SCEGGS Darlinghurst

2010
HSC Assessment 2
1st June, 2010

Mathematics Extension 2

Outcomes Assessed: E2, E3, E4, E6, E8 and E9

General Instructions
- Time allowed – 80 minutes
- This paper has **four** questions
- Attempt **all** questions and show necessary working
- Write your Student Number at the top of each page
- Start each question on a **new page**
- Marks may be deducted for careless or badly arranged work
- Write using blue or black pen; **diagrams in pencil**
- Approved calculators, mathematical templates and geometrical instruments may be used

Question	Communication	Calculus	Reasoning	Marks
1				/12
2				/16
3				/13
4				/13
TOTAL				/54

Average: ____________ **Standard Deviation:** ____________ **Rank:** ________

Total marks – 54
Attempt Questions 1–4

Answer each question on the pad paper provided.
Write your student number at the top of each page.
Begin each question on a NEW page.

Marks

Question 1 (12 marks)

(a) Find $\displaystyle\int \frac{x}{\left(1+3x^2\right)^2}\,dx$ **2**

(b) Find $\displaystyle\int \sin^5 x\ dx$ **2**

(c) Use completing the square to find: **2**

$$\int \frac{1}{\sqrt{x^2+4x+6}}\,dx$$

(d) Use the substitution $x = 2\tan\theta$ to find: **3**

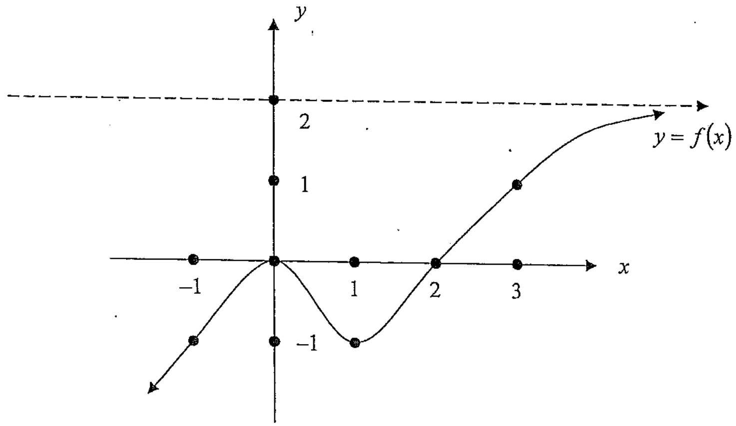
$$\int \frac{x}{\sqrt{4+x^2}}\,dx$$

(e) Find $\displaystyle\int x\tan^{-1}x\ dx$ **3**

End of Question 1

Question 2 (16 marks) Begin a NEW page. **Marks**

(a) The diagram shows the graph of $y = f(x)$.



On the answer page provided, draw separate graphs of the following.

(i) $y = f(|x|)$ **1**

(ii) $y = \dfrac{1}{f(x)}$ **2**

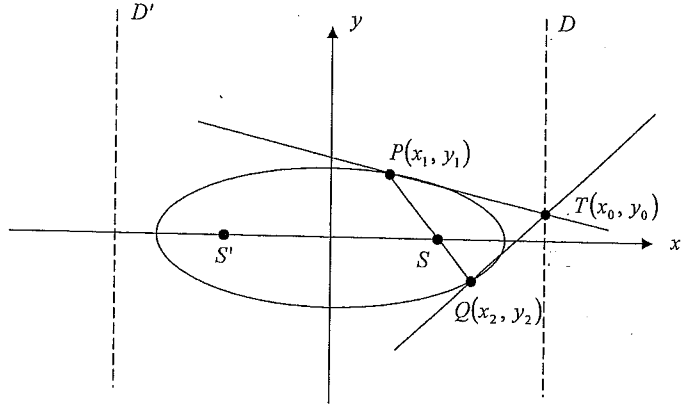
(iii) $y^2 = f(x)$ **2**

(iv) $y = e^{f(x)}$ **2**

Question 2 continues on page 4

Question 2 (continued) **Marks**

(b)



The ellipse E with equation $\dfrac{x^2}{25} + \dfrac{y^2}{16} = 1$ has directrix D as shown in the diagram.
The points $P(x_1, y_1)$ and $Q(x_2, y_2)$ lie on the ellipse E.
The external point $T(x_0, y_0)$ lies on the directrix D.
PQ is the chord of contact from $T(x_0, y_0)$.

(i) Find the eccentricity, e. **1**

(ii) Find the equation of the directrix D. **1**

(iii) Given that the equation of the tangent at P is $\dfrac{x_1 x}{25} + \dfrac{y_1 y}{16} = 1$, show that the chord of contact from $T(x_0, y_0)$ is a focal chord. **2**

(iv) Show that $PS + PS'$ is independent of the position of P on the ellipse, E. **2**

(c) Find the equation of the normal to the curve $x^4 + 3xy - 2y^2 + 9 = 0$ at the point $(-1, 2)$. **3**

End of Question 2

Question 3 (13 marks) Begin a NEW page. **Marks**

(a) Consider the curve $y = \dfrac{x}{(x+1)(2x+1)}$.

(i) Find the x values of any stationary points. **1**

(ii) Sketch the graph of $y = \dfrac{x}{(x+1)(2x+1)}$ clearly indicating any asymptotes and any points where the graph meets the axes. **3**

(iii) Given that $\dfrac{x}{(x+1)(2x+1)}$ can be written as $\dfrac{A}{x+1} + \dfrac{B}{2x+1}$, where A and B are real numbers, find the values of A and B. **2**

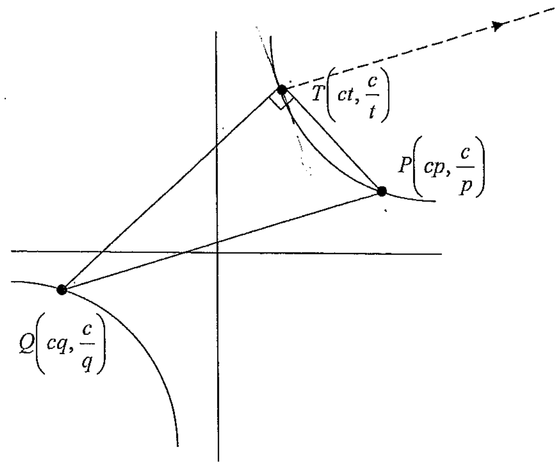
(iv) Hence find the area bounded by the curve $y = \dfrac{x}{(x+1)(2x+1)}$ and the x-axis between $x = 0$ and $x = 1$. **2**

Leave your answer in exact form.

Question 3 continues on page 6

Question 3 (continued) **Marks**

(b)



$P\left(cp, \dfrac{c}{p}\right)$ and $Q\left(cq, \dfrac{c}{q}\right)$ are two variable points on different branches of the rectangular hyperbola $xy = c^2$.

The point $T\left(ct, \dfrac{c}{t}\right)$ lies on the hyperbola such that $\angle QTP$ is a right angle.

(i) Show that the gradient of PT is given by $\dfrac{-1}{tp}$ **1**

(ii) Deduce that $t^2 = \dfrac{-1}{pq}$. **2**

(iii) Hence prove that PQ is parallel to the normal at T. **2**

End of Question 3

Question 4 (13 marks) Begin a NEW page. **Marks**

(a) (i) If $I_n = \int_0^1 (1+x^2)^n \, dx$, for $n = 0, 1, 2, \ldots$ **3**

show that

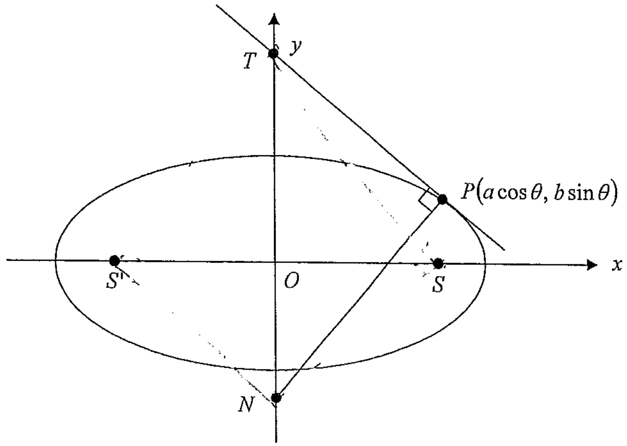
$$I_n = \frac{1}{2n+1}\left(2^n + 2n\, I_{n-1}\right) \quad \text{for } n = 1, 2, 3, \ldots$$

(ii) Hence evaluate I_3 **2**

Question 4 continues on page 8

Question 4 (continued) **Marks**

(b)



The ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ has foci $S(ae, 0)$ and $S'(-ae, 0)$ where e is the eccentricity.
The point $P(a\cos\theta, b\sin\theta)$ is on the ellipse.
The tangent and normal at P meet the y-axis at T and N respectively.

(i) Given that the equations of the tangent and normal at P are **2**

$$\frac{x\cos\theta}{a} + \frac{y\sin\theta}{b} = 1 \quad \text{and} \quad \frac{ax}{\cos\theta} - \frac{by}{\sin\theta} = a^2 - b^2 \text{ respectively,}$$

find the co-ordinates of T and N.

(ii) Show that TS is perpendicular to NS and hence deduce that TS' is perpendicular to NS'. **3**

(iii) Hence explain why T, P, S, N and S' are concyclic. **3**

End of Paper

Maths Ext② Task #2 June 2010.

Q1 cont'

Question 1

Calc /12

a) $\int \frac{x}{(1+3x^2)^2}\,dx$

$= \frac{1}{6}\int 6x(1+3x^2)^{-2}\,dx$ ✓

$= \frac{1}{6} \times \frac{(1+3x^2)^{-1}}{-1} + C$ ✓

$= \frac{-1}{6(1+3x^2)} + C$

(RCR) $\int f'(x) f(x)^n\,dx = \frac{(f(x))^{n+1}}{n+1} + C$

You can also do this by using the substitution $u = 1+3x^2$

b) $\int \sin^5 x\,dx$

$= \int \sin x\,(\sin^2 x)^2\,dx$

$= \int \sin x\,(1-\cos^2 x)^2\,dx$

$= \int \sin x\,(1 - 2\cos^2 x + \cos^4 x)\,dx$

$= \int (\sin x - 2\sin x\cos^2 x + \sin x\cos^4 x)\,dx$ ✓

$= -\cos x + \frac{2}{3}\cos^3 x - \frac{1}{5}\cos^5 x + C$ ✓

- split up
- substitute for $\sin^2 x$
- Integrate using RCR.

✱ Watch those signs! Don't lose marks for careless expanding.

c) $\int \frac{1}{\sqrt{x^2+4x+6}}\,dx$

$= \int \frac{1}{\sqrt{(x+2)^2+2}}\,dx$

$= \int \frac{1}{\sqrt{(x+2)^2+(\sqrt{2})^2}}\,dx$ ✓

Using standard integrals

$= \ln\left((x+2) + \sqrt{(x+2)^2+2}\right) + C$ ✓

Directly from the standard integrals. Match it carefully!!! and then doublecheck!

d) $\int \frac{x}{\sqrt{4+x^2}}\,dx$

$x = 2\tan\theta$
$dx = 2\sec^2\theta\,d\theta$
$\theta = \tan^{-1}\frac{x}{2}$

$= \int \frac{2\tan\theta\, 2\sec^2\theta\,d\theta}{\sqrt{4+4\tan^2\theta}}$

$= \int \frac{4\tan\theta\sec^2\theta\,d\theta}{\sqrt{4(1+\tan^2\theta)}}$ ✓

$= \int \frac{4\tan\theta\sec^2\theta}{2\sqrt{\sec^2\theta}}\,d\theta$

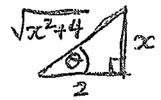
$= \int \frac{4\tan\theta\sec^2\theta}{2\sec\theta}\,d\theta$

$= \int 2\tan\theta\sec\theta\,d\theta$

$= 2\sec\theta + C$ ✓

$= 2 \times \frac{\sqrt{x^2+4}}{2} + C$ ✓

$= \sqrt{x^2+4} + C$



$\cos\theta = \frac{2}{\sqrt{x^2+4}}$

$\sec\theta = \frac{\sqrt{x^2+4}}{2}$

Don't forget to rewrite back in terms of x.

e) $\int x\tan^{-1}x\,dx$

using I.B.P.
$u = \tan^{-1}x \qquad v' = x$
$u' = \frac{1}{1+x^2} \qquad v = \frac{x^2}{2}$
$\int uv' = uv - \int vu'$

$= \frac{x^2}{2}\tan^{-1}x - \int \frac{x^2}{2} \times \frac{1}{1+x^2}\,dx$ ✓

$= \frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\left(\int \frac{1+x^2-1}{1+x^2}\,dx\right)$

$= \frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\left(\int 1 - \frac{1}{1+x^2}\,dx\right)$ ✓

$= \frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\int 1\,dx + \frac{1}{2}\int \frac{1}{1+x^2}\,dx$

$= \frac{x^2}{2}\tan^{-1}x - \frac{1}{2}x + \frac{1}{2}\tan^{-1}x + C$ ✓

(Some careless work with fractions and powers and signs. Please be careful.)

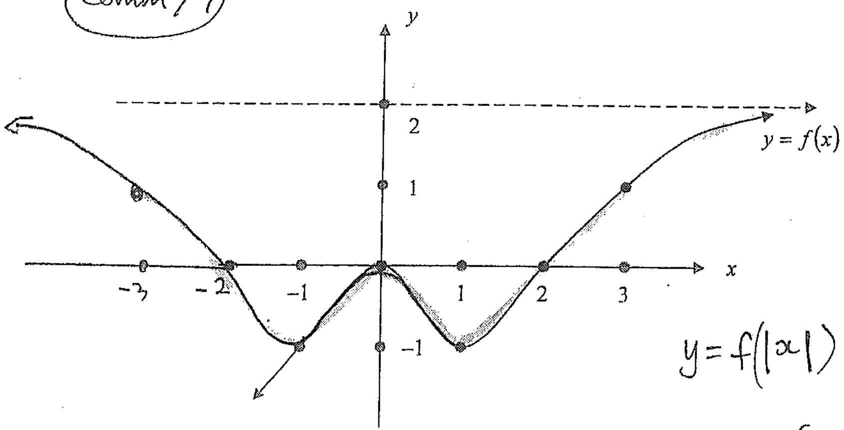
(A straight forward question)

Overall in Q1. it was obvious that everyone knew their work BUT you can't afford to lose marks for being careless.

Comm /7 Calc /3 Reas /4

Question 2.

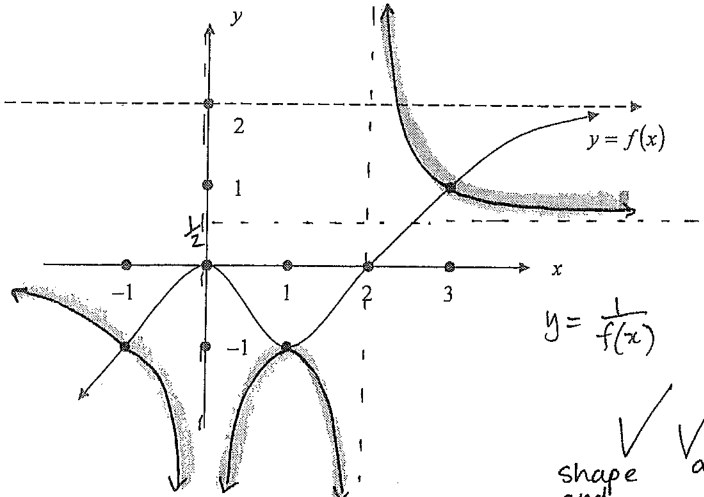
(a) (i) Comm /7



$y=f(|x|)$ ✓

(well done easy!)

(ii)



$y=\frac{1}{f(x)}$

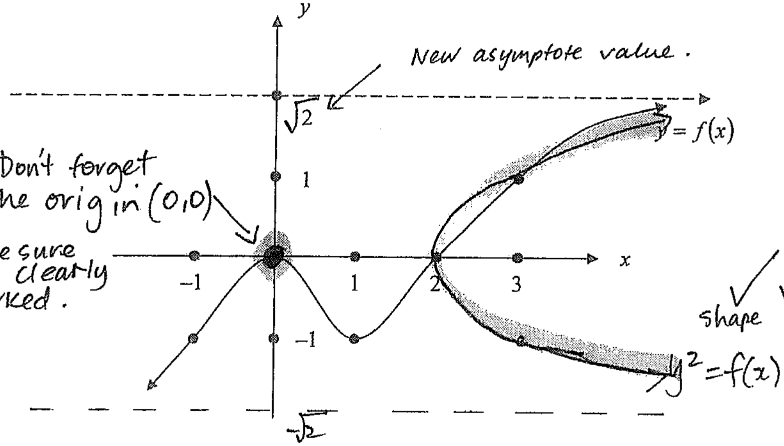
✓ shape and position ✓ asymptote $x\to\infty$ $y\to\frac{1}{2}$.

(iii)

New asymptote value.

Don't forget the origin (0,0)

Make sure it's clearly marked.



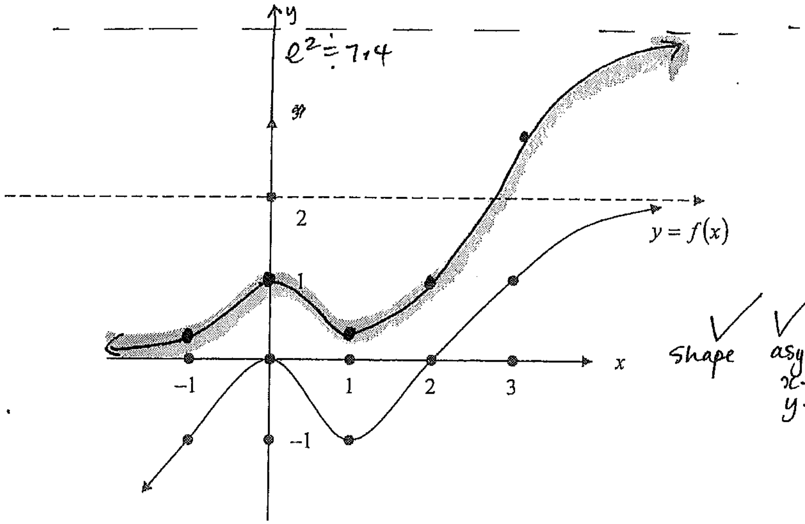
✓ shape ✓ origin and asymptote $y=\pm\sqrt{2}$

(iv)

$e^{-1}\doteqdot 0.4$

$e^{1}\doteqdot 2.7$

$e^{2}\doteqdot 7.4$ asymptote.



✓ shape ✓ asymptote $x\to\infty$ $y\to e^2$

In general.
Everyone knew the correct shapes, so well done, BUT you need to watch the fine details such as asymptotes.
(*) Make your graph stands out for the marker in the HSC.

Q2 continued.

b) i) $\frac{x^2}{25} + \frac{y^2}{16} = 1$

$a = 5,\ b = 4$

$b^2 = a^2(1-e^2)$

$16 = 25(1-e^2)$

$\frac{16}{25} = 1 - e^2$

$e^2 = \frac{9}{25}$

$e = \frac{3}{5}$ ✓

Well done (easy!)

ii) Directrix ~~D~~

$x = \frac{a}{e}$

$x = \frac{5}{\frac{3}{5}}$

$x = \frac{25}{3}$

$x = 8\frac{1}{3}$ ✓

Well done (easy!) Note the location of D. You don't need $x = \pm 8\frac{1}{3}$ for D and D′

iii) Tangent at P

$\frac{x_1 x}{25} + \frac{y_1 y}{16} = 1$

(Generally this was well done.)

Chord of contact from $T(x_0, y_0)$ has equation

$\frac{x_0 x}{25} + \frac{y_0 y}{16} = 1$

You can just quote this result. The question doesn't ask you to prove it.

Test if this chord passes through the focus $S(ae, 0) = S(3, 0)$ ✓

The external point $T(x_0, y_0)$ lies on the directrix so it has the coordinates $T(8\frac{1}{3}, y_0)$

Substitute these points S and T into the chord of contact.

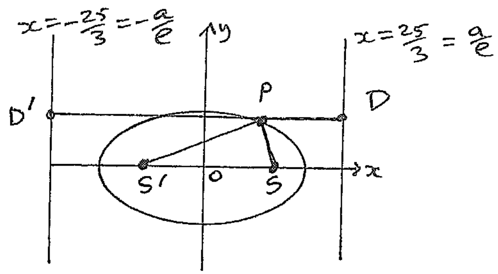
$LHS = \frac{8\frac{1}{3} \times 3}{25} + \frac{y_0 \times 0}{16} = 1 = RHS$

$\therefore$ the chord of contact is a focal chord. ✓

Reas 2

Q3 cont′.

iv)



Using the definition of an ellipse

$PS = e \cdot PD$

$PS' = e \cdot PD'$

$PS + PS' = e(PD + PD')$

$= e(DD')$

$= e \times 2 \times \frac{a}{e}$

$= 2a$ ✓

$\therefore PS + PS' = 2 \times 5$

$= 10$ units in this case

$\therefore$ Since this is a constant, it is independent of the position of P on the ellipse.

This was very well done but you should use the letters given in the diagram instead of the proof you learned off by heart. Where the directrix is called M, M′. ✓

Reas 2

Q2 (cont')

ⓒ $x^4 + 3xy - 2y^2 + 9 = 0$

use implicit differentiation w.r.t x

product rule   chain rule

$4x^3 + 3\left(x\frac{dy}{dx} + y.1\right) - 4y.\frac{dy}{dx} = 0$

$4x^3 + 3x\frac{dy}{dx} + 3y - 4y\frac{dy}{dx} = 0$

$\frac{dy}{dx}(4y - 3x) = 4x^3 + 3y$

$\frac{dy}{dx} = \frac{4x^3+3y}{4y-3x}$

Gradient of tangent at point $(-1,2)$

$m_T = \frac{-4+6}{8+3}$

$= \frac{2}{11}$

Gradient normal

$m_N = -\frac{11}{2}$

Equation of normal at $(-1,2)$

$y - y_1 = m(x - x_1)$

$y - 2 = -\frac{11}{2}(x+1)$

$2y - 4 = -11x - 11$

$11x + 2y + 7 = 0$.

(*) Using implicit differentiation you should recognise.

EASY ONES

$\frac{d}{dx}(x^n) = nx^{n-1}$

PRODUCT RULE

$\frac{d}{dx}(xy) = x\frac{dy}{dx} + y$

CHAIN RULE

$\frac{d}{dx}(y^n) = ny^{n-1} \times \frac{dy}{dx}$

QUOTIENT RULE

$\frac{d}{dx}\left(\frac{x}{y}\right) = \frac{y.1 - x.\frac{dy}{dx}}{y^2}$

A surprising number of arithmetic and algebraic mistakes in this easy question

Calc 3

Question 3.   /3   /3   /4

a) i) $y = \frac{x}{(x+1)(2x+1)}$

$= \frac{x}{2x^2+3x+1}$

Using the quotient rule

$y' = \frac{vu' - uv'}{v^2}$

$= \frac{(2x^2+3x+1).1 - x(4x+3)}{(2x^2+3x+1)^2}$

$= \frac{2x^2+3x+1-4x^2-3x}{(2x^2+3x+1)^2}$

$= \frac{-2x^2+1}{(2x^2+3x+1)^2}$

Stationary points. $y' = 0$

$-2x^2 + 1 = 0$

$x^2 = \frac{1}{2}$

$x = \pm\frac{1}{\sqrt{2}}$

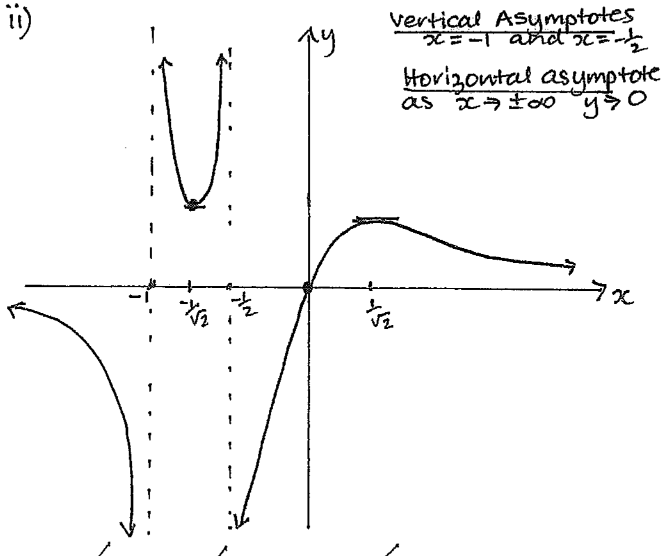
$x \doteq \pm 0.7$

This is a 2U technique and you've been using the quotient rule for ages so it's surprising that some found this challenging.

Only the x-values where required but the y values will also help with the location on your graph.

Calc 1

ii)



Vertical Asymptotes $x = -1$ and $x = -\frac{1}{2}$

Horizontal asymptote as $x \to \pm\infty$ $y \to 0$

Sign Diagram (this will help.)

$y = \frac{x}{(x+1)(2x+1)}$

$x<-1$	$-1<x<-\frac{1}{2}$	$-\frac{1}{2}<x<0$	$x>0$
$\frac{(-)}{(-)(-)}$	$\frac{(-)}{(+)(-)}$	$\frac{(-)}{(+)(+)}$	$\frac{+}{++}$
−	+	−	+
	Stat pt at $x = -\frac{1}{\sqrt{2}}$		Stat pt at $x = \frac{1}{\sqrt{2}}$

(*) There are only 2 vertical asymptotes

(*) Check points on your calculator.

Comm 3

Q3 cont'

iii) $\dfrac{x}{(x+1)(2x+1)} = \dfrac{A}{x+1} + \dfrac{B}{2x+1}$

Mental method

put $x=-1$ $\quad A = \frac{-1}{-1} = 1$

put $x = -\frac{1}{2}$ $\quad B = \dfrac{-\frac{1}{2}}{\frac{1}{2}} = -1$

OR Solving simultaneously/substitution

$x = A(2x+1) + B(x+1)$

put $x=-1$ $\quad -1 = A \times -1$, $A = 1$

put $x = -\frac{1}{2}$ $\quad -\frac{1}{2} = B\frac{1}{2}$, $B = -1$

This part was well done. (easy)

iv) Area lies above the x axis

$$\text{Area} = \int_0^1 \frac{x}{(x+1)(2x+1)}\,dx$$

$$= \int_0^1 \frac{1}{x+1} - \frac{1}{2x+1}\,dx$$

$$= \int_0^1 \frac{1}{x+1}\,dx - \frac{1}{2}\int \frac{2}{2x+1}\,dx$$

$$= \left[\ln(x+1) - \tfrac{1}{2}\ln(2x+1)\right]_0^1$$

$$= \left[\ln\left(\frac{x+1}{\sqrt{2x+1}}\right)\right]_0^1$$

$$= \ln\frac{2}{\sqrt{3}} - \ln\frac{1}{\sqrt{1}}$$

$$= \ln\frac{2}{\sqrt{3}}$$

Generally well done if you read the question properly.

Make sure you show this line where it's split into partial fractions.

Some surprising mistakes with this easy integration and evaluation of the definite integral.

Calc 2

Q3 cont'.

b) i) $P(cp, \frac{c}{p})$ $\quad T(ct, \frac{c}{t})$

Gradient PT

$$m_{TP} = \frac{\frac{c}{t} - \frac{c}{p}}{ct - cp}$$

$$= \frac{c(\frac{1}{t} - \frac{1}{p})}{c(t-p)}$$

$$= \frac{p-t}{tp} \times \frac{1}{t-p}$$

$$= \frac{-(t-p)}{tp(t-p)}$$

$$= \frac{-1}{tp}$$

Part b) was very well done.

It's pretty easy to get it all correct.

ii) Since $\angle QTP = 90^\circ$

$$m_{TP} \times m_{QT} = -1$$

Similarly as above

$$m_{QT} = \frac{-1}{tq}$$

$$\therefore \frac{-1}{tp} \times \frac{-1}{tq} = -1$$

$$\frac{1}{t^2pq} = -1$$

$$\frac{1}{t^2} = -pq$$

$$t^2 = -\frac{1}{pq}$$

Reas 2

iii) $xy = c^2$

$$y = \frac{c^2}{x} = c^2x^{-1}$$

$$y' = -c^2x^{-2}$$

$$= -\frac{c^2}{x^2}$$

Q3 cont'

Gradient tangent at $T(ct, \frac{c}{t})$

$$m_T = -\frac{c^2}{(ct)^2}$$
$$= \frac{-c^2}{c^2t^2}$$
$$= -\frac{1}{t^2}$$

Gradient normal at T

$$m_N = t^2$$

Gradient of interval PQ

$$m_{PQ} = -\frac{1}{pq}$$
$$= t^2$$
from part (ii)

$\therefore m_N = m_{PQ}$

$\therefore$ PQ is parallel to the normal at T.

Reas 2

Question 4 a) /5 /6

a) i) $I_n = \int_0^1 (1+x^2)^n \, dx \quad ; n = 0, 1, 2, \ldots$

$$= \int_0^1 1.(1+x^2)^n \, dx$$

Using IBP	$u = (1+x^2)^n$ $u' = n(1+x^2)^{n-1}.2x$	$v' = 1$ $v = x$
	$\int uv' = uv - \int vu'$	

$$= \left[x(1+x^2)^n\right]_0^1 - \int_0^1 2x^2 n (1+x^2)^{n-1} \, dx$$

$$= \{1 \times 2^n - 0\} - 2n \int_0^1 x^2(1+x^2)^{n-1} \, dx$$

replace this by $x^2 = 1+x^2-1$

$$= 2^n - 2n \int_0^1 (1+x^2-1)(1+x^2)^{n-1} \, dx$$

$$= 2^n - 2n \int_0^1 (1+x^2)^n \, dx + 2n \int_0^1 (1+x^2)^{n-1} \, dx$$

$$I_n = 2^n - 2n I_n + 2n I_{n-1}$$

$$(2n+1) I_n = 2^n + 2n I_{n-1}$$

$$\therefore I_n = \frac{1}{2n+1}\left(2^n + 2n I_{n-1}\right)$$

Pretty well done if you started off with good choices for u & v'.

→ These choices for u and v' are easiest method.

← This is a standard replacement.

Calc 3

Q4 cont'.

ii) $I_n = \frac{1}{2n+1}\left(2^n + 2n I_{n-1}\right)$

$I_3 = \frac{1}{7}\left(2^3 + 6 I_2\right)$

$= \frac{2^3}{7} + \frac{6}{7} I_2$

$= \frac{8}{7} + \frac{6}{7}\left(\frac{1}{5}\left(2^2 + 4 I_1\right)\right)$

$= \frac{8}{7} + \frac{6}{7}\left(\frac{4}{5} + \frac{4}{5} I_1\right)$

$= \frac{8}{7} + \frac{6}{7}\times\frac{4}{5} + \frac{6}{7}\times\frac{4}{5}\times I_1$

I_1 is easy to find

$\int_0^1 (1+x^2)^1\,dx$

$= \left[x + \frac{x^3}{3}\right]_0^1$

$= (1+\frac{1}{3}) - 0$

$= \frac{4}{3}$

$\therefore I_3 = \frac{8}{7} + \frac{6}{7}\times\frac{4}{5} + \frac{6}{7}\times\frac{4}{5}\times\frac{4}{3}$

$= \frac{96}{35}$

$= 2\frac{26}{35}$

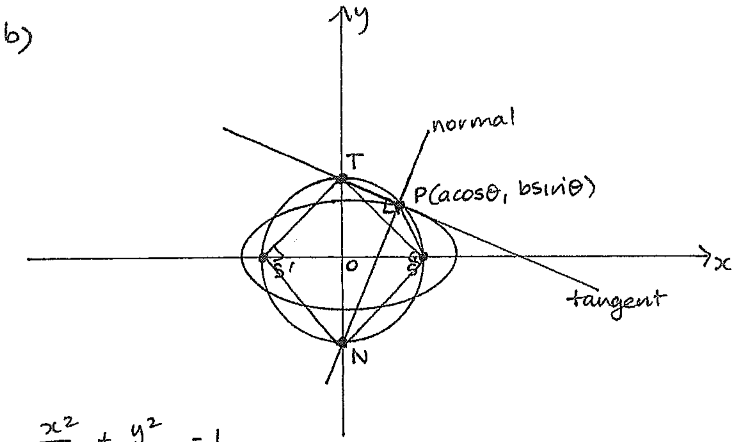
This part was very well done.

You should do this even if you can't do part a) i).

Calc 2

Q4 continued.

b)



$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$S(ae,0)$ $S'(-ae,0)$

i) Tangent cuts yaxis at $x=0$ at T

$\frac{x\cos\theta}{a} + \frac{y\sin\theta}{b} = 1$

$\frac{y\sin\theta}{b} = 1$

$y = \frac{b}{\sin\theta}$

$\therefore T\left(0, \frac{b}{\sin\theta}\right)$

Normal cuts yaxis at $x=0$ at N

$\frac{ax}{\cos\theta} - \frac{by}{\sin\theta} = a^2 - b^2$

$-\frac{by}{\sin\theta} = a^2 - b^2$

$y = -\frac{\sin\theta}{b}(a^2 - b^2)$

$\therefore N\left(0, -\frac{(a^2-b^2)\sin\theta}{b}\right)$

This part was easy. & very well done.

Q4 cont'.

ii) $T(0, \frac{b}{\sin\theta})$ $S(ae, 0)$

$N(0, -\frac{(a^2-b^2)\sin\theta}{b})$ $S'(-ae, 0)$

Find gradients.

$$m_{TS} = \frac{\frac{b}{\sin\theta} - 0}{0 - ae}$$

$$= -\frac{b}{ae\sin\theta}$$

$$m_{NS} = \frac{-\frac{(a^2-b^2)\sin\theta}{b} - 0}{0 - ae}$$

$$= \frac{(a^2-b^2)\sin\theta}{bae}$$

$$m_{TS} \times m_{NS} = \frac{-\cancel{b}}{ae\cancel{\sin\theta}} \times \frac{(a^2-b^2)\cancel{\sin\theta}}{\cancel{b}ae}$$

$$= \frac{-(a^2-b^2)}{a^2e^2}$$

for ellipse $b^2 = a^2(1-e^2)$

(rearrange however you like!)

$b^2 = a^2 - a^2e^2$

$a^2e^2 = a^2 - b^2$

$$= \frac{-(a^2-b^2)}{a^2-b^2}$$

$$= -1$$

$\therefore TS \perp NS$

This change can be made earlier.

Q4

ii) (cont'.)

Similarly using the points T, S' and N it can be shown that

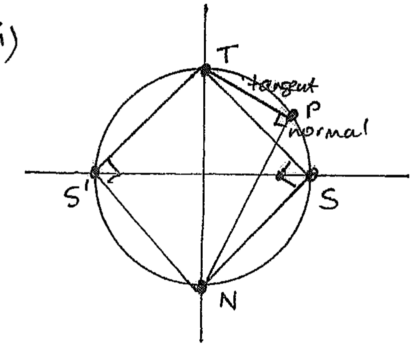
$$m_{TS'} \times m_{NS'} = -1$$

$\therefore TS' \perp NS'$

Don't forget to do this part as the question asks you to deduce it.

Reas 3

iii)



$\angle TS'N = 90^\circ$
$\angle TPN = 90^\circ$

$\therefore TS'NP$ are concyclic (opposite angles are supplementary in a cyclic quadrilateral)

$\angle TSN = 90^\circ$
$\angle TS'N = 90^\circ$

$\therefore TSNS'$ are concyclic (opposite angles are supplementary in a cyclic quadrilateral)

$\therefore$ The five points T, P, S, N and S' are concyclic.

If you got this far and had time to do it, congratulations. It wasn't really very difficult.

Reas 3