



Name: \_\_\_\_\_

Teacher: \_\_\_\_\_

### Question One (10 marks)

- Answer on the pad paper provided
- Write your name and your teacher's name at the top of the page
- Start each question on a new page
- Clearly label each question

SCEGGS Darlinghurst

HSC Assessment 1  
22 November, 2002

# Mathematics

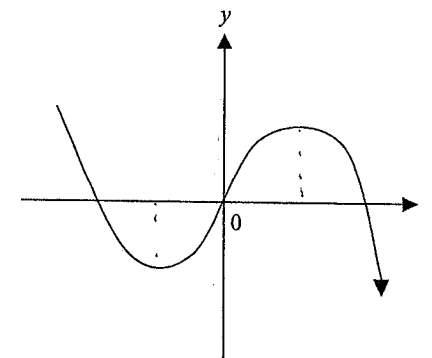
## General Instructions

- Time allowed: 60 minutes
- Weighting 15%
- This paper has four questions
- Attempt all questions and show all necessary working
- Marks will be deducted for careless or badly arranged work
- Write using blue or black pen, diagrams in pencil
- Start each question on a new page
- Write your name and your teacher's name on each page
- Approved calculators, mathematical templates and geometrical instruments may be used

Question		Marks	Communication	Reasoning	Calculus
1	Applications of Derivative	9 /10	1(d) 2 /2		1 7 /8
2	Series	7 /10		2(c) 3 /6	
3	Series	9 /10		3(c)(i) 2 /2	
4	Application of Derivative	3 /10			4 3 /10
	<b>TOTAL</b>	<b>28</b>			

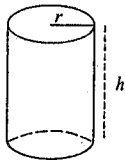
Marks

- (a) Differentiate the following functions;
- (i)  $y = 3x + \frac{4}{\sqrt{x}}$  1
- (ii)  $y = \frac{x^2 + 3}{x + 5}$  2
- (b) Find the equation of the normal to the curve  $y = (3x - 2)^2$  at  $x = 0$ . 3
- (c) For what values of  $x$  is the curve  $y = 6x^2 - x^3$  concave down? 2
- (d) Copy or trace this graph onto your answer page. Either on the same axes or directly underneath your copy, draw the graph of  $f'(x)$ . 2



### Question Two (10 marks)

- START A NEW PAGE

- |   | Marks |
|---|-------|
| (a) Evaluate:<br>$\sum_{r=1}^{20} (4 - 3r)$   | 2     |
| (b) Find the exact value of the tenth term in the series:<br>216, 108, 54, ...  | 2     |
| (c) A Pepsi can is in the shape of a closed cylinder with height $h$ cm and radius $r$ cm, as shown in the diagram.<br><div style="text-align: center; margin: 10px 0;">  </div> |       |
| (i) The volume of the can is $375\text{cm}^3$ .<br>Find an expression for $h$ in terms of $r$ .   | 1     |
| (ii) Show that the surface area of the can is given by:<br>$S = 2\pi r^2 + \frac{750}{r}$   | 1     |
| (iii) If the area of the metal used to make the can is to be minimised, find the radius of the can. (Correct to 1 decimal place.)   | 4     |

### Question Three (10 marks)

- START A NEW PAGE

- |   | Marks      |
|---|------------|
| (a) For what value(s) of $x$ does the infinite Geometric series<br>$1 + 2x + 4x^2 + 8x^3 + \dots$ have a limiting sum?  | 2          |
| (b) The weekly sales of a car retailer increase by 2 cars per week. In the first week of January, 24 vehicles are sold.<br><br>(i) How many are sold in the last week of December?<br><br>(ii) In which week is the 500th car sold?   | 1<br><br>3 |
| (c) Judi deposits \$600 on the same day each year in an investment account which pays interest at 10% per annum.<br><br>(i) If $A_n$ represents the amount in the account after $n$ years, show that<br>$A_{20} = 600(1.1^{20} + 1.1^{19} + \dots + 1.1^1).$<br><br>(ii) Find the amount in the account after 20 years. | 2<br><br>2 |

**Question Four (10 marks)**

• START A NEW PAGE

Marks

Consider the function  $y = (x-2)(x+2)^3$

The first derivative of this function is given by the expression:

$$\frac{dy}{dx} = 4(x-1)(x+2)^2$$

- |     |  |   |
|-----|--|---|
| (a) | Find the co-ordinates of any stationary points and determine their nature. | 4 |
| (b) | Find any points of inflexion.  | 3 |
| (c) | Sketch the curve, showing all important features.                          | 2 |
| (d) | For which value(s) of $x$ is the curve increasing?                         | 1 |

End of Paper

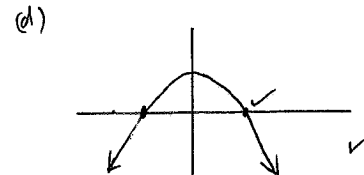
turn one

(a) (i)  $y = 3x + 4x^{-\frac{1}{2}}$   
 $y' = 3 - 2x^{-\frac{3}{2}}$   
 $= 3 - \frac{2}{\sqrt{x^3}}$  ✓

(ii)  $y = \frac{x^2+3}{x+5}$   
 $y' = \frac{(x+5)(2x) - (x^2+3)(1)}{(x+5)^2}$  ✓  
 $= \frac{x^2+10x-3}{(x+5)^2}$  ✓

(b)  $y = (3x-2)^2$   
 $y' = 2(3x-2) \cdot 3$   
 $= 6(3x-2)$  ✓  
 At  $x=0$   $y' = -12$  ✓  $\therefore$  slope of normal =  $\frac{1}{12}$   
 $\therefore$  Pt  $(0, 4)$  slope =  $\frac{1}{12}$   
 $y - 4 = \frac{1}{12}(x - 0)$   
 $y = \frac{1}{12}x + 4$  ✓

(c)  $y = 6x^2 - x^3$   
 $y' = 12x - 3x^2$   
 $y'' = 12 - 6x$  ✓  
 Concave down  $\rightarrow y'' < 0$   
 $12 - 6x < 0$   
 $x > 2$  ✓



Comments

Either index or radical form

cfpe ↷

1 mark for placing the turning points on the  $x$  axis

1 mark for showing Point of int. at  $x=0$  as max. of  $f'(x)$  (or nearby)

Question 2.

a)  $\sum_{n=1}^{20} 4-3n = 1-2-5-8-\dots-56$   
 $= \frac{20}{2} (2 + 19 \times -3)$

$= -550$

b) 216, 108, 54...

$T_{10} = 216 \times (\frac{1}{2})^9$

$= \frac{27}{64} \text{ (or } 0.421875)$

c) (i)  $\pi R^2 H = 375$

$H = \frac{375}{\pi R^2}$

(ii)  $S = 2\pi R^2 + 2\pi R H$

$= 2\pi R^2 + 2\pi R \times \frac{375}{\pi R^2}$

$= 2\pi R^2 + \frac{750}{R}$

(iii)  $\frac{dS}{dR} = 4\pi R - \frac{750}{R^2}$

if  $4\pi R - \frac{750}{R^2} = 0$

$4\pi R^3 = 750$

$R^3 = \frac{750}{4\pi}$

$R \approx 3.9 \text{ (1 d.p.)}$

Testing:

R	1	$\sqrt[3]{\frac{750}{4\pi}}$	4
S	-	0	+

a) mostly well done apart from careless errors.

1 for concept of sum  
1 for answer.

1 mark for  $T_{10}$

1 mark for either answer  
Personal preference for the fraction.

(i) Find H in terms of R  
not R in terms of H

(ii) Substitute carefully  
**LEARN FORMULAE!!**

(iii) Use the given expression for S. Keep going!  
Differentiate carefully  
Very poor algebra skill

1 mark for differentiating

2 marks for solving correctly to  $R=3.9$

1 mark for test.

$\therefore$  minimum surface area occurs when the radius is 3.9 cm (1 d.p.)

Question 3

a)  $-1 < 2x < 1$

$-\frac{1}{2} < x < \frac{1}{2}$

b) (i) 24, 26, 28...

There are 52 weeks in a year!

$T_{52} = 24 + 51 \times 2$

$= 126$  cars sold

(ii)  $S_n = \frac{n}{2} (48 + (n-1)2)$

$\therefore 500 = \frac{n}{2} (46 + 2n)$

$n^2 + 23n - 500 = 0$

$n = \frac{-23 \pm \sqrt{23^2 + 2000}}{2}$

$n \approx 13.6$  or  $-36.6$

$\therefore$  500th car sold in the 14th week

c) (i) First amount deposited becomes  $600(1.1)^{20}$

Second amount becomes  $600(1.1)^{19}$

Third amount becomes  $600(1.1)^{18}$

Final (20th) amount  $600(1.1)$

$\therefore$  Total Amount after 20 years

$A_{20} = 600(1.1^{20} + 1.1^{19} + \dots + 1.1)$

i)  $A_{20} = \frac{600(1.1)(1.1^{20}-1)}{1.1-1}$

$= 37801.50$  (nearest cent)

Answer the question properly. Note 3.9 is not the minimum radius

a) Formula not known by many.

(i) must use  $T_n$  here.

(ii) Very poor work.

- Must use  $S_n$  formula
- Careful algebra needed
- Untidy work small resulted in errors.

1 mark for equation  
1 mark for 2 solutions  
1 mark for 14th (not 13th) week.

c) try to explain more carefully.

NOTE:

$A_1 = 600(1.1)^{20}$

$A_2 = A_1 + 600(1.1)^{19}$

$A_3 = A_2 + 600(1.1)^{18}$  etc

many students lost 1 mark because they thought

$A_2 = 600(1.1)^{19}$  etc.

Question 4

(a)  $4(x-1)(x+2)^2 = 0$

$x = 1$  and  $-2$  ✓

Stationary points at  $(1, -27)$   $(-2, 0)$  ✓

$$\frac{d^2y}{dx^2} = 4 \left[ (x-1) \cdot 2(x+2) + (x+2)^2 \cdot 1 \right]$$

$$= 4(x+2)(2x-2+x+2)$$

$$= 4(x+2)(3x)$$

$$= 12x(x+2)$$

At  $x = 1$   $\frac{d^2y}{dx^2}$  is +ve  $(1, -27)$  Minimum ✓

$x = -2$   $\frac{d^2y}{dx^2}$  is 0  $(-2, 0)$  poss. pt. of inf. ✓

test on either side

$x$	$-2^-$	$-2$	$-2^+$
$\frac{d^2y}{dx^2}$	$+$	$0$	$-$

∴  $(-2, 0)$  is a confirmed point of inflexion (horizontal) ✓

(b)  $y'' = 0 \Rightarrow 12x(x+2) = 0$  ✓

Two solutions  $x = 0$  and  $x = -2$

$(-2, 0)$  is the horizontal inflexion found above ✓

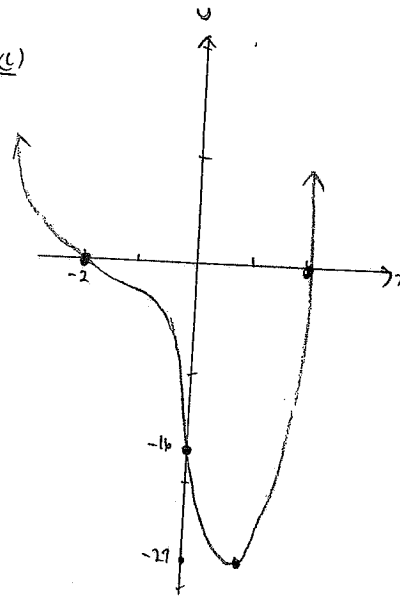
$(0, -16)$  is a poss. pt. of inflexion. ✓

test on either side

$x$	$0^-$	$0$	$0^+$
$\frac{d^2y}{dx^2}$	$-$	$0$	$+$

$(0, -16)$  is a confirmed point of inflexion (non. horizontal) ✓

Question 4(c)



1 for point of inf/sta p  
1 for "integrity" of sketch

one mark for obtaining two correct solutions for  $x$

one mark for obtaining two correct 'y' values

one mark for establishing minimum

one mark for establishing pt. of inflexion.

(d)  $x > 1$  ✓

cfpe

one mark for  $y'' = 0$

one mark for identifying point of inflexion

one mark for identifying non-horizontal inflexion