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# SCEGGS Darlinghurst

Assessment Task 3
Wednesday $10^{th}$ June, 2009

## Mathematics

- Outcomes: P2-P8, H1- H9
- Weighting 25%

General Instructions

- Time allowed – 75 minutes
- This paper has 4 questions
- Answer on the pad paper provided
- Start each question on a new page
- Write your student number at the top of each page
- Attempt all questions and show all necessary working
- Marks may be deducted for careless or badly arranged work
- Approved calculators, mathematical templates and geometrical equipment may be used
- A table of standard integrals is provided at the back of this paper

| Question | Calculus | Communication | Reasoning | Marks |
|---|---|---|---|---|
| 1 | /5 | | /2 | /13 |
| 2 | /6 | /3 | | /12 |
| 3 | /2 | | /8 | /13 |
| 4 | /8 | /4 | | /12 |
| Total | /21 | /7 | /10 | /50 |

23

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**Start a new page**

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**Question 1 (13 marks)**

(a) Convert $\frac{8\pi}{15}$ radians to degrees — 1

(b) Evaluate $\log_3 11$ correct to 2 decimal places — 1

(c) Differentiate:

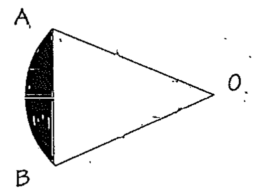
i) $\frac{2}{e^x}$ — 1

ii) $2x^2 \ln x$ — 2

iii) $\frac{\tan 3x}{3x+2}$ — 2

(d) Given $\log_3 2 \approx 0.63$ and $\log_3 7 \approx 1.77$, find $\log_3 56$ — 2

(e) In the figure below OA and OB are radii of a circle with centre O. They both measure 10cm in length. The arc AB subtends an angle of $\frac{\pi}{3}$ radians at O. AB is a chord of the circle.




i) Calculate the exact area of the sector AOB — 2

ii) Calculate the exact area of the triangle AOB and hence find the area of the shaded segment of the circle. — 2

**Start a new page**

**Question 2 (12 marks)**

(a) Solve: $e^{x-2} - 1 = 0$ — 1

(b) i) Sketch the curve $y = 2\cos x$ for $0 \le x \le \pi$ — 2

ii) On the diagram for part (i) shade the region enclosed by the curve $y = 2\cos x$ and the $x$ axis from $x = 0$ to $x = \pi$ — 1

iii) Find the area of the shaded region in part (ii) — 2

(c) Find:

i) $\int (2\sin x - \sec^2 2x)\, dx$ — 2

ii) $\int_0^2 e^{\frac{x}{2} - 1}\, dx$ — 2

iii) $\int_1^4 \frac{x}{x^2 + 4}\, dx$ — 2

**Start a new page**

**Question 3 (13 marks)**

(a) Fully Simplify — 2

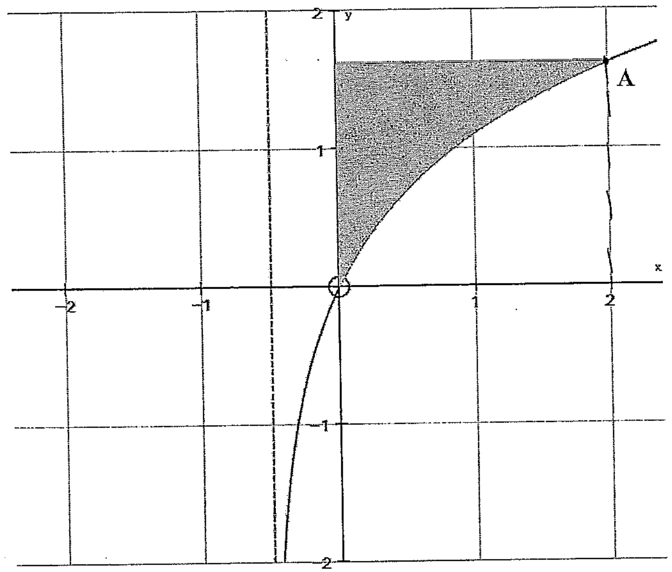
$$\log_{\frac{1}{a}} a^2 - \log_{\frac{1}{a}} a^3$$

(b) Find the equation of the tangent to the curve $y = e^{2x}$ at the point on the curve where $x = 1$ (leave your answer in terms of $e$) — 2

(c) Solve: $2\ln x = \ln(2x + 3)$ — 3

**Question 3 continues on the next page**

(d) The graph below shows the function $y=\ln(2x+1)$




i) Find the coordinates of point A, shown on the diagram above. 1

ii) Show that the area of the shaded region is given by $\int_0^{\ln 5} \frac{e^y-1}{2}\,dy$ 2

iii) Show that $\int_0^{\ln 5} \frac{e^y-1}{2}\,dy$ has exact value $2-\frac{\ln 5}{2}$ 2

iv) Hence find the exact value of $\int_0^2 \ln(2x+1)\,dx$ 1

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**Start a new page**

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**Question 4 (12 marks)**

(a) Find the volume generated when the area between the curve $y=\frac{1}{\sqrt{x+1}}$, the x-axis and the lines at $x=0$ and $x=4$ is rotated about the x-axis, leaving your answer in exact form 3

(b) Differentiate $y=x\ln x$ and hence find $\int \ln x\,dx$ 2

(c) Consider the function $y=xe^{-x}$

i) Show that $\frac{dy}{dx}=e^{-x}(1-x)$, hence show that there is a maximum turning point at $\left(1,\frac{1}{e}\right)$ 3

ii) Describe the behaviour of the function as $x$: 2

a) approaches very large positive values
b) approaches very large negative values

iii) Sketch the curve of this function, clearly showing all important features 2

**End of paper**

# Year 12 Assessment task 3 June 2009 - Solutions

## Question One (13 marks)

a) $96^\circ$ ✓

b) 2.18 ✓

c) i- $2e^{-x}$ $\quad \frac{d}{dx} = -2e^{-x}$ ✓ — Calc 5

ii- $2x^2 \ln x \quad u = 2x^2 \quad v = \ln x$

$u' = 4x \quad v' = \frac{1}{x}$

$\frac{d}{dx} = 4x\ln x + 2x$ ✓

iii- $\frac{\tan 3x}{3x+2} \quad u = \tan 3x \quad v = 3x+2$

$u' = 3\sec^2 3x \quad v' = 3$

$\frac{d}{dx} = \frac{3\sec^2 3x(3x+2) - 3\tan 3x}{(3x+2)^2}$ ✓✓

Please remember you can't cancel either side of a plus or minus sign!

d) $\log_3 56 = \log_3(7 \times 2^3)$ ✓ — Reas 2

$= \log_3 7 + 3\log_3 2$

$= 1.77 + 3 \times 0.63$

$= 3.66$ ✓

Many students need to revise log laws. You are throwing away easy marks!

e) i- $A = \frac{1}{2}r^2\theta$

$= \frac{1}{2} \times 10^2 \times \frac{\pi}{3}$ ✓

$= \frac{50\pi}{3}$ cm² ✓

ii- $A_\triangle = \frac{1}{2} \times 10^2 \times \sin\frac{\pi}{3}$

$= 50 \times \frac{\sqrt{3}}{2}$

$= 25\sqrt{3}$ cm² ✓

* $A$ = area of sector − area of triangle

$= \frac{50\pi}{3} - 25\sqrt{3}$

$= 9.06$ cm² ✓

## Question 2 (12 marks)

a) $e^{x-2} - 1 = 0$

$e^{x-2} = 1$

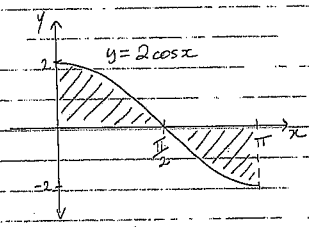
$e^{x-2} = e^0$

$x - 2 = 0$

$x = 2$ ✓

This part was well done by those who did it this way. Less successful for those whose use logs both sides or change of base law

b) i-




Com 3 ✓ Range ✓ Period

ii) ✓ shading

iii- $A = 2 \times \int_0^{\frac{\pi}{2}} 2\cos x \, dx$

$= 2\left[2\sin x\right]_0^{\frac{\pi}{2}}$ ✓

$= 2 \times 2$

$= 4u^2$ ✓

It was surprising to see anyone get this incorrect. Use your page of standard integrals

c) i- $\int (2\sin x - \sec^2 2x)\,dx$ — Calc 6

$= -2\cos x - \frac{\tan 2x}{2}$ ✓ $+ C$

Basic work here. Use the standard integrals. Don't forget +C.

ii- $\int_0^2 e^{\frac{x}{2}-1}\,dx$

$= \left[2e^{\frac{x}{2}-1}\right]_0^2$ ✓

$= \left[2e^0 - 2e^{-1}\right]$

$= 2 - \frac{2}{e}$ ✓

Some mistakes with $\frac{1}{2}e^{\frac{x}{2}-1}$ instead of $\frac{1}{\frac{1}{2}} \to 2e^{\frac{x}{2}-1}$. Check the standard integral page!!!

iii- $\int_1^4 \frac{x}{x^2+4}\,dx = \frac{1}{2}\int_1^4 \frac{2x}{x^2+4}\,dx$

$= \frac{1}{2}\left[\log_e(x^2+4)\right]_1^4$ ✓

$= \frac{1}{2}(\log_e 20 - \log_e 5)$

$= \frac{1}{2}\log_e 4$ ✓

You must show correct use of a log-law to get this mark.

Note this can go further but wasn't marked $= \log_e 4^{\frac{1}{2}}$ $= \log_e 2$

## Question 3 (12 marks)

a) $\log_{\frac{1}{a}}\left(\frac{a^2}{a^3}\right) = \log_{\frac{1}{a}}\left(\frac{1}{a}\right)$ ✓ — Reas 2

$= 1$ ✓

b) $y = e^{2x}, \; x = 1, \; y = e^2$

$y' = 2e^{2x}$ ✓

$m_T = 2e^2$

Eqn: $y - e^2 = 2e^2(x-1)$

$y = 2e^2x - 2e^2 + e^2$

$y = 2e^2x - e^2$

$y = e^2(2x-1)$ ✓

c) $2\ln x = \ln(2x+3), \; x > 0$ — Reas 3

$\ln x^2 = \ln(2x+3)$

$x^2 = 2x+3$ ✓

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = 3, \; x = -1$

but $x = -1$ is not a solution ✓

as $x > 0 \; \therefore x = 3$ ✓

Remember the domain for $\log_a x$

## Question 4 (13 marks)

a) $y = \frac{1}{\sqrt{x+1}}$ — Calc 3

$y^2 = \frac{1}{x+1}$

$V = \pi \int_0^4 \frac{1}{x+1}\, dx$ ✓

$V = \pi \left[\log_e (x+1)\right]_0^4$ ✓

$V = \pi \left(\log_e 5 - \log_e 1\right)$

$V = \pi \log_e 5 \; u^3$ ✓

* Well Done. For the final line some students wrote $\log_e 5\pi$. Please don't move the $\pi$ as this could be confused for $\log_e(5\pi)$ and is not necessary.

b) $y = x \ln x$ $\quad u = x \quad v = \ln x$ — Calc 2

$y' = \ln x + 1$ ✓ $\quad u' = 1 \quad v' = \frac{1}{x}$

* Most achieved the first mark

$\therefore \int \ln x\, dx = \int (\ln x + 1 - 1)\, dx$

$= \int (\ln x + 1)\, dx - \int 1\, dx$

$= x \ln x - x + c$ ✓

* Hence means use your answer from the previous question. If the derivative of $x\ln x$ is $\ln x + 1$ then we are able to find the integral of $\ln x + 1$ and therefore the integral of $\ln x$ using this result.

c) $y = x e^{-x}$ $\quad u = x \quad v = e^{-x}$, $\quad u' = 1 \quad v' = -e^{-x}$

i- $\frac{dy}{dx} = e^{-x} - x e^{-x}$ — Calc 3

$= e^{-x}(1 - x)$ ✓

turning point when $\frac{dy}{dx} = 0$

$0 = e^{-x}(1-x)$

$e^{-x} = 0$ has no solution — * This must be stated

$1 - x = 0$

$x = 1$ is the only solution ✓

at $x = 1$, $y = \frac{1}{e}$ test nature — * Use of table to test $y'$ was also accepted here

$y'' = -e^{-x} - e^{-x} + x e^{-x}$

$= -2e^{-x} + x e^{-x}$

$= e^{-x}(x - 2)$

at $x = 1$

$y'' = e^{-1}(1-2)$

$y'' = -\frac{1}{e}$ ∩ ✓ or table

$\therefore \left(1, \frac{1}{e}\right)$ is a maximum turning point

ii- a) as $x \to \infty$ $\quad e^{-x} \to 0$

$\therefore x e^{-x} \to 0$ ✓ — Com 2

b) as $x \to -\infty$ $\quad e^{-x} \to \infty$

$\therefore x e^{-x} \to -\infty$ ✓

* you must consider which part of the function is most powerful in each case and determine the effect this has.

d) i- $A(2, \ln 5)$ ✓

ii- $y = \ln(2x+1)$ — Res 2

$e^y = 2x + 1$ ✓

$2x = e^y - 1$

$x = \frac{e^y - 1}{2}$ ✓

required region is

$\int_0^{\ln 5} \frac{e^y - 1}{2}\, dy$

iii- $\int_0^{\ln 5} \frac{e^y - 1}{2}\, dy = \int_0^{\ln 5} \left(\frac{e^y}{2} - \frac{1}{2}\right) dy$ — Calc 2

$= \left[\frac{e^y}{2} - \frac{y}{2}\right]_0^{\ln 5}$ ✓ — don't change the variable to $x$

$= \left(\frac{e^{\ln 5}}{2} - \frac{\ln 5}{2}\right) - \left(\frac{e^0}{2} - \frac{0}{2}\right)$

$= \frac{5}{2} - \frac{\ln 5}{2} - \frac{1}{2}$ ✓

$= 2 - \frac{\ln 5}{2}$

ii- $\int_0^2 \log_e(2x+1)\, dx = 2 \times \ln 5 - \int_0^{\ln 5} \frac{e^y - 1}{2}\, dy$ — Res 1

$= 2\ln 5 - \left(2 - \frac{\ln 5}{2}\right)$ — watch these

$= \frac{4\ln 5 - 4 + \ln 5}{2} = \frac{5\ln 5}{2} - 2$ ✓

d) (ii) "Show" requires a reason for changing the subject to $x$.

eg. Area is bounded by the curve and the y axis.

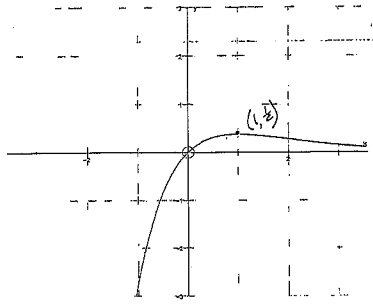
or Area = $\int x\, dy$.

then $e^y = 2x + 1$

$2x = e^y - 1$

$x = \frac{e^y - 1}{2}$ etc.

iii –




Corn 2

- turning point at $(1, \frac{1}{e})$
- goes through origin
- correct shape

√√ all three

√ one mistake

* Must show turning point, passing through the origin and the asymptote as $x \to \infty$ — 2 marks

* One mark off for each mistake

Year 12 Assessment task 3 June, 2009 - Solutions

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Reas 2

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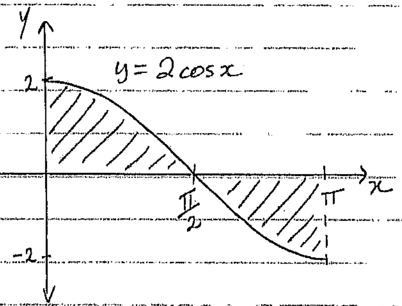


Com 3

✓ Range

✓ Period

ii) ✓ shading

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It was surprising to see anyone get this incorrect. Use your page of standard integrals.

c)

Calc 6

i - $\int (2\sin x - \sec^2 2x)\,dx$

$= -2\cos x - \frac{\tan 2x}{2} + c$ ✓ ✓

Basic work here. Use the standard integrals. Don't forget +c.

ii - $\int_0^2 e^{\frac{x}{2}-1}\,dx$

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$= \frac{1}{2}\left(\log_e 20 - \log_e 5\right)$

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Reas 2

b) $y = e^{2x}$, $x = 1$, $y = e^2$

$y' = 2e^{2x}$ ✓

$m_T = 2e^2$

Eqn: $y - e^2 = 2e^2(x-1)$

$y = 2e^2x - 2e^2 + e^2$

$y = 2e^2x - e^2$

$y = e^2(2x-1)$ ✓

c) $2\ln x = \ln(2x+3)$, $x > 0$

Reas 3

$\ln x^2 = \ln(2x+3)$

$x^2 = 2x+3$ ✓

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = 3$, $x = -1$ ✓

but $x = -1$ is not a solution as $x > 0$, $\therefore x = 3$ ✓

Remember the domain for $\log_a x$

d) (ii) "Show" requires a reason for changing the subject to $x$.

eg. Area is bounded by the curve and the $y$ axis.

or Area $= \int x\,dy$.

then $e^y = 2x+1$

$2x = e^y - 1$

$x = \frac{e^y - 1}{2}$ etc.

d) i – $A(2, \ln 5)$ ✓

ii – $y = \ln(2x+1)$ — Reas 2

$e^y = 2x+1$ ✓

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required region is

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iii – $\int_0^{\ln 5} \frac{e^y - 1}{2}\,dy = \int_0^{\ln 5}\left(\frac{e^y}{2} - \frac{1}{2}\right)dy$ — Calc 2

$= \left[\frac{e^y}{2} - \frac{y}{2}\right]_0^{\ln 5}$ ✓ — don't change the variable to $x$

$= \left[\left(\frac{e^{\ln 5}}{2} - \frac{\ln 5}{2}\right) - \left(\frac{e^0}{2} - \frac{0}{2}\right)\right]$

$= \frac{5}{2} - \frac{\ln 5}{2} - \frac{1}{2}$ ✓

$= 2 - \frac{\ln 5}{2}$

ii – $\int_0^2 \log_e(2x+1)\,dx = 2 \times \ln 5 - \int_0^{\ln 5} \frac{e^y - 1}{2}\,dy$ — Reas 1

$= 2\ln 5 - \left(2 - \frac{\ln 5}{2}\right)$ — watch bracket

$= \frac{4\ln 5 - 4 + \ln 5}{2} = \frac{5\ln 5}{2} - 2$ ✓

## Question 4 (13 marks)

a) $y = \dfrac{1}{\sqrt{x+1}}$ Calc 3.

$y^2 = \dfrac{1}{x+1}$

$V = \pi \int_0^4 \dfrac{1}{x+1}\, dx$ ✓

$V = \pi \left[ \log_e (x+1) \right]_0^4$ ✓

$V = \pi (\log_e 5 - \log_e 1)$

$V = \pi \log_e 5\ u^3$ ✓

* Well Done
For the final line some students wrote $\log_e 5\pi$. Please don't move the $\pi$ as this could be confused for $\log_e(5\pi)$ and is not necessary

b) $y = x \ln x$ $u = x$ $v = \ln x$ Calc 2

$y' = \ln x + 1$ ✓ $u' = 1$ $v' = \frac{1}{x}$

* Most achieved the first mark

$\therefore \int \ln x\, dx = \int (\ln x + 1 - 1)\, dx$

$= \int (\ln x + 1)\, dx - \int 1\, dx$

$= x \ln x - x + c$ ✓

* Hence means use your answer from the previous question
If the derivative of $x \ln x$ is $\ln x + 1$ then we are able to find the integral of $\ln x + 1$ and therefore the integral of $\ln x$ using this result.

c) $y = x e^{-x}$ $u = x$ $v = e^{-x}$
$u' = 1$ $v' = -e^{-x}$

i- $\dfrac{dy}{dx} = e^{-x} - x e^{-x}$ Calc 3

$= e^{-x}(1 - x)$ ✓

turning point when $\dfrac{dy}{dx} = 0$

$0 = e^{-x}(1-x)$

$e^{-x} = 0$ has no solution * This must be stated

$1 - x = 0$

$x = 1$ is the only solution ✓

at $x = 1$, $y = \frac{1}{e}$ test nature

* Use of table to test $y'$ was also accepted here

$y'' = -e^{-x} - e^{-x} + x e^{-x}$

$= -2e^{-x} + x e^{-x}$

$= e^{-x}(x - 2)$

at $x = 1$

$y'' = e^{-1}(1-2)$

$y'' = -\frac{1}{e}$ ∩ ✓ or table

$\therefore (1, \frac{1}{e})$ is a maximum turning point

ii- a) as $x \to \infty$ $e^{-x} \to 0$ Com 2

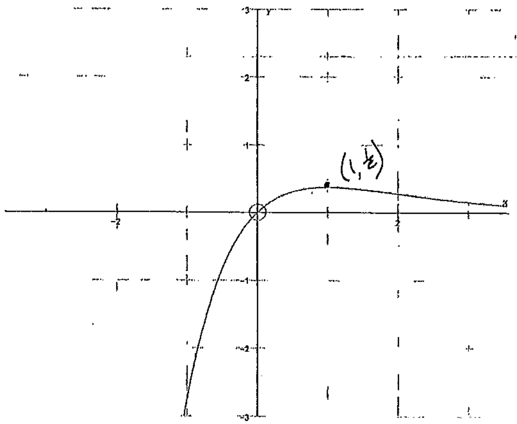
$\therefore x e^{-x} \to 0$ ✓

b) as $x \to -\infty$ $e^{-x} \to \infty$

$\therefore x e^{-x} \to -\infty$ ✓

* you must consider which part of the function is most powerful in each case and determine the effect this has.

iii -




Corr 2

- turning point at $(1, \frac{1}{e})$
- goes through origin
- correct shape

✓✓ all three

✓ one mistake

* Must show turning point, passing through the origin and the asymptote as $x \to \infty$ 2 marks

* One mark off for each mistake