NAME: _____

SCEGGS DARLINGHURST

Higher School Certificate

Assessment Task 1

November, 2001

Mathematics

Weighting 15%

TIME ALLOWED: 70 minutes

DIRECTIONS TO CANDIDATES:

- Attempt all questions.
- All part marks are shown on the paper
- START EACH QUESTION ON A NEW PAGE.
- Write your answers on the paper provided.
- Write your name and your teacher's name on each page.
- Approved scientific calculators should be used.
- All necessary working should be shown. Marks may be deducted for careless or badly arranged work.
- Mathematical templates and geometrical instruments may be used.

Question 1 **START A NEW PAGE** **10 Marks**

a) i) Find the sum of the first 13 terms of the geometric series 5, 10, 20..... 2

ii) Which term of the series is 640? 1

b) i) Find the derivative of $y = 2(3x-5)^5$ 1

ii) Find the equation of the tangent to the curve $y = \frac{x}{x+2}$ at the point where x = 1. 2

c) i) Write the recurring decimal 0.474747..... as an infinite geometric series. 1

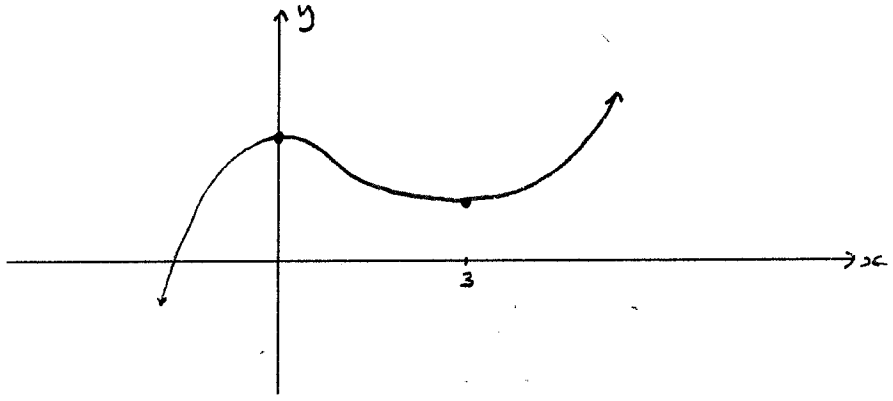
ii) Hence, express it as a fraction in its lowest terms. 1

d) A student is asked to sketch the curve of a function. She finds the stationary points, determines their nature and finds any points of inflexion. What else should she consider before sketching the curve? 2

Question 2 **START A NEW PAGE** **10 Marks**

a) A series has $T_1=3$ and $T_6=96$. Could this series arithmetic or geometric or both? Explain your reasoning. 2

b)



i) For what values of x is the curve increasing? 2

ii) What would be the highest power of x in this function and why? 1

iii) Sketch the gradient function. 1

c) A curator in an art gallery receives a large shipment of paintings. She decides to hang a painting 10 metres from the entrance and then a new painting every 2 metres. All of the paintings were delivered to the entrance and as she can only carry one painting at a time, she needs to walk back to the entrance each time before hanging the next painting.

i) Show that the total distance she walks to hang n pictures is $18n + 2n^2$ 2

ii) If she walks 3080 metres in total to hang all the pictures, how many pictures were there? 2

Question 3 **START A NEW PAGE** **10 Marks**

a) Consider the curve given by $y = x^3 - 3x + 5$

(i) Find the stationary points and determine their nature. 3

(ii) Find the point of inflexion. 2

(iii) Sketch the curve. 1

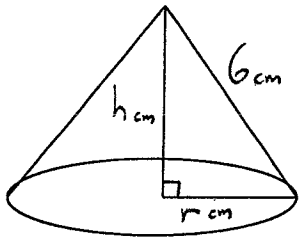
b) On the 1st of January 1977, Michelle joins a superannuation fund by investing \$3000 at 9% p.a compounded annually. A similar amount is invested at the beginning of each subsequent year until Michelle retires on 31st December 2004.

Show that the accumulated value of the investment at the date of return is \$369 406 correct to the nearest dollar.

4

Question 4 **START A NEW PAGE** **10 Marks**

a) The slant edge of a right circular cone of height h cm and base radius r cm, is 6 cm.



i) Write down an equation linking r and h 1

ii) Show that the volume of the cone is given by the equation $V = 12\pi h - \frac{1}{3}\pi h^3$ 1

iii) Find the height of the cone which gives a maximum volume. 2

b) A loan of \$10 000 was to be repaid in equal monthly instalments over 5 years with repayments commencing at the end of the first month of the loan. Interest, at the rate of 12% p.a. is calculated each month on the balance owing at the beginning of the month and added to that balance. A student started the calculation for the repayments as follows:

Let An be the amount owing after n months and M be the amount of each monthly instalment.

$A_1 = 10000(1.12)^1 - M$

$A_2 = A_1(1.12) - M$

$= (10000(1.12)^2 - M)1.12 - M$

$= 10000(1.12)^2 - 1.12M - M$

$= 10000(1.12)^2 - M(1 + 1.12)$

$A_3 = A_2(1.12) - M$

$= 10000(1.12)^3 - M(1 + 1.12 + 1.12^2)$

i) The student has made a fundamental error in the working above. Write down the correct expression for A_3. 1

ii) Calculate the correct amount of each monthly instalment. 4

iii) If you had continued with the working of the student, would the amount of the monthly instalment be larger or smaller than the amount calculated with the corrected working. Without calculations, justify your response. 1

END OF PAPER

Question 1

Year 11/12
HSC Assessment
Task #1

a) i) $a = 5$
$r = 2$

$S_n = \frac{a(r^n - 1)}{r - 1}$

$S_{13} = \frac{5(2^{13} - 1)}{2 - 1}$ ✓

$= 40955$ ✓ 2

ii) $T_n = ar^{n-1}$

$640 = 5 \times 2^{n-1}$

$128 = 2^{n-1}$

$7 = n - 1$

$\therefore n = 8$

it is the 8th term ✓ 1

b) i) $y = 2(3x - 5)^5$

$y' = 10(3x - 5)^4 \times 3$

$= 30(3x - 5)^4$ ✓ 1

ii) $y = \frac{x}{x + 2}$

$y' = \frac{(x+2) \cdot 1 - x \times 1}{(x+2)^2}$ ✓

$= \frac{2}{(x+2)^2}$

when $x = 1$,

$y = \frac{1}{3}$

$y' = \frac{2}{9}$

$y - y_1 = m(x - x_1)$

$y - \frac{1}{3} = \frac{2}{9}(x - 1)$

$y = \frac{2}{9}x - \frac{2}{9} + \frac{1}{3}$

$y = \frac{2}{9}x + \frac{1}{9}$ ✓ 2

(or general form $9y - 2x - 1 = 0$)

c) i) $0.\dot{4}\dot{7} = \frac{47}{100} + \frac{47}{10000} + \frac{47}{1000000} + \cdots$ ✓

ii) $a = \frac{47}{100}$

$r = \frac{1}{100}$

$S_\infty = \frac{a}{1 - r}$

$= \frac{\frac{47}{100}}{1 - \frac{1}{100}}$

$= \frac{47}{100} \times \frac{100}{99}$

$= 47/99$ ✓

d) She should consider

* intercepts, let $x = 0$, let $y = 0$
* asymptotes → do these exist for this curve
* domain and range of the function
* what happens as x is really large or really small.
* odd + even function
* test nature of point of inflexion to make sure it is

2

Question 2

a) This could be both arithmetic and geometric. You could find an arithmetic series.

eg
$T_n = a + (n-1)d$
$96 = 3 + (6-1)d$
$93 = 5d$
$d = 18.6$

could say, once you have shown it can be geometric, it can be arithmetic as difference does not need to be a whole number

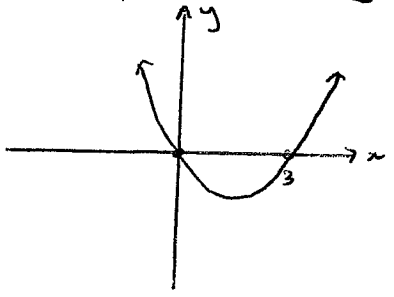
or a geometric series
$T_n = ar^{n-1}$
$96 = 3 \cdot r^{6-1}$
$\frac{96}{3} = r$
$r = \sqrt[5]{32}$
$= 2$
∴ Could be both.

2R

b) i) $x > 3, x < 0$ ✓✓

iii) cubic, from the shape of the graph ✓

iv)



✓

4

c) Distance walked
10 10 12 12 14 14 ....
This is an arithmetic series repeated twice
10, 12, 14 .....

$a = 10$
$d = 2$

Distance one way!

i) $S_n = \frac{n}{2}(2a + (n-1)d)$
$= \frac{n}{2}(2 \times 10 + (n-1) \times 2)$
$= \frac{n}{2}(20 + 2n - 2)$
$= \frac{n}{2}(18 + 2n)$ ✓

∴ Total distance
$= S_n \times 2$
$= \frac{n}{2}(18 + 2n) \times 2$ ✓
$= 18n + 2n^2$

2R

ii)
$3080 = 18n + 2n^2$
$0 = 2n^2 + 18n - 3080$
$0 = 2(n^2 + 9n - 1540)$
$0 = n^2 + 9n - 1540$
$n = \frac{-9 \pm \sqrt{9^2 - 4 \times 1 \times -1540}}{2}$ ✓
$= \frac{-9 \pm 79}{2}$
$= \frac{-9+79}{2}, \frac{-9-79}{2}$
n can't be negative
$= 35$ ✓

∴ There are 35 paintings

2

Question 3

a) $y = x^3 - 3x + 5$

$y' = 3x^2 - 3$

$y'' = 6x$

(i) Stationary points let $y' = 0$

$0 = 3x^2 - 3$

$0 = 3(x^2 - 1)$

$0 = (x-1)(x+1)$

$\therefore x = 1 \quad x = -1$

$y = 3 \quad y = 10$

$(1,3) \quad (-1,7)$

Test nature

when $x = 1$ $y'' = 6 \times 1$
$= 6 > 0 \therefore$ min ^{m}

when $x = -1$ $y'' = 6 \times -1$
$= -6 < 0 \therefore$ max ^{m}

$\therefore$ $(1,3)$ is a minimum stationary point

$(-1,7)$ is a maximum stationary point

(ii) Let $y'' = 0$

$0 = 6x$

$0 = x$

$\therefore y = 5 \quad (0,5)$

test

when $x > 0$ $y'' > 0$

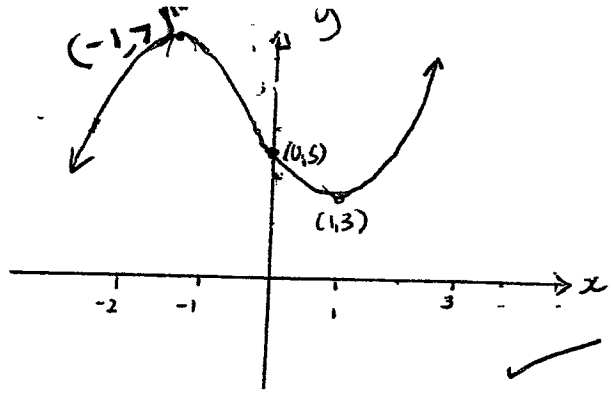
when $x < 0$ $y'' < 0$

$\therefore$ concavity changes

$(0,5)$ is a point of inflexion.

(iv) intercepts when $x = 0$ $y = 5$

no asymptotes.



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b)

the first amount invested earns interest for 28 years.

$A_{28} = \$3000(1.09)^{28}$

the second amount invested earns interest for 27 years

$A_{27} = \$3000(1.09)^{27}$

the final amount invested earns interest for 1 year.

$A_1 = \$3000(1.09)^1$

the total value of the investment is

$\$3000(1.09) + \$3000(1.09)^2 + \ldots +$

$\$3000(1.09)^{27} + \$3000(1.$

$= \$3000(1.09)(1 + 1.09 + \ldots . 1.09^{27})$

This is a geometric series with

$a = \$3000(1.09) \quad r = 1.09 \quad n = 28$

$S_n = \frac{a(r^n - 1)}{r - 1}$

$S_{28} = \frac{3000(1.09)(1.09^{28} - 1)}{1.09 - 1}$

$= \$369406$

4

Question 4

b)

i) $A_3 = 10000(1.01)^3 - M(1 + 1.01 + 1.01^2)$ ✓R

ii) $\therefore$

$A_{60} = 10000(1.01)^{60} - (M(1 + 1.01 + \ldots 1.01^{59}))$

This is a geometric series with $a = 1$ $r = 1.01$ $n = 60$

$S_n = \frac{a(r^n - 1)}{r - 1}$

$S_{60} = \frac{1(1.01^{60} - 1)}{1.01 - 1}$ ✓

$\therefore$

$A_{60} = 10000(1.01)^{60} - M \times \frac{(1.01^{60} - 1)}{0.01}$ ✓

but the loan is repaid after 60 month so $A_{60} = 0$ ✓

$0 = 10000(1.01)^{60} - M\left(\frac{1.01^{60} - 1}{0.01}\right)$

$M\left(\frac{1.01^{60} - 1}{0.01}\right) = 10000(1.01)^{60}$

$M = \frac{10000 \times 0.01 \times (1.01)^{60}}{1.01^{60} - 1}$

$= \$222.44$

$\therefore$ Monthly repayment is $222.44 ✓

(iii) The monthly repayment would be much greater as the interest rate is much higher 12% instead of 1% a month. ✓R

a) i) $r^2 + h^2 = 6^2$

$r^2 + h^2 = 36$ ✓

ii) $V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi \times (\sqrt{36 - h^2})^2 h$

$= \frac{1}{3}\pi \times (36 - h^2) \times h$

$= 12\pi h - \frac{1}{3}\pi h^3$

iii) $V = 12\pi h - \frac{1}{3}\pi h^3$

$V' = 12\pi - \pi h^2$

$V'' = -2\pi h$ ✓

Let $V' = 0$

$0 = 12\pi - \pi h^2$

$\pi h^2 = 12\pi$

$h^2 = 12$

$h = \pm\sqrt{12}$

only sensible answer is $h = 2\sqrt{3}$

Test

when $h = 2\sqrt{3}$ $V'' < 0$ ✓

$\therefore$ maximum.

The height of the cone for maximum volume is $2\sqrt{3}$ cm. ✓