution near $x=2.7$.

Use this value of x and Newton's Method once to find a more accurate solution, correct to 3 significant figures.

(b) The tangent to the parabola $4ay=x^2$ at the point $P(2ap, ap^2)$ meets the y axis at T. M is the foot of the perpendicular from P to the y axis.



(i) Prove that the equation of the tangent is: 2

$$px-y-ap^2=0$$

(ii) Hence prove that the origin O is the midpoint of the interval MT. 2

(c) (i) How many different arrangements are there for the letters of the word COMMITTEE? 1

(ii) If one arrangement is chosen at random, find the probability that the Es are together. 2

Question 1 continues on the next page

Question 2 (10 marks) **Marks**

(a) The polynomial $P(x) = 2x^3 + 4x^2 - 3x - 6$ has roots α, β and γ.
Evaluate:

(i) $\alpha\beta\gamma$ **1**

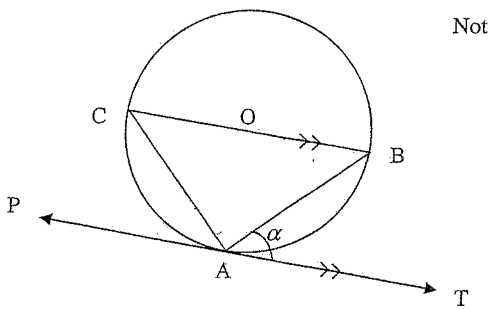
(ii) $(\alpha-1)(\beta-1)(\gamma-1)$ **3**

(b) A committee of 5 is to be formed from a group of 5 boys and 6 girls.
How many committees are possible if:

(i) there are no restrictions? **1**

(ii) there must be a majority of girls? **2**

(c) **3**

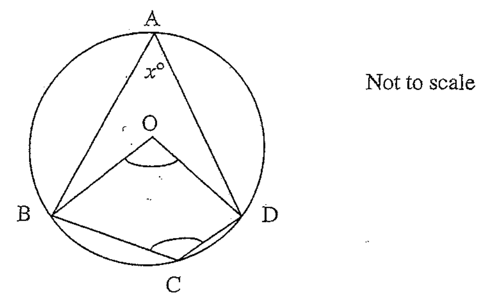


A, B and C are points on the circle centre O.
PT is a tangent to the circle at A.
Diameter BC is parallel to PT.
$\angle BAT = \alpha$.

Find the value of α giving clear reasons.

Question 1 (continued) **Marks**

(d) **3**

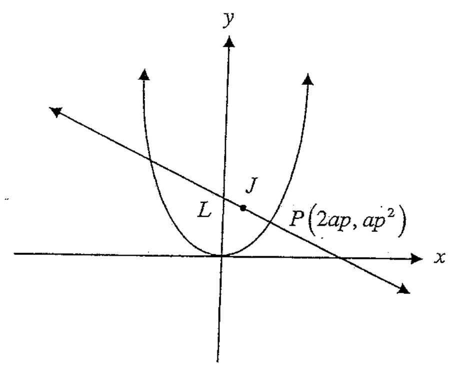


O is the centre of the circle $\angle BOD = \angle BCD$.

If $\angle BAD = x°$, evaluate $x°$ giving clear reasons.

Question 3 (10 marks) **Marks**

(a)



PL is the normal to the parabola $4ay = x^2$ at the point $P(2ap, ap^2)$.
The equation of PL is $x + py = 2ap + ap^3$.
L is on the y axis.

(i) Find the co-ordinates of L. **1**

(ii) Find the co-ordinates of J, the midpoint of PL. **1**

(iii) Prove that the locus of J is a parabola and find its vertex. **3**

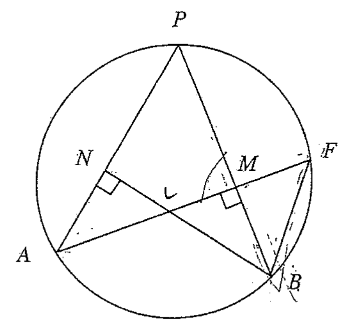
(b) (i) Prove that the coefficient of x^3 in the expansion of $\left(x + \frac{1}{x}\right)^7$ is 21. **2**

(ii) A student was asked to find the coefficient of x^3 in the expansion of $(2x^2+3)\left(x+\frac{1}{x}\right)^7$. **3**
Her incorrect answer was 63.
Why did she think this was the answer?
Without further calculation, explain the method she should have used.

Question 4 (10 marks) **Marks**

(a) At Virginia's birthday party, her 9 guests are to be seated with her around a circular table. How many arrangements are possible if Virginia must sit next to her special friend Annabel? **2**

(b)



Not to scale

Given $BN \perp AP$ and $AF \perp PB$ at point M, prove that:

(i) $MNAB$ is a cyclic quadrilateral. **1**

(ii) PB bisects $\angle NBF$. **2**

(c) The polynomial $P(x) = (x-2)(x+3)Q(x) + Ax - 1$ gives a remainder of 8 when divided by $x+3$.

Find the remainder when $P(x)$ is divided by $(x-2)(x+3)$. **2**

(d) Use the expansion of $(2+x)^n$ to prove that **3**

$$2^n = \binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \cdots + \binom{n}{r} + \cdots \binom{n}{n}$$

End of Assessment

Extension 1 Assessment 1 2005.

① a) $P(x) = x^3 - 2x^2 - 2x$ 12

$P'(x) = 3x^2 - 4x - 2$

$P(2.7) = (2.7)^3 - 2(2.7)^2 - 2(2.7)$

$= -0.297$

$P'(2.7) = 3(2.7)^2 - 4(2.7) - 2$

$= 9.07$

2nd approximation $= 2.7 + \frac{0.297}{9.07}$

$= 2.73$ (3 significant figures). ✓✓

b) (i) $y = \frac{x^2}{4a}$

$\frac{dy}{dx} = \frac{2x}{4a} = \frac{x}{2a}$

at P, $m = \frac{2ap}{2a} = p$. ✓

tangent: $y - ap^2 = p(x - 2ap)$

$= px - 2ap^2$.

$\therefore$ $px - y - ap^2 = 0$ is tangent at P. ✓

(ii) if $x = 0$, $-y = ap^2$

$\therefore$ $y = -ap^2$

$\therefore$ T is $(0, -ap^2)$ ✓

But M is $(0, ap^2)$ ✓

$\therefore$ O is the midpoint of MT.

c) (i) no. of arrangements $= \frac{9!}{2! \times 2! \times 2!} = 45360$. ✓

(ii) no. with Es together $= \frac{8!}{2! \, 2!} = 10080$. ✓

Probability $= \frac{10080}{45360} = \frac{2}{9}$ ✓

d) $\angle BOD = \angle BCD$ (given)

$\angle BOD = 2\angle BAD = 2x^\circ$ (angle at centre is twice angle at circumference if subtended by same arc) ✓

$\angle BAD + \angle BCD = 180^\circ$ (opp. angles of a cyclic quad. are supp.) ✓

But $\angle BAD + \angle BCD = x + 2x$

$= 3x^\circ$

$\therefore$ $x^\circ = 60^\circ$ ✓

② a) $P(x) = 2x^3 + 4x^2 - 3x - 6$ 10

$\alpha + \beta + \gamma = -2$, $\alpha\beta + \alpha\gamma + \beta\gamma = -\frac{3}{2}$, $\alpha\beta\gamma = 3$.

(i) $\alpha\beta\gamma = 3$. ✓

(ii) $(\alpha - 1)(\beta - 1)(\gamma - 1) = (\alpha\beta - \alpha - \beta + 1)(\gamma - 1)$

$= \alpha\beta\gamma - \alpha\gamma - \beta\gamma + \gamma - \alpha\beta + \alpha + \beta - 1$

$= \alpha\beta\gamma - (\alpha\beta + \alpha\gamma + \beta\gamma) + (\alpha + \beta + \gamma) - 1$ ✓

$= 3 + \frac{3}{2} - 2 - 1$

$= 1\frac{1}{2}$.

b) (i) no. of committees $= \binom{11}{5} = 462$. ✓

(ii) 3 girls 2 boys $= \binom{6}{3}\binom{5}{2} = 20 \times 10 = 200$

concept ✓

4 girls 1 boy $= \binom{6}{4}\binom{5}{1} = 15 \times 5 = 75$

5 girls $= \binom{6}{5} = 6$

Total no. is 281. ✓

c) $\angle CAB = 90^\circ$ (angle in a semicircle is 90°) ✓

$\angle BAT = \angle ABC = \alpha$ (alt $\angle$'s equal, BC || PT) ✓

$\angle BAT = \angle ACB = \alpha$ (angle between tangent and chord equals angle in alternate segment) ✓

$\angle CAB + \angle ABC + \angle ACB = 180^\circ$ (angle sum of $\triangle ACB$ is 180°)

$\therefore$ $90 + 2\alpha = 180^\circ$

$\alpha = 45^\circ$.

③ a) (i) if $x=0$, $py = 2ap + ap^3$ 1/10

$y = 2a + ap^2$.

$\therefore$ L is $(0, 2a+ap^2)$ ✓

(ii) J : $\left(\frac{2ap}{2}, \frac{ap^2+2a+ap^2}{2}\right)$

$= (ap, a+ap^2)$ ✓

(iii) Let $x = ap$, $y = ap^2 + a$

$p = \frac{x}{a}$, $y = a \times \frac{x^2}{a^2} + a$

$y = \frac{x^2}{a} + a$ ✓✓

This equation is of the form of a parabola $y = ax^2+bx+c$

vertex is $(0, a)$ ✓

b) (i) $T_{k+1} = \binom{7}{k} x^{7-k} (x^{-1})^k$

$= \binom{7}{k} x^{7-2k}$ ✓

if $7-2k = 3$ (where term is in x^3)

$2k = 4$

$k = 2$.

$C_3 = \binom{7}{2} = 21$. ✓

(ii) she multiplied 3 by 21.
she should have found the coefficient of x in the expansion of $(x+\frac{1}{x})^7$, multiplied this coefficient by 2 then added 21×3

④ a) 10 altogether ✓ ✓

no. of arrangements $8! \times 2 = 80640$

b) (i) $\angle ANB = \angle AMB$ (given)

$\therefore$ MNAB is cyclic (if angles subtended by the same arc are equal the 4 points must be concyclic) ✓

(ii) $\angle FBP = \angle FAP$ (angles subtended by the same arc in the larger circle are equal) ✓

$\angle NAM = \angle NBM$ (angles subtended by the same arc in the smaller circle are equal) ✓

$\therefore \angle FBP = \angle NBM$

i.e. PB bisects $\angle NBF$

c) $P(x) = (x-2)(x+3)Q(x) + Ax - 1$

$P(-3) = 0 - 3A - 1 = 8$

$-3A = 9$

$A = -3$. ✓

$\therefore P(x) = (x-2)(x+3)Q(x) - 3x - 1$.

remainder is $-3x-1$. ✓

d) $(2+x)^n = \binom{n}{0}2^n + \binom{n}{1}2^{n-1}x + \binom{n}{2}2^{n-2}x^2 + \cdots + \binom{n}{n}x^n$. ✓

if $x=2$, $4^n = \binom{n}{0}2^n + \binom{n}{1}2^n + \binom{n}{2}2^n + \cdots + \binom{n}{n}2^n$. ✓

$2^{2n} = 2^n\left[\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \cdots + \binom{n}{n}\right]$.

$\therefore 2^{2n} \div 2^n = \binom{n}{0} + \binom{n}{1} + \cdots + \binom{n}{n}$

$2^n =$ " " " . ✓