[redacted]
Centre Number

Student Number

SCEGGS Darlinghurst

2008
Higher School Certificate
Assessment Task 3

Mathematics

Task Weighting: 25%

Outcomes Assessed: H2 – H6, H8 & H9

General Instructions
- Time allowed – 70 minutes
- **Start each question on a new page.**
- Attempt **all** questions and show all necessary working.
- Answer **Question 3 (a)** on the answer sheet provided
- Write your student number at the top of each page.
- Marks can be deducted for careless or badly arranged work
- Mathematical templates, geometrical equipment and scientific calculators may be used.

Question	Reasoning	Communication	Calculus	Total
1	/2	/2	/7	/13
2	/6	/1	/4	/13
3	/2	/4	/5	/12
4	/5		/8	/13
Total	/15	/7	/24	/51

Parent's Signature____

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Marks

Question 1 (13 Marks)

a) Convert 200° to radians and leave your answer in terms of π **1**

b) If $\log_a 2 = 0.356$ and $\log_a 5 = 0.827$ find the value of:

i) $\log_a \frac{2}{5}$ **1**

ii) $\log_a 20a$ **2**

c) Differentiate:

i) $y = \frac{\tan x}{x}$ **2**

ii) $y = \cos^2 x$ **2**

d) Find:

i) $\int \sec^2 3x \, dx$ **1**

ii) $\int \sqrt{e^x} \, dx$ **2**

e) Olivia was asked the following question in class: **2**

"In a bag there are 3 green marbles, 6 yellow marbles and 4 blue marbles.
What is the probability of choosing, at random, a blue marble?"

Olivia answered: "Because there are 3 colours the probability of choosing a blue marble is $\frac{1}{3}$"

Is Olivia correct? Explain

Marks

Question 2 (13 Marks)

a) Find the equation of the tangent to the curve $y = e^{2x}$ at the point where $x = 1$. **3**

b) Consider $y = e^x + e^{-x}$

i) Find $\frac{d^2y}{dx^2}$ **1**

ii) Using your result from part i) explain why the graph of $y = e^x + e^{-x}$ is always concave up. **1**

c) Solve for x: **3**

$$2\log x = \log(x+6)$$

d) Andrew catches the "All stations to Cronulla" train to work each day. After travelling on the train for 2 years Andrew has established that the probability his train will arrive late at Cronulla is 0.7.

i) What is the probability that his train arrives on time? **1**

ii) Calculate the probability that his train arrives late 3 days in a row. **1**

iii) Find the number of days Andrew would have to travel on the train for him to have a 90% chance that his train will arrive on time at least once. **3**

Marks

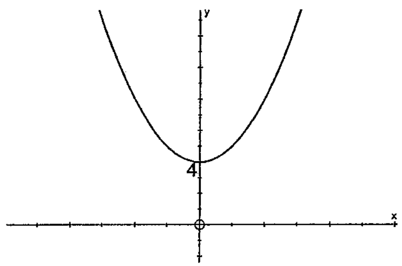
Question 3 (12 Marks)

a) i) On the answer sheet provided is a graph of $y = \cos x$. On the same axes graph $y = \sin 2x$ for $0 \le x \le 2\pi$ — **2**

ii) The first two points of intersection after the origin are at $x = \frac{\pi}{6}$ and $x = \frac{\pi}{2}$. — **3**

Find the area bounded by the curves $y = \cos x$ and $y = \sin 2x$ between these points of intersection.

b)



i) Copy the sketch of $y = x^2 + 4$ onto your paper.

ii) Find the points of intersection between $y = x^2 + 4$ and $y = x + 4$ — **1**

iii) Draw the line $y = x + 4$ on your diagram showing points of intersection and shade the area between $y = x^2 + 4$ and $y = x + 4$ — **2**

iv) Show that the volume of the solid generated by this area rotated around the y – axis can be found by the integral: — **2**

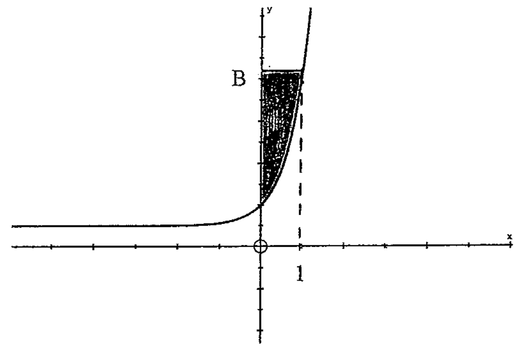
$$V = \pi \int_4^5 9y - 20 - y^2 \, dy$$

v) Find the volume in terms of π — **2**

Marks

Question 4 (13 Marks)

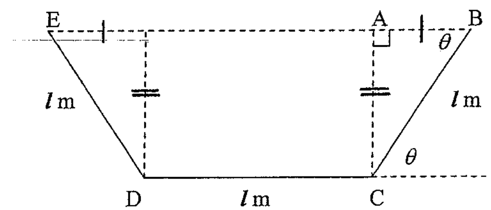
a) The diagram below shows the graph of $y = e^{2x} + 1$



i) Find the coordinate of B — **1**

ii) Find the area of the shaded region — **3**

b) An irrigation channel has a cross section in the shape of a trapezium as in the accompanying figure. The bottom and sides are l metres long. Suppose that the sides of the channel make an angle of $\theta \le \frac{\pi}{2}$ to the horizontal



i) Find an expression for AB and AC in terms of l and θ — **2**

Question 4 continues on the next page

Question 4 continues on the next page

Marks

Question 4 continued

ii) Hence show that the area of the cross section is given by: **2**

$$A = l^2 \sin\theta(1+\cos\theta)$$

iii) Show that $\dfrac{dA}{d\theta} = l^2(2\cos^2\theta + \cos\theta - 1)$ **2**

iv) For what angle θ is the area of the cross section a maximum? **3**

End Of Paper

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LEE

13/13

Rees: 2
Com: 2
Calc: 7

Question 1

(a) 200°

$= \frac{200}{180} \times \pi$

$= \frac{10\pi}{9}$

(b) $\log_a 2 = 0.356 \quad \log_a 5 = 0.827$

i. $\log_a \frac{2}{5} = \log_a 2 - \log_a 5$

$= 0.356 - 0.827$

$= -0.471$

ii. $\log_a 20a = \log_a a + \log_a 20$

$= \log_a a + \log_a 2 + \log_a 2 + \log_a 5$

$= 1 + 0.356 + 0.356 + 0.827$

$= 2.539$

(c)

i. $y = \frac{\tan x}{x}$

$u = \tan x \quad v = x$

$u' = \sec^2 x \quad v' = 1$

$y' = \frac{x.\sec^2 x - \tan x}{x^2}$

$= \frac{x \sec^2 x - \tan x}{x^2}$

ii. $y = \cos^2 x$

$= (\cos x)^2$

$= \cancel{2(\ldots\sin x)}\ 2(\cos x) \times -\sin x$

$= -2 \sin x \cos x$

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(d)

i. $\int \sec^2 3x \, dx$

$= \frac{1}{3} \tan 3x + C$

ii. $\int \sqrt{e^x} \, dx$

$= \int (e^x)^{\frac{1}{2}} dx$

$= \int e^{\frac{1}{2}x} dx$

$= \frac{1}{\frac{1}{2}} e^{\frac{1}{2}x} + C$

$= 2\sqrt{e^x} + C$

(e)

3 Green
6 yellow
4 blue

No, Olivia is incorrect because although there are three colours of marbles, there are an uneven amount of each coloured marble. In total ther are 13 marbles, therefore the probability of choosing a blue marble is $\frac{4}{13}$.

11/13

Calc 4
com 1
reas 4

Question 2

(a) $y = e^{2x}$

$y' = 2e^{2x}$

at $x=1$.

$y' = 2e^2$

$m = 2e^2$

when $x=1$, $y=e^2$

$y - e^2 = 2e^2(x-1)$

$y - e^2 = 2xe^2 - 2e^2$

$2xe^2 - y - 2e^2 + e^2 = 0$

$2e^2x - y - e^2 = 0$

(b) $y = e^x + e^{-x}$

i. $\frac{dy}{dx} = e^x - e^{-x}$

$\frac{d^2y}{dx^2} = e^x + e^{-x}$

ii. The graph of $y = e^x + e^{-x}$ is always concave up because the 2^{nd} derivative $\left(\frac{d^2y}{dx^2} = e^x + e^{-x}\right)$ is always positive, which means the graph is concave up.

(c) $2\log x = \log(x+6)$

$\log x^2 = \log(x+6)$

$x^2 = x+6$

$x^2 - x - 6 = 0$ $\quad \begin{matrix} x & -6 \\ + & -1 \end{matrix}$ $-3, 2$

$(x-3)(x+2) = 0$

$x=3$ and/or $x=-2$

$\therefore x = 3$

$x=-2$ cannot be a solution ~~since log~~ because you cannot have a negative log i.e. $2\log(-2)$ is undefined.

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(d)

i. P(train arriving on time) $= 1 - 0.7$
$= 0.3$

ii. P(arriving late) $= 0.7 \times 0.7 \times 0.7$
$= 0.343$

iii. P(arriving on time) after ~~n~~ days $= (0.7)^n$

$1 - (0.7)^n > 0.9$

~~$\log 0.7^n > \log 0.9$~~

~~$n \log 0.7 > \log 0.9$~~

~~$n > \frac{\log 0.9}{\log 0.7}$~~

$n > 0.295 \times 100$

~~$n > 29.54$~~

~~∴ After 30 days Andrew will have a 90% chance that his train will arrive on time at least once.~~

∴ After one day.

$1 - 0.7^n > 0.9$

$0.7^n < 0.1$

$n \ln 0.7 < \ln 0.1$

$\therefore n > \frac{\ln 0.1}{\ln 0.7} \doteqdot 6.5$

$(\because \ln 0.7 < 0)$

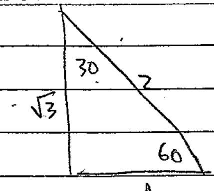
$n = 7$ days.

$\frac{10}{12}$ | com 5, Reas 2, Cal 4 | 19863435

Question 3

(a)

ii. $A = \int_{\pi/6}^{\pi/2} \sin 2x - \int_{\pi/6}^{\pi/2} \cos x \, dx$



$= \left[-\frac{1}{2}\cos 2x - \sin x \right]_{\pi/6}^{\pi/2}$

$= \left(-\frac{1}{2}\cos\pi - \sin\frac{\pi}{2}\right) - \left(-\frac{1}{2}\cos\frac{\pi}{3} - \sin\frac{\pi}{6}\right)$

$= \left(-\frac{1}{2}\times -1 - 1\right) - \left(-\frac{1}{2}\times\frac{1}{2} - \frac{1}{2}\right)$

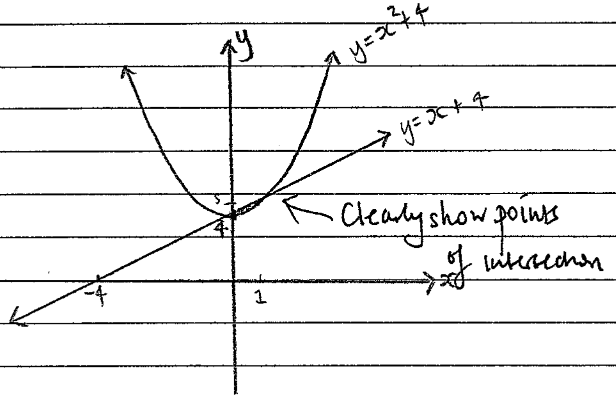
$= -\frac{1}{2} + \frac{3}{4}$

$= 1\frac{1}{4}$ units^2 × Added incorrectly

a) com 2, Calc 2 (4)

b) com 1, reas 2, calc 2 (6)

(b)



Clearly show points of intersection

ii. $x^2+4 = x+4$

$x^2 - x = 0$

$x(x-1) = 0$

$x = 0$ and $x = 1$

$y = 4$ $y = 5$

Write as an ordered pair

$y = 0$, $x+4 = 0$

$x = -4$

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iv. $V = \pi \int_4^5 y^2 - 8y + 16 - (y-4)\, dy$ $y = x^2+4$

$y - 4 = x^2$

$= \pi \int_4^5 y^2 - 9y + 20 \, dy$

$y = x+4$

$y - 4 = x$

$x^2 = (y-4)^2$

$= y^2 - 8y + 16$

$V = \pi \int_4^5 (y-4) - (y^2-8y+16)\, dy$

$= \pi \int_4^5 y - 4 - y^2 + 8y - 16 \, dy$

$y - 4 - (y^2 - 8y + 16)$

$y - 4 - y^2 + 8y - 16$

$= 9y - 20 - y^2$

$= \pi \int_4^5 9y - 20 - y^2 \, dy$

v. $V = \pi \left[\frac{9y^2}{2} - 20y - \frac{y^3}{3}\right]_4^5$

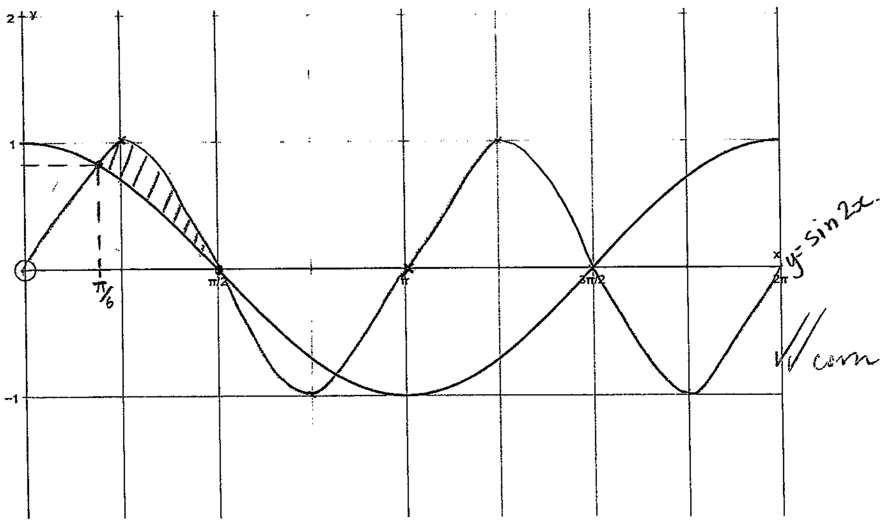
$= \pi \left[\left(112\frac{1}{2} - 100 - 41\frac{2}{3}\right) - \left(72 - 80 - 21\frac{1}{3}\right)\right]$

$= \pi\left(-29\frac{1}{6} + \left(29\frac{1}{3}\right)\right)$

$= \pi \times \frac{1}{6}$

$V = \frac{\pi}{6}$ units^3

Question 3 a) – Answer Sheet Student Number 1986 3433



$y = \sin 2x$

period $= \frac{2\pi}{2}$

$= \pi$

11/13 R 7/5 Calc 7/8 1986 3433

Question 4

(a) $y = e^{2x} + 1$

when $x = 1$

$y = e^2 + 1$

$= 8.39$

i. B (0, 8.39) × B lies on the y axis.

zero

ii. $\cancel{A = \int_2^{8.39} e^{2x} + 1 \, dx}$

$A = \int_0^1 e^{2x} + 1 \, dx$

$= \left[\frac{1}{2}e^{2x} + x\right]_0^1$

$= \left(\frac{1}{2}e^2 + 1\right) - \left(\frac{1}{2}\right)$

$= \frac{1}{2}e^2 + \frac{1}{2}$

$= \frac{1}{2}(e^2 + 1)$ ✓

$y = e^{2x} + 1$

$(y-1) = e^{2x}$

$\log_e(y-1) = 2x$

$x = \frac{\ln(y-1)}{2}$

Area = area of rectangle minus area below curve.

Area of rectangle $= 1 \times (e^2 + 1)$

$= e^2 + 1$

Area $= (e^2+1) - \left(\frac{1}{2}e^2 + \frac{1}{2}\right)$ ✓

$= e^2 - \frac{1}{2}e^2 + \frac{1}{2}$

$= e^2 - \frac{e^2}{2} + \frac{1}{2}$ ✓

$= \frac{1}{2}e^2 + \frac{1}{2}$

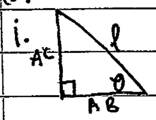
$= \frac{1}{2}(e^2+1)$ units^2

$\doteqdot 4.19$ units^2.

Calc 3

(b)

i.



$\cos\theta = \frac{AB}{l}$

$AB = l\cos\theta$ ✓

$\sin\theta = \frac{AC}{l}$

R2

$AC = l\sin\theta$ ✓

ii. Area of trapezium $\qquad EB = l + 2(l\cos\theta)$

$= \frac{1}{2}(a+b)h$

$= \frac{1}{2}(l + l + 2(l\cos\theta)) \times l\sin\theta$. ✓

$= \frac{1}{2}(2l + 2l\cos\theta)\, l\sin\theta$

$= \cancel{\frac{1}{2}\, l\sin\theta\,(2l}$

$\frac{1}{2} \times (2l^2\sin\theta + 2l^2\cos\theta\,\sin\theta)$

no bracket.

$= \frac{1}{2} \times 2l^2(\sin\theta + \cos\theta\, \sin\theta)$

$= \cancel{l^2(\sin\theta + \cos\theta)}$

$A = l^2\sin\theta(1+\cos\theta)$ # ✓

R2

iii. $\cancel{\frac{dA}{d\theta}}$ $A = l^2\sin\theta(1+\cos\theta)$

$u = l^2\sin\theta$

$u' = l^2\cos\theta$

$\frac{dA}{d\theta} = l^2\sin\theta \times -\sin\theta + (1+\cos\theta) \times l^2\cos\theta$ ✓

bracket please

$v = 1 + \cos\theta$

$v' = -\sin\theta$.

$= \cancel{l^2\sin\theta}$

$-l^2\sin^2\theta + l^2\cos\theta + l^2\cos^2\theta$

$= l^2(-\sin^2\theta + \cos\theta + \cos^2\theta)$

$1 - \sin^2\theta = \cos^2\theta$.

$= l^2(\cos^2\theta - 1 + \cos\theta + \cos^2\theta)$ ✓

$-\sin^2\theta = \cos^2\theta - 1$

$= l^2(2\cos^2\theta + \cos\theta - 1)$ #

Calc 2

iv. find t.p.

$l^2(2\cos^2\theta + \cos\theta - 1) = 0$

$2\cos^2\theta + \cos\theta - 1 = 0$

$\cancel{\cos\theta(2\cos\theta + 1) = 1}$

$\cancel{2\cos\theta + 1 = \frac{1}{\cos\theta}}$

$\cancel{\cos\theta = \frac{\sec\theta + 1}{2}}$

let $\cos\theta = x$

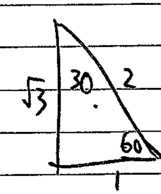
$2x^2 + x - 1 = 0 \qquad \begin{matrix} x & -2 \\ + & 1 \end{matrix} \qquad -1;\ 2$

$(2x-1)(x+1) = 0$

$2x - 1 = 0 \qquad x + 1 = 0$

$2x = 1 \qquad x = -1$

$x = \frac{1}{2} \qquad \cos\theta = -1$ ✓



$\cos\theta = \frac{1}{2}$ $\qquad \theta = \pi$ ← cannot be solution because $\theta \le \frac{\pi}{2}$.

$\cos 60 = \frac{1}{2}$ ✓

$\cos\frac{\pi}{3} = \frac{1}{2}$

$\boxed{\theta = \frac{\pi}{3}}$ $\qquad$ use 2nd derivative to test.

but $\frac{d^2A}{d\theta^2} = \ell^2(-4\cos\theta\sin\theta - \sin\theta)$

$2x^2 + x - 1$

$y' = 4x + 1$

$\frac{d^2A}{d\theta^2} = \ell^2(4\cos\theta + 1)$

$u = \ell^2 \quad v = 2\cos^2\theta + \cos\theta - 1$

$u' = 0 \quad v' = 4\cos\theta + 1$

Incorrect second derivative.

at $\theta = \pi/3$

$\frac{d^2A}{d\theta^2} = -ve \Rightarrow$ maximum t.p.

$\theta = \pi/3$.

Calc 2

$\frac{dA}{d\theta} = \ell^2(2\cos^2\theta + \cos\theta - 1)$

$\frac{d^2A}{d\theta^2} = \ell^2(4\cos\theta . (-\sin\theta) - \sin\theta)$

$= -\ell^2(4\cos\theta\sin\theta + \sin\theta)$

< 0 for $\theta = \frac{\pi}{3}$ $\therefore$ maximum value.