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SCEGGS Darlinghurst

HSC Assessment Task 2
Thursday 20 March, 2003

Class Av 27.5/40

Mathematics

$\bar{x} = \frac{29}{40}$

General Instructions

- Time allowed: 60 minutes
- Weighting 20%
- This paper has **four** questions
- Attempt **all** questions and show all necessary working
- Marks will be deducted for careless or badly arranged work
- Start each question on a new page
- Write your name at the top of each page
- Approved scientific calculators should be used
- Mathematical templates and geometrical instruments may be used
- A table of Standard Integrals is provided

Question	Communication	Reasoning	Total	Av
1	4 /4		10 /10	8
2		2 /2	10 /10	8
3	4 /4	1 /1	10 /10	6.
4		5 /5	10 /10	4.
Total	8	8	40 /40	

Question 1 (10 marks)

- Answer on the pad paper provided
- Write your name at the top of the page
- Start each question on a new page
- Clearly label each question

Marks

(a) Find:

(i) $\int 4x^3 + \sqrt{x}\, dx$ **2**

(ii) $\int \frac{x^7 + x^4}{x^6}\, dx$ **2**

(b) Use the table of standard integrals to find $\int 2 \sec 3x \tan 3x\, dx$ **2**

(c) Sally was asked to find the gradient of the normal to the curve **2**

$y = \sin \frac{x}{2}$ at the point $x = 2\pi$

This is her solution:

Line 1: $y = \sin \frac{x}{2}$

Line 2: $\frac{dy}{dx} = 2 \cos \frac{x}{2}$

Line 3: $\therefore$ At $x = 2\pi, \frac{dy}{dx} = 2 \cos \pi$

Line 4: $\therefore \frac{dy}{dx} = -2$

Line 5: $\therefore$ Grad of normal is -2

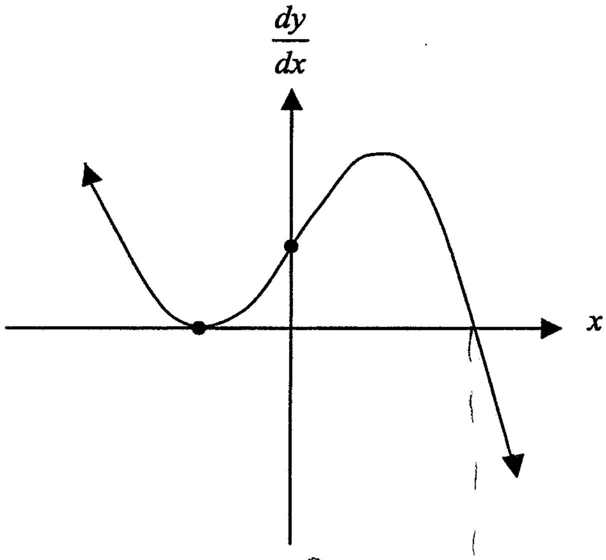
Her solution contains TWO major errors. Describe these errors.

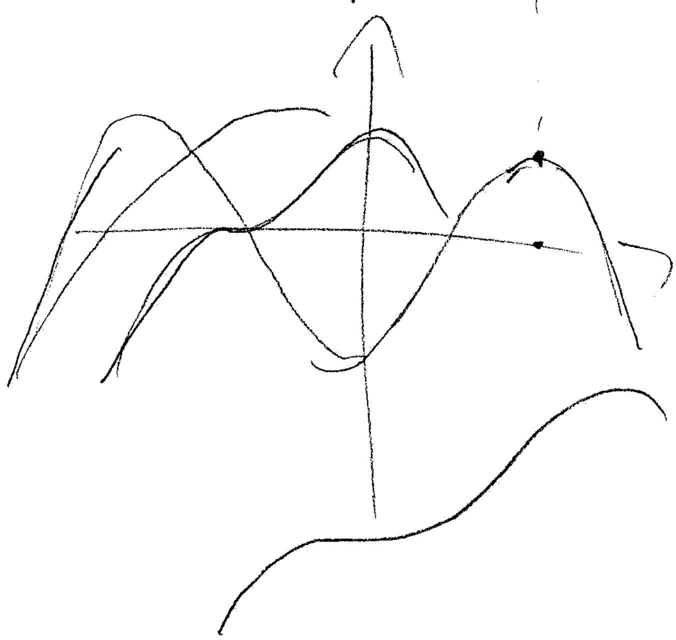
Question 1 continues on page 3

Question 1 (continued)

(d) The graph of $y = f\,'(x)$ is shown. **2**

Copy or trace this graph and sketch a possible graph of $y = f(x)$





Question 2 (10 marks)

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Marks

(a) The following are function values for a graph $y = f(x)$. **3**

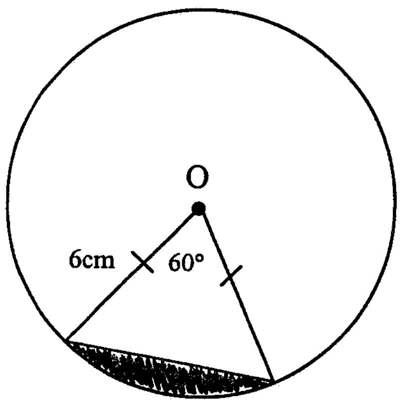
x	1	1·25	1·5	1·75	2
$f(x)$	0·50	0·40	0·33	0·29	0·25

Use Simpson's Rule with 5 function values to estimate $\int_1^2 f(x)\,dx$.
Answer correct to 2 decimal places.

(b) $\dfrac{dy}{dx} = (2x+1)^3$. Find the equation of the curve if it passes through the point $\left(-\dfrac{1}{2}, 1\right)$ **2**

(c) O is the centre of the circle. **3**

Show that the exact value of the shaded area is $6\pi - 9\sqrt{3}$ units^2.



(d) Find the value of k if $\int_0^3 kx^2\,dx = 4$. **2**

Question 3 (10 marks)

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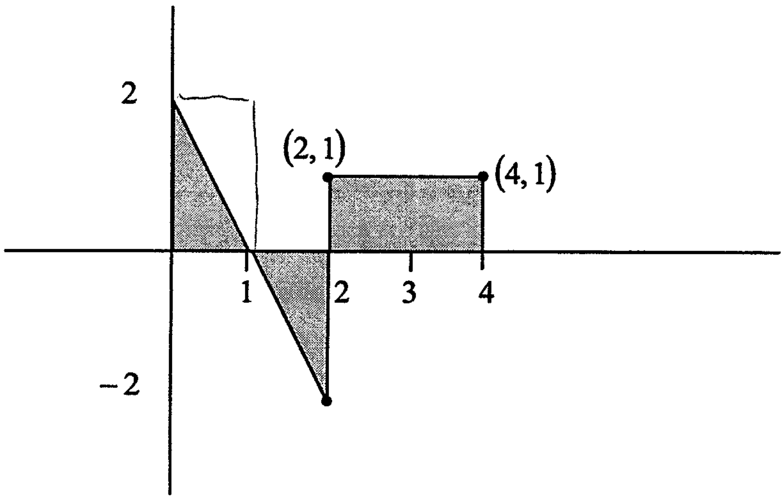
Marks

(a)



(i) Explain why $\int_0^4 f(x)\,dx$ does not give the value of the shaded area. **1**

(ii) Evaluate the shaded area. **1**

(b) (i) Sketch $y = 3\cos 2x$ in the domain $-\pi \le x \le \pi$. **2**

(ii) **On the same graph**, sketch $y = \dfrac{3x}{2}$. **1**

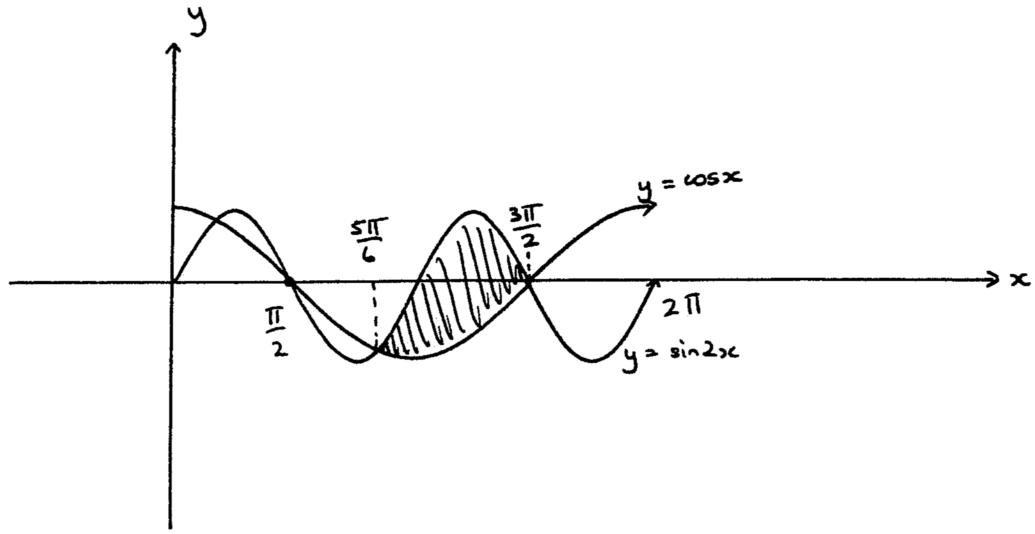
(iii) Use your graph to find the number of solutions to the equation **1**

$$3\cos 2x = \frac{3x}{2}.$$

Question 3 continues on page 6

Question 3 (continued)

(c) The following graph shows $y = \sin 2x$ and $y = \cos x$ between $x = 0$ and $x = 2\pi$. Find the value of the shaded area. **4**



Question 4 (10 marks)

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Marks

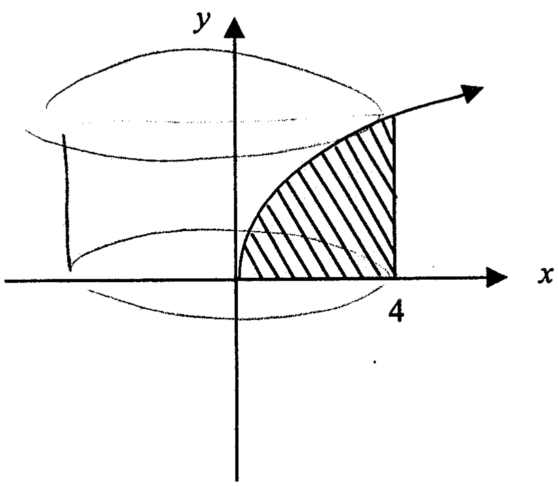
(a) (i) Differentiate $y = \sin^5 x$. **1**

(ii) Hence find $\int \sin^4 x \cos x \, dx$ **1**

(b) (i) Show that the only point of intersection between the curves $y = (x-2)^2$ and $y = (x-6)^2$ is the point $(4, 4)$. **1**

(ii) Find the area bounded by the curves $y = (x-2)^2$, $y = (x-6)^2$ and the x axis. **3**

(c) The area bounded by the curve $x = y^2$, the x axis and the line $x = 4$ in the first quadrant is rotated about the y axis. Find the volume of the solid of revolution formed. **4**



$4^2 \times \pi \times 2$

End of Paper

YEAR 12 : MATHEMATICS

ASS 2 MARCH 2003

SOLUTIONS + MARKING SCALE

QUESTION 1: 10 marks MF

Comments: Com /4

(a) (i) $\int 4x^3 + \sqrt{x}\, dx$

$= x^4 + \frac{2x^{3/2}}{3} + C$ ✓✓+C

Must have +C for 2nd mark.

(ii) $\int \frac{x^7 + x^4}{x^6}\, dx$

$= \int x + x^{-2}\, dx$ ✓

$= \frac{x^2}{2} + \frac{x^{-1}}{-1} + C$ ✓

$= \frac{x^2}{2} - \frac{1}{x} + C$

Don't forget to split into pieces over denominator then use index rules to cancel.

(b) $\int 2 \sec 3x \tan 3x\, dx$

$= 2 \times \frac{1}{3} \sec 3x + C$ ✓✓

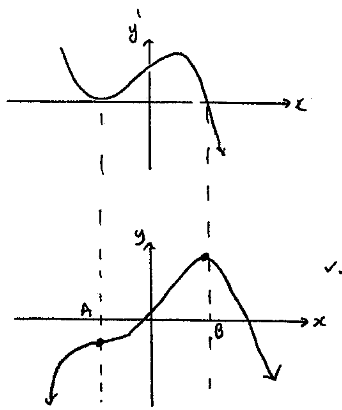
$= \frac{2}{3} \sec 3x + C$

(c) Line 2: $\frac{dy}{dx} = \frac{1}{2} \cos \frac{x}{2}$ ✓

and Line 5: Grad normal is negative reciprocal ∴ $\frac{1}{2}$ ✓

Com Many students integrated instead of differentiating. You must show the correct derivative for the mark.

(d)



✓✓

Com Draw the graph of the primitive not the derivative

One for max t.p. at B and smooth curve

One for horizontal poi. at A.

Question 2: 10 marks CB

Comments: Reas /2

(a) $h = 0.25$ ✓

$\int_1^2 f(x)\,dx \doteq \frac{0.25}{3}\left(0.5 + 4\times 0.4 + 2\times 0.33 + 4\times 0.29 + 0.25\right)$

$= \frac{0.25}{3} \times 4.17$ ✓Σ

$\doteq 0.35 \text{ units}^2$ ✓

Some confusion as to format of rule with "odds" and "evens". Omission of 'n' in the formula $h = \frac{b-a}{n}$

(b) $\frac{dy}{dx} = (2x+1)^3$

$\therefore y = \frac{(2x+1)^4}{4\times 2} + C$ ✓

∴ When $x = -\frac{1}{2}$, $y = 1$

$1 = \frac{(2\times -\frac{1}{2} + 1)^4}{8} + C$

$\therefore C = 1$

$\therefore y = \frac{(2x+1)^4}{8} + 1$ ✓

Well done – But those who chose to expand rather than apply the chain rule tended to make algebraic errors

(c) Area $= \frac{1}{2} \times 6^2 \times \left(\frac{\pi}{3} - \sin\frac{\pi}{3}\right)$ ✓✓

$= 18\left(\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right)$ ✓

$= 6\pi - 9\sqrt{3}$ units^2

Radians conversion + substitution

Some students could not evaluate $\sin\frac{\pi}{3}$ in exact form

(d) $\int_0^3 kx^2\, dx = \left[\frac{kx^3}{3}\right]_0^3$

$= 9k - 0$

$= 9k$ ✓

$\therefore 9k = 4$

$k = \frac{4}{9}$ ✓

Reas Some students did not know what to do with 'k' when they integrated.

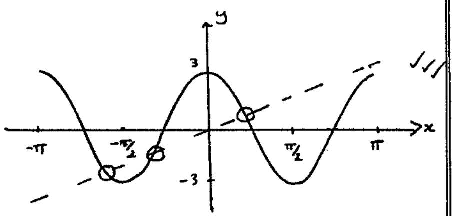
QUESTION 3: (10 marks) KB

COMMENTS: Com /4 Reas /1

(a) (i) $\int_0^4 f(x)\,dx$ gives the value of the signed area and since $f(x)$ is negative $1<x<2$, this is not equal to the area. ✓

(ii) Area $= 1+1+2$
$= 4\,u^2$ ✓

(b)



✓✓✓

(iii) 3 points of intersection, ✓
∴ 3 solutions

(c)
$$\text{Area} = \int_{\frac{5\pi}{6}}^{\frac{3\pi}{2}} \sin 2x - \cos x \, dx$$ ✓

$$= \left[-\tfrac{1}{2}\cos 2x - \sin x \right]_{\frac{5\pi}{6}}^{\frac{3\pi}{2}}$$ ✓

$$= \left(-\tfrac{1}{2}\cos 3\pi - \sin\tfrac{3\pi}{2}\right) - \left(-\tfrac{1}{2}\cos\tfrac{5\pi}{3} - \sin\tfrac{5\pi}{6}\right)$$

$$= \left(\tfrac{1}{2} \;-\, -1\right) - \left(-\tfrac{1}{2}\times\tfrac{1}{2} - \tfrac{1}{2}\right)$$

$$= \tfrac{1}{2} + 1 + \tfrac{1}{4} + \tfrac{1}{2}$$ ✓

$$= 2\tfrac{1}{4}\,u^2$$

Comments:

(Com) Use the word "negative" in the explanation..

(Com) Two for $y = 3\cos 2x$ – amplitude + period
One for $y = \frac{3}{2}x$ on same scale.
Very poor graphs. Should be larger, use pencil, label axes fully
Try to improve the shape of the curve.

(Reas) Any correct answer from (i)+(ii)

• Lack of understanding of the use of absolute value signs.

• Use the standard integral sheet!

✓• Fraction cancelling poor.

• If you cannot do this work learn to use your calculator properly!

QUESTION 4: (10 marks) HG

COMMENTS: Reas /5

(a) (i) $y = \sin^5 x$
$$\frac{dy}{dx} = 5\sin^4 x \cos x$$ ✓

(ii) $\therefore \int 5\sin^4 x \cos x \, dx = \sin^5 x + C$

$\therefore \int \sin^4 x \cos x \, dx = \frac{1}{5}\sin^5 x + C$ ✓

(b) (i) $(x-2)^2 = (x-6)^2$

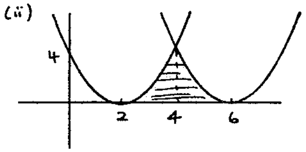
$\cancel{x^2} - 4x + 4 = \cancel{x^2} - 12x + 36$ ✓

$8x = 32$

$\therefore x = 4$

$\therefore y = 4$

(ii)



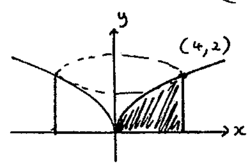
$$\text{Area} = \int_2^4 (x-2)^2\,dx + \int_4^6 (x-6)^2\,dx$$ ✓

$$= \left[\frac{(x-2)^3}{3}\right]_2^4 + \left[\frac{(x-6)^3}{3}\right]_4^6$$ ✓

$$= \left(\frac{8}{3} - 0\right) + \left(0 - -\frac{8}{3}\right)$$

$$= \frac{16}{3}\,u^2.$$ ✓

(c)



(Interior)
$V_1 = \pi\int_0^2 (y^2)^2\,dy$ ✓
$= \pi\int_0^2 y^4\,dy$
$= \pi\left[\frac{y^5}{5}\right]_0^2$
$= \frac{32\pi}{5}$ ✓

$V_2 = \text{cylinder} = \pi \times 4^2 \times 2 = 32\pi$ ✓

$\therefore \text{Volume} = V_2 - V_1 = 32\pi - \frac{32\pi}{5}$
$= \frac{128\pi}{5}$ units^3 ✓

Comments:

*NOT TRUE that "$\sin^5 x = 5\sin^4 x\cos x$"
Be careful about how you write the relationship between these equations/curves.

(Reas) Don't forget little things like dx, $+C$

Well done ✓.

* Must draw a diagram.
* Areas are adjacent ∴ Need to add them together
* Must be done as two separate integrals as they have different limits + curve boundaries.
(or by expanding first)

* Evaluate carefully.
* Don't forget units^2.

(Reas) * Make sure limits are for the y-axis. 0→2 not 0→4.
* Don't forget π, and to square the function.
* Most forgot to do CYLINDER – INTERIOR
* Cylinder formula: $\pi r^2 h$ or by $\pi\int_0^2 (4)^2\,dy$
* Units^3 should be there too!!