Name: Lauren Chua

Teacher: Mrs. Bailey

SCEGGS Darlinghurst

HSC Assessment 2
9th March, 2006

Mathematics

General Instructions

- Time allowed: 60 minutes
- Weighting 20%
- This paper has **four** questions
- Attempt **all** questions and show all necessary working
- Marks will be deducted for careless or badly arranged work
- Write using blue or black pen, diagrams in pencil
- Start each question on a new page
- Write your name and your teacher's name at the top of each page
- Approved calculators, mathematical templates and geometrical instruments may be used
- A table of standard integrals is provided at the back of this paper

Questions	Marks	Communication	Reasoning	Calculus
1	10 /10			(a) 7 /7
2	9 /9	(b) 2 /2	(d) 3 /3	
3	11 /11	(b) (i) /2 (c) (i) /2 4	(a) 2 /2	(b) (ii) /3 (c) (ii) /2 5
4	9 /10	(c) (iii) 0 /1	(b) 3 /3	(a) 2 /2
TOTAL	39 /40	6 /7	8 /8	14 /14

Instructions

- Attempt **all** questions on the pad paper provided
- Start each question on a new page

Question 1 (10 marks) **Marks**

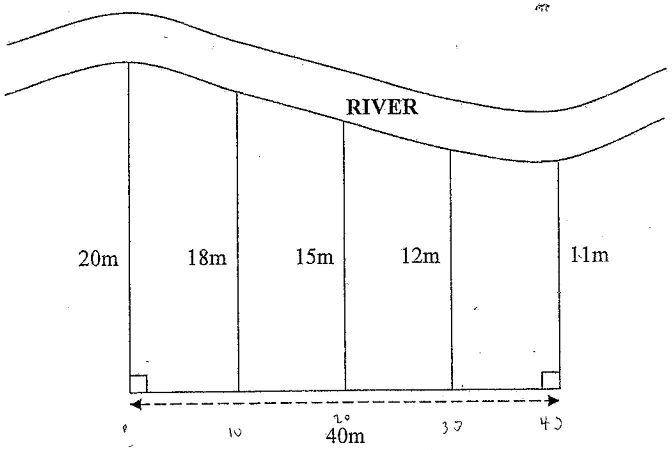
(a) Find:

(i) $\int 3x^2 - 4x + 1 \quad dx$ **2**

(ii) $\int \frac{x^7 + 12x}{x^5} \quad dx$ **2**

(b) Find the exact value of $\int_0^4 \sqrt{2x+1} \quad dx$ **3**

(c) **3**



NOT TO SCALE

Use Simpson' Rule with 5 function values to approximate the area of the paddock above. Give your answer correct to 1 decimal place.

- **Start a new page**

Question 2 (9 marks) **Marks**

(a) Convert $\frac{5\pi}{6}$ radians to degrees. **1**

(b) Scott was asked to answer the following question: **2**

"From a pile of cards numbered 3 to 18, one card is chosen at random. Find the probability that the card is a multiple of 3 or a multiple of 5."

Here is Scott's solution:

There are 3 multiples of 5 (5, 10 and 15).
There are 6 multiple of 3 (3, 6, 9, 12, 15 and 18)
$\therefore \quad P(\text{multiples of 3 or 5}) = \frac{9}{15} = \frac{3}{5}$

Scott has made TWO errors. Describe them.

(c) (i) The arc length of a sector is 1.8cm and the angle subtended at the centre of the circle by this arc is $72°$. Find the radius of the circle, correct to 1 decimal place. **2**

(ii) Hence find the area of the sector correct to 1 decimal place. **1**

(d) The area bounded by the curve $y = 3x^2$, the x-axis, the lines $x = 1$ and $x = k$ (where k is a positive integer) is 26 units squared. Find the value of k. **3**

■ **Start a new page**

Question 3 (11 marks) **Marks**

(a) The game *Time Waster* is played with two identical tetrahedral (4-sided) dice. Each dice has the numbers 1, 3, 4 and 6 on its faces. Each player takes a turn at rolling the dice and their score is the sum of the two numbers on the dice. **2**

Find the probability that a player's score would be less than 7.

(b) (i) Sketch the curve $y = x^2$ and the line $y = 2x + 3$ on the same set of axes and find the x coordinate(s) of the point(s) of intersection. **2**

(ii) Hence find the area bounded by $y = x^2$ and $y = 2x + 3$. **3**

(c) Ethel was solving a problem related to areas and curves.
The first line of her <u>correct</u> working was:

$$\text{Area} = \int_0^1 (x+1)^2 \, dx + \int_1^3 (x-3)^2 \, dx$$

(i) Draw a sketch, clearly shading the area that Ethel was asked to find. **2**

(ii) Complete Ethel's working out to find the area. **2**

■ **Start a new page**

Question 4 (10 marks) **Marks**

(a) The curve $y = f(x)$ passes through the point $(1, 8)$ and $f'(x) = 3x^2 + 5$. Find $y = f(x)$. **2**

(b) The region bounded by the curve $y = 2\sqrt{x}$, the y-axis and the line $y = 6$ is rotated about the y axis. **3**

Find the exact value of the solid of revolution formed.

(c) Bag I contains 3 red balls and 2 blue balls.
Bag II contains 5 red balls and 3 green balls.

A ball is chosen from Bag I and not replaced. If this first ball is red, then the second ball is also chosen from Bag I. If the first ball is blue, then the second ball is chosen from Bag II.

(i) Draw a probability tree and hence find the probability that exactly one of the two balls chosen is red. **2**

(ii) Find the probability that at least one of the two balls chosen is red. **2**

(iii) Comment on the validity of the statement: **1**

"There is a total of 8 red balls, 3 green balls and 2 blue balls.

$\therefore$ *The probability of the first ball being red is* $\frac{8}{13}$*."*

End of paper

Lauren Chua
Mrs. Bailey.

Question One

10

Calc 7

a) i) $\int 3x^2 - 4x + 1$

$= \left[\frac{\cancel{3}x^3}{\cancel{3}} - \frac{{}^2\cancel{4}x^2}{\cancel{2}_1} + x\right] + c$

$= x^3 - 2x^2 + x + c.$ ✓✓

ii) $\int \frac{x^7 + 12x}{x^5}\,dx$

$= \int x^2 + 12x^{-4}\,dx$

$= \left[\frac{x^3}{3} + \frac{12x^{-3}}{-3}\right] + c$

$= \frac{x^3}{3} - 4x^{-3} + c.$

$= \frac{x^3}{3} - \frac{4}{x^3} + c$ ✓✓

b) $\int_0^4 (2x+1)^{1/2}\,dx$

$= \left[\frac{(2x+1)^{3/2}}{3/2 \times 2}\right]_0^4$

$= \left[\frac{(2x+1)^{3/2}}{3}\right]_0^4$

$= [9 - \frac{1}{3}]$

$= 8\frac{2}{3}$ ✓✓✓

c)

x	0	10	20	30	40
$f(x)$	20	18	15	12	11
	y_0	y_1	y_2	y_3	y_4

$A \doteq \frac{h}{3}[(y_0+y_4) + 4(y_1+y_3) + 2(y_2)]$

$\doteq \frac{10}{3}[(20+11) + 4(18+12) + 2(15)]$

$\doteq 603.3\,m^2$ (to 1 dp) ✓✓✓

9

Excellent, Lauren! :)

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Question Two

a) $\frac{5\pi}{6} \times \frac{180}{\pi} = 150°$ ✓

Cm = 2
R = 3

b) 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18

Scott's errors:

1. He has counted the no. 15 twice in finding the P, as it is a multiple of both 3 + 5. (overlapping) ✓
2. He has counted the cards from 3-18 as 15, where it is actually 16 cards (inclusive of 3 + 18) ✓

c) i) $72° = \frac{2\pi}{5}$

$l = R\theta^c$

$1.8 = R\,\frac{2\pi}{5}$ ✓

$R = \frac{1.8}{\frac{2\pi}{5}}$

$\doteq 1.4$ cm (to 1 dp). ✓

ii) $A = \frac{1}{2} r^2 \theta^c$

$= \frac{1}{2} \times 1.4^2 \times \frac{2\pi}{5}$

$\doteq 1.3u^2$ (to 1 dp) ✓

d) $A = \int_1^k 3x^2\,dx$

$26 = \left[\frac{3x^3}{3}\right]_1^k$ ✓

$= [x^3]_1^k$

$= k^3 - 1$ ✓

$27 = k^3$

$\therefore k = 3$ ✓

11/11 Comm = 4|4
Reas = 2|2
Calc = 5|5

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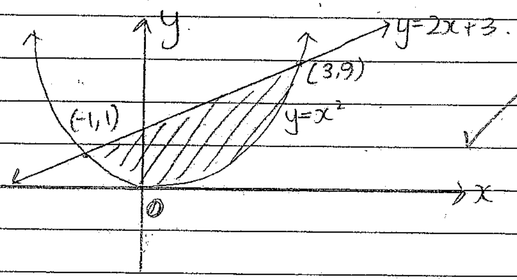
Question Three

a)

	1	3	4	6
1	2	4	5	7
3	4	6	7	9
4	5	7	8	10
6	7	9	10	12

$P = \frac{6}{16} = \frac{3}{8}$

b) i)



$y = x^2$
$y = 2x+3$
$x^2 = 2x+3$
$x^2 - 2x - 3 = 0$
$(x+1)(x-3) = 0$
$\therefore x = -1, 3$
$y = 1, 9$
$\therefore$ p.o.i = $(-1,1)$, $(3,9)$

ii) $A = \int_a^b f(x) - g(x)\, dx$
$= \int_{-1}^{3} 2x+3 - x^2 \, dx$
$= \left[\frac{\cancel{2}x^2}{\cancel{2}} + 3x - \frac{x^3}{3}\right]_{-1}^{3}$
$= \left[x^2 + 3x - \frac{x^3}{3}\right]_{-1}^{3}$
$= [(9+9-9) - (1-3+1/3)]$
$= 10\,2/3\,u^2$

Three continued...

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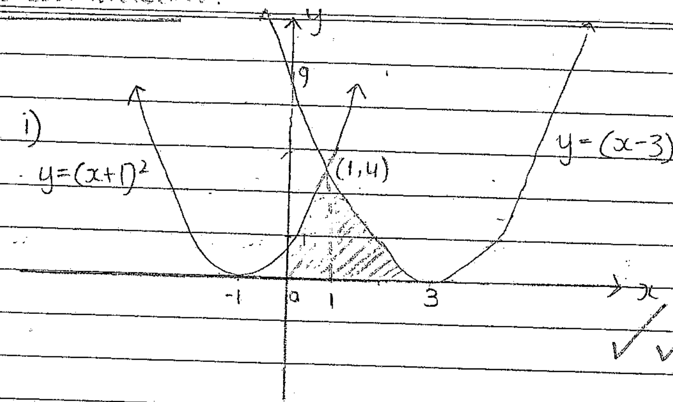
c) i)



$(x+1)^2$
$= (x+1)(x+1)$
$= x^2 + x + x + 1$
$= x^2 + 2x + 1.$

$(x-3)^2$
$= (x-3)(x-3)$
$= x^2 - 3x - 3x + 9$
$= x^2 - 6x + 9.$

ii) $A = \int_0^1 (x+1)^2\,dx + \int_1^3 (x-3)^2\,dx$
$= \left[\frac{(x+1)^3}{3}\right]_0^1 + \left[\frac{(x-3)^3}{3}\right]_1^3$
$= [(2\,2/3) - 1/3)] + [(0) - (-2\,2/3)]$
$= 5u^2$

$\frac{9}{10}$ Com 0

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Question Four

a) $f'(x) = 3x^2 + 5$

$f(x) = \frac{\cancel{3}x^3}{\cancel{3}} + 5x + c$

$= x^3 + 5x + c.$

when $x = 1, y = 8$

$8 = (1)^3 + 5(1) + c$

$= 1 + 5 + c$

$= 6 + c$

$\therefore c = 2.$

$\therefore y = x^3 + 5x + 2$ ✓✓

b) $V = \pi \int_c^d x^2 \cdot dy$

$y = 2x^{1/2}.$

$x^{1/2} = y/2$

$x = y^2/4. \quad \therefore x^2 = \frac{y^4}{16}$

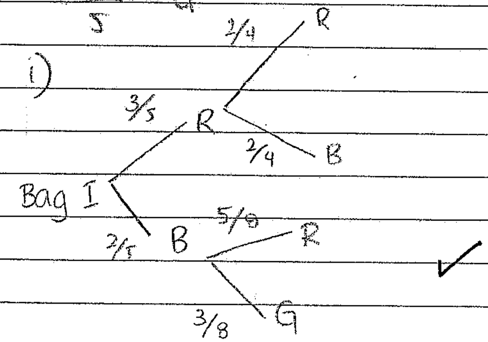
$V = \pi \int_0^6 \frac{y^4}{16} \cdot dy$

$= \pi \left[\frac{y^5}{80} \right]_0^6$

$= \pi [(97\tfrac{1}{5}) - 0]$

$\therefore V = \frac{486\pi}{5} u^3$ ✓ 3

c) i)



✓

$P(\text{one red}) = \left(\frac{3}{5} \times \frac{2}{4}\right) + \left(\frac{2}{5} \times \frac{5}{8}\right) = \frac{11}{20}$

✓ 2

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ii) $P(\text{at least 1 red}) = \left(\frac{3}{5} \times \frac{2}{4}\right) + \left(\frac{2}{5} \times \frac{5}{8}\right) + \left(\frac{3}{5} \times \frac{2}{4}\right)$

$= \frac{17}{20}$ ✓✓

iii) ~~The comment is correct as there is a total of 13 balls (hence the denominator) + a probability of 8 being red, therefore $\frac{8}{13}$~~ ✗

$\frac{0}{1}$ Com

(iii) In the above context, this statement or comment is INCORRECT because there are 2 bags and the probability of drawing a red ball from bag 1 is $\frac{3}{5}$ and from bag 2 is $\frac{5}{8}$.