SCEGGS DARLINGHURST

TRIAL EXAMINATION - 2000

Mathematics

Year 12

2/3 Unit

TIME ALLOWED: 3 hours (plus 5 minutes reading time)

This is a TRIAL PAPER only and does not necessarily reflect the content or format of the Higher School Certificate Examination for this subject.

DIRECTIONS TO CANDIDATES:

- Attempt all ten questions. All questions are of equal value..
- Ensure that your student number is on this paper.
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Approved calculators should be used. Mathematical templates and geometrical instruments may be used.
- Begin each question on a new page - write your number at the top of each page.
- The table of standard integrals is printed on the last page.

Question 1. (12 Marks) **Marks**

a) Find, to two significant figures, the value of $\sqrt{\dfrac{3.5^2-4}{4.6-2.5}}$. 2

b) Factorise $4x^2-36$. 2

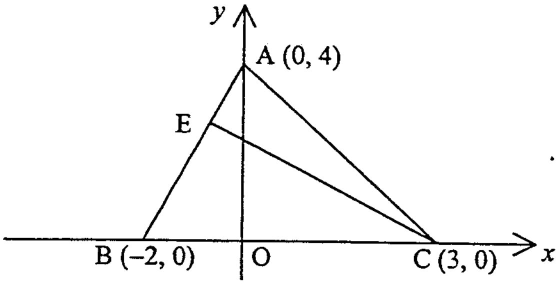
c) Simplify: $3-2(3x+4)+13x$. 2

d) Solve $15x^2=10x$. 2

e) Show the solution to $|x-4|<5$ on a number line. 2

f) Show that the reciprocal of $(\sqrt{3}-\sqrt{2})$ is $(\sqrt{3}+\sqrt{2})$ 2

Question 2. (12 Marks) ***Begin a new page.***



In the figure shown, ABC is a triangle with vertices A (0, 4), B (−2, 0) and C (3, 0). AO and CE are altitudes of the triangle (that is, AO is perpendicular to BC and EC is perpendicular to AB).

Copy the diagram onto your writing paper.

a) State the equation of the altitude AO. 1

b) Show that the equation of EC is $x+2y-3=0$. 2

c) Find the co-ordinates of H, the point of intersection of EC and AO. 1

d) Find the ratio, OH : AH. 1

e) Explain why the equation of the line AB is $y=2x+4$. 2

f) Find the length EC, that is, the perpendicular from C to E. 2

g) Find the area of the triangle ABC using AB as the base and EC as the height. Check your answer by finding the same area another way. 3

Question 3 begins on the next page.

Question 3. (12 Marks) ***Begin a new page.***

Marks

a) Differentiate: 3

i) $\tan(5x-3)$

ii) $\dfrac{2}{3x^2}$

b) i) Rewrite $\log_e\left(\dfrac{2x-1}{4x+1}\right)$ using the Logarithm laws. 3

ii) Hence differentiate $\log_e\left(\dfrac{2x-1}{4x+1}\right)$

c) Find 4

i) $\int(3x+2)^4\,dx$

ii) $\int_0^1 3e^{2x}\,dx$

d) A body is increasing its velocity at a decreasing rate. Draw a sketch of the velocity-time graph for this body. 2

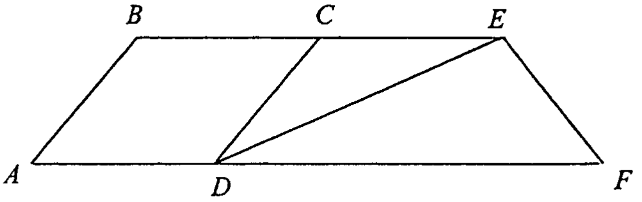
Question 4. (12 marks) ***Begin a new page.***

a) 'When tossing three coins there are four possible results. They are: 2
3 tails, 2 tails and 1 head, 2 heads and 1 tail, 3 heads.

Thus the probability of obtaining 3 tails when tossing three coins is $\frac{1}{4}$.'

Comment on the validity of this statement, justifying your response.

b) In the figure below $ABCD$ is a rhombus. E is a point on BC produced such that C is the midpoint of BE. F is a point on AD produced such that $\angle DEF = 90°$. Also $\angle BAD = 72°$. 4



i) Copy the diagram and clearly show this information.

ii) Find, giving reasons, the size of $\angle CDE$.

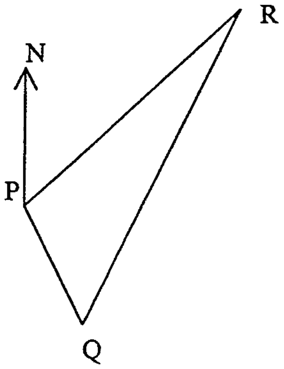
iii) Find, giving reasons, the size of $\angle DFE$.

Question 4 continues on the next page.

Question 4. ***(continued)***

Marks

c) A ship leaves a port P, sailing on a bearing of 170°, and reaches a port Q, 50 kilometres from P. The ship's captain then realises that they should have sailed from P on a bearing of 50° to reach R, 170 km from P. The diagram is shown below. 6

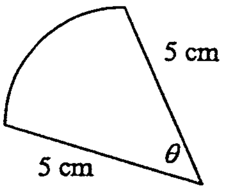


i) Copy the diagram and show on it the lengths of PR and PQ, as well as the size of the angle RPQ.
ii) Calculate the distance from Q to R, giving your answer to the nearest kilometre.
iii) Find $\angle$PQR to the nearest degree, and hence the bearing on which the ship will need to sail so that it reaches R from Q.

Question 5. (12 Marks) *Begin a new page.*

a) A parabola has equation $(x-4)^2 = 8(y+3)$. 4
i) Find the co-ordinates of its focus.
ii) Find the equation of its directrix.
iii) Describe the locus in words.

b) The perimeter of the sector shown below is 18 cm. 4



i) Find the exact value of the angle θ in radians.
ii) Hence find the area of this sector.

c) Consider the equation $x^2 + 6x + c = 0$. 4

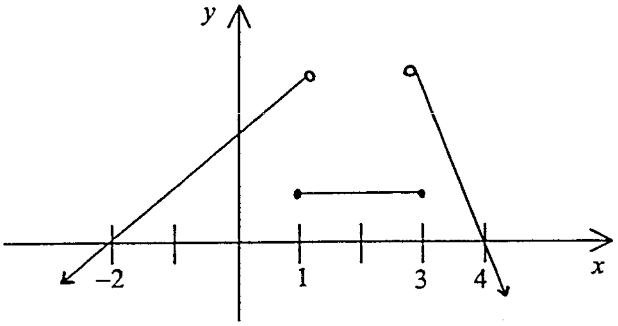
i) Find the range of values of c for which there are no real solutions to this equation.
ii) Explain how you can use your answer in c) part i) to help you determine the values of x for which $x^2 + 6x + 20 > 0$, and what this tells you about the graph of $y = x^2 + 6x + 20$. (You do not need to draw this graph).

Question 6 begins on the next page.

Question 6. (12 Marks) ***Begin a new page.***

Marks

a) The graph of $y = f(x)$ is below. 3



Draw a sketch of $y = f'(x)$ for this function.

b) For the curve $y = x^3 - 3x^2 + 1$; 9

i) Find any stationary points and determine their nature.
ii) Find any points of inflexion.
iii) Find the values of x for which the curve is concave down.
iv) Sketch the curve for $-1 \le x \le 4$.

Question 7. (12 marks) ***Begin a new page.***

a) Solve the inequality $-6 \le 2x \le 5 - 2x$. 2

b) A box contains 12 light bulbs that look the same, but two of them are defective. Two light bulbs are drawn at random without replacement. 4

i) What is the probability that the first light bulb drawn is defective?
ii) What is the probability that, of the two light bulbs drawn, at least one is defective?

c) Explain how you can tell that a function has a horizontal point of inflexion. Give an example of the equation for such a function. 2

d) Given that $\dfrac{dy}{dx} = \dfrac{x}{x^2 - 4}$; 4

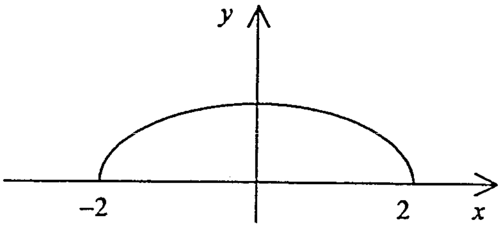
i) Find y in terms of x, given that $y = 0$ when $x = 3$.
ii) State the set of x values for which y exists.

Question 8 begins on the next page.

Question 8. (12 Marks) ***Begin a new page***

Marks

a) The diagram below shows the graph of the curve $y = \frac{1}{2}\sqrt{4 - x^2}$. 6



i) Use Simpson's Rule with five function values to estimate the area enclosed by the curve and the x axis.
ii) Use integration to find the exact value of the volume of the solid of revolution formed when the region enclosed by this curve and the x axis is rotated about the x axis.

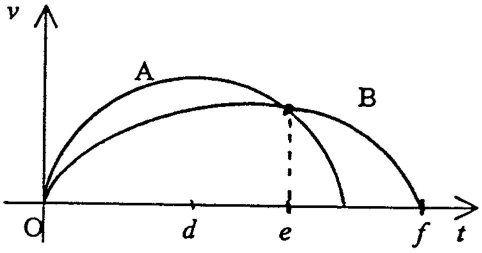
b) The amount in grams of a radioactive substance present after t years is given by $A = 80e^{-0.025t}$. 6

i) Find the amount of radioactive substance present after 10 years.
ii) What is the half-life of this radioactive substance?
iii) At what rate is this substance decaying after 8 years?

Question 9. (12 Marks) ***Begin a new page***

a) i) Draw the graphs of $y = 2\sin x$ and $y = \tan x$, for $0 \le x \le 2\pi$. 6
ii) Use your graphs to determine the number of solutions to $2\sin x = \tan x$ within this domain.
iii) From the equation $2\sin x = \tan x$, show that $\sin x(2\cos x - 1) = 0$.
iv) Find all values of x, where $0 \le x \le 2\pi$, that are solutions to the equation $2\sin x = \tan x$. (Give values in exact form.)

b) Two particles A and B are moving in a straight line. The graphs below show the velocities (in m/s) of each particle at time t seconds. 6



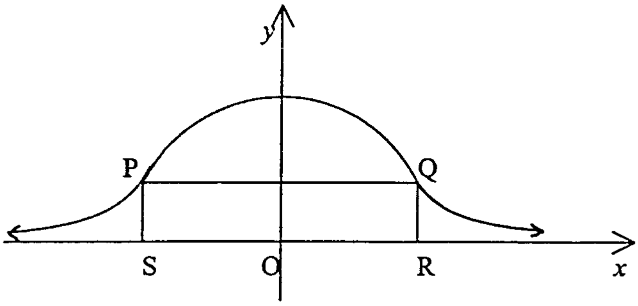
i) Which particle shows the greater acceleration in the first d seconds? Why?
ii) Describe the motion of each particle at time e seconds.
iii) Draw a graph showing the displacement of particle B, as a function of time t, from $t = 0$ to $t = f$.

Question 10 begins on the next page.

Question 10. (12 Marks) *Begin a new page.*

Marks

a) The diagram shows a rectangle PQRS, where P and Q are on the curve $y = e^{-x^2}$ and R and S are on the x axis. The point O is the origin and OS = OR. 5



i) Let the length OR = x. Show that the area of the rectangle PQRS is represented by $A = 2xe^{-x^2}$.

ii) Find the value of x, in exact form, for which PQRS has maximum area.

b) If $f(x) = 2 - \log_e x$, 7

i) Explain why the graph of the curve $y = f(x)$ is always decreasing.

ii) Find the x intercept of $y = f(x)$.

iii) Sketch the graph of $y = f(x)$.

iv) Write down the integral that enables us to find the area enclosed by the curve $y = 2 - \log_e x$ the line $y = 2$ and the x-axis. (You do not have to evaluate this integral).

End of Examination.

1 a) $\sqrt{\frac{3.5^2-4}{4.6-2.5}} \doteqdot 1.98206\ldots$ ✓

$= 2.0$ (to 2 sig figs) ✓

b) $4x^2-36 = 4(x^2-9)$ ✓

$= 4(x-3)(x+3)$ ✓

c) $3-2(3x+4)+13x$

$= 3-6x-8+13x$ ✓

$= 7x-5$ ✓

d) $15x^2=10x$

$15x^2-10x=0$

$5x(3x-2)=0$ ✓

$x=0, \quad 3x-2=0$

$3x=2$

$x=\frac{2}{3}$ ✓

$\therefore$ Solutions are $x=0$, $x=\frac{2}{3}$

e) $|x-4|<5$

$-5<x-4<5$

$-1<x<9$ ✓

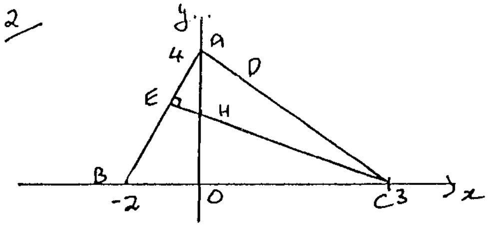
✓

f) $\boxed{=2}$ $\frac{1}{\sqrt3-\sqrt2} = \frac{1}{\sqrt3-\sqrt2}\times\frac{\sqrt3+\sqrt2}{\sqrt3+\sqrt2}$ ✓

$= \frac{\sqrt3+\sqrt2}{3-2}$ ✓

$= \sqrt3+\sqrt2$ //

2



a) AO: $x=0$ ✓

b) $m_{AB} = \frac{0-4}{-2-0} = 2$

$\therefore m_{EC} = -\frac{1}{2}$ ✓

$\therefore y-0 = -\frac{1}{2}(x-3)$

$2y = -x+3$

$x+2y-3=0$ ✓

c) $x=0, \quad 3x-4y+6=0$

$\therefore \quad -4y+6=0$ ✓

$-4y=-6$

$y=\frac{6}{4}=\frac{3}{2}$

$\therefore H=(0,1\frac{1}{2})$

d) $OH:AH = \frac{3}{2}:(4-1\frac{1}{2})$

$= \frac{3}{2}:\frac{5}{2}$ ✓

$= 3:5$

e) $\boxed{C=2}$ $m_{AB} = \frac{4}{2}=2 \rightarrow y=mx+b$ ✓

$\therefore y=2x+b$

$b=4$ (y-intercept) ✓

$\therefore y=2x+4$

f) $(3,0) \quad 2x-y+4=0$

$EC = \left|\frac{2\times3-1\times0+4}{\sqrt{2^2+1^2}}\right|$ ✓

$= \left|\frac{6+4}{\sqrt5}\right|$

$= \frac{10}{\sqrt5} = \frac{10\sqrt5}{5}$ or $2\sqrt5$ ✓

g) $AB = \sqrt{4^2+2^2}$

$\boxed{PS=3}$ $= \sqrt{20} = 2\sqrt5$ ✓

2g) ctd Area $\triangle ABC = \frac{1}{2}\times AB\times EC$

$= \frac{1}{2}\times2\sqrt5\times2\sqrt5$ ✓

$= 10$ units^2

Area $\triangle ABC = \frac{1}{2}\times BC\times AO$

$= \frac{1}{2}\times5\times4$ ✓

$= 10$ units^2

3 a) i) $\frac{d}{dx}\tan(5x-3) = 5\sec^2(5x-3)$ ✓

ii) $\frac{d}{dx}\left(\frac{2}{3x^2}\right) = \frac{d}{dx}\left(\frac{2}{3}x^{-2}\right)$ ✓

$= \frac{2}{3}\times-2x^{-3}$ ✓

$= \frac{-4}{3x^3}$

b) i) $\log_e\frac{(2x-1)}{4x+1} = \log_e(2x-1) - \log_e(4x+1)$ ✓

ii) $\frac{d}{dx}\log_e\left(\frac{2x-1}{4x+1}\right) =$

$\frac{d}{dx}\left\{\log_e(2x-1)-\log_e(4x+1)\right\}$

$= \frac{2}{2x-1} - \frac{4}{4x+1} = \frac{6}{(2x-1)(4x+1)}$ ✓✓

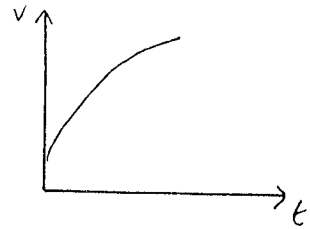
c) i) $\int(3x+2)^4\,dx = \frac{(3x+2)^5}{5\times3}+C$ ✓✓

$= \frac{1}{15}(3x+2)^5+C$

ii) $\int_0^1 3e^{2x}\,dx = 3\left[\frac{e^{2x}}{2}\right]_0^1$ ✓

$= 3\left(\frac{e^2}{2}-\frac{e^0}{2}\right)$ ✓

c)



✓✓ (1- in

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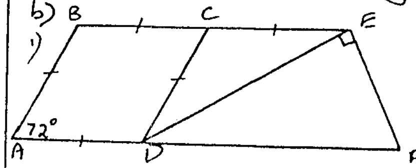
$\boxed{C=2}$

4

a) This statement is not vali since the probability of obtaining each of these out is not the same – the outco are not equally likely. ✓

$\boxed{C=2}$

b) i)



✓

$\boxed{C=($

ii) $\angle BCD = 72°$ (opp ∠s in rhombus equal

$\angle DCE = 180°-72°$ (supplementar

$= 108°$ ✓

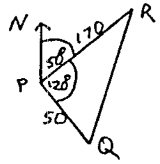
$\therefore \triangle CDE = \frac{1}{2}(180°-108°)$ (∠s in isosceles triangle ✓

$\therefore \angle CDE = 36°$

iii) $\angle DFE = 90°-36°$ (∠ sum o right angled $\triangle DEF$

$= 54°$ ✓

c) i)



✓

ii) $QR^2 = 50^2+170^2-2\times50\times170\cos$ ✓

$= 2500+28900-17000\times$

$= 31400+8500$

$= 39900$ ✓

$QR \doteqdot 199.7498\ldots$

$= 200$ km (to nearest

~ new page

c

1) $\frac{170}{\sin\angle PQR} = \frac{200}{\sin 120^\circ}$ ✓

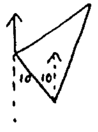
$\sin\angle PQR = \frac{170\sin 120^\circ}{200}$

$= \frac{17}{20} \times \frac{\sqrt{3}}{2}$

$= 0.7361\ldots$ ✓

$\boxed{=3}$ $\angle PQR = 47^\circ 24'$

$= 47^\circ$ (to nearest degree)



$\therefore$ Bearing $= 037^\circ$ ✓

a) 1) $(x-4)^2 = 8(y+3)$

Vertex $= (4,-3)$

$4a = 8$

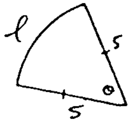
$a = 2$ ✓

$\therefore$ Focus $= (4,-1)$ ✓

ii) directrix $y = -5$ ✓

iii) This is the locus of all points that are the same distance from $(4,-1)$ as they are from $y=-5$ ✓

b)



1) $l = r\theta = 5\theta$ ✓

Perimeter

$= 5+5+5\theta$

$\therefore 10 + 5\theta = 18$ ✓

$5\theta = 8$

$\theta = \frac{8}{5}^c$ ✓

ii) Area $= \frac{1}{2} 5^2 \times \frac{8}{5}$ ✓

$= \frac{1}{2} \times 25 \times \frac{8}{5}$

$= 20 \text{ cm}^2$

c) $x^2 + 6x + c = 0$

1) $\Delta = b^2 - 4ac$

$= 36 - 4 \times 1 \times c$

$= 36 - 4c$

No real solns, $\Delta < 0$ ✓

$\therefore 36 - 4c < 0$

$36 < 4c$ ✓

$c > 9$

ii) For $x^2+6x+20$, this quadratic expression has the same form $\boxed{c=2}$ as x^2+6x+c, and $c>9$, thus there are no real solutions to $x^2+6x+20=0$.

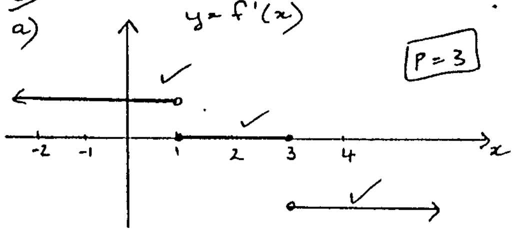
Also the parabola is upright since $a>0$ ✓

$\therefore x^2+6x+20 > 0$ for all x.

The graph of $y = x^2+6x+20$ is thus completely above the x axis and upright. ✓

6

a)



$\boxed{P=3}$

b) $y = x^3 - 3x^2 + 1$

1) $y' = 3x^2 - 6x$

$= 3x(x-2)$

For stat pts $y' = 0$

ie $3x(x-2) = 0$ ✓

$x = 0, x = 2$ ✓

$y'' = 6x - 6$

when $x = 0$, $y'' = -6$ $\therefore$ max ✓

when $x = 2$, $y'' = 6$ $\therefore$ min

y values, when $x = 0$, $y = 1$

when $x = 2$, $y = 8 - 12 + 1$ ✓

$= -3$

$\therefore (0,1)$ is a max turning pt

$(2,-3)$ is a min turning pt

6 b) ii) Pts of inflexion, $y'' = 0$

$6x - 6 = 0$

$x = 1$ ✓

x	1^-	1	1^+
y''	$-$	0	$+$

✓

when $x = 1$, $y = 1 - 3 + 1$

$= -1$

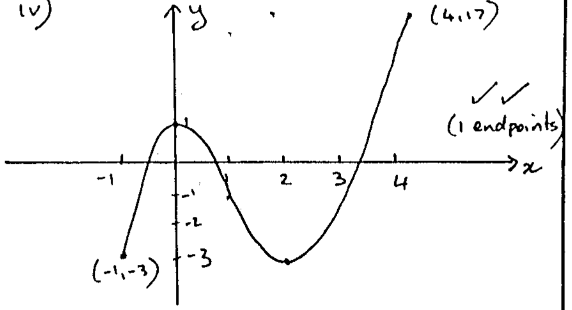
$\therefore (1,-1)$ is a pt of inflexion

iii) Concave down, $y'' < 0$

$6x - 6 < 0$

$x < 1$ ✓

iv)



✓✓ (1 endpoints)

$x = -1$, $y = -1 - 3 + 1$

$= -3$

$x = 4$, $y = 64 - 48 + 1$

$= 17$

7 a) $-6 \le 2x \le 5 - 2x$

$-6 \le 2x$, $\quad 2x \le 5 - 2x$ ✓

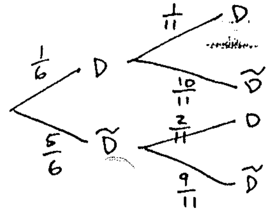
$x \ge -3$ $\quad 4x \le 5$

$x \le \frac{5}{4}$

$\therefore -3 \le x \le \frac{5}{4}$ ✓

b) 1) $P(D) = \frac{2}{12} = \frac{1}{6}$ (1st draw) ✓

ii)



✓

$P(\text{at least } 1D) = 1 - P(\tilde{D}\tilde{D})$

$= 1 - \frac{5}{6} \times \frac{9}{11}$ ✓

$= 1 - \frac{15}{22}$

$= \frac{7}{22}$ ✓

c) For horizontal pt of inflex

$f'(x) = 0$ and $f''(x) = 0$

An example of a functio that has a horiz pt of inflex is $f(x) = x^3$ ✓

$\boxed{C=2}$

d) 1) $\frac{dy}{dx} = \frac{x}{x^2-4}$

$y = \int \frac{x}{x^2-4}\,dx$ ✓

$= \frac{1}{2}\int \frac{2x}{x^2-4}\,dx$

$= \frac{1}{2}\ln(x^2-4) + ($

$y = 0$ when $x = 3$

$\therefore 0 = \frac{1}{2}\ln(9-4) + C$

$0 = \frac{1}{2}\ln 5 + C$

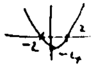
$C = -\frac{1}{2}\ln 5$ ✓

$\therefore y = \frac{1}{2}\ln(x^2-4) - \frac{1}{2}$

ii) $x^2 - 4 > 0$

$\therefore x^2 > 4$

ie $x < -2$, $x > 2$



8 a) $y = \frac{1}{2}\sqrt{4-x^2}$

1)

x	-2	-1	0	1	2
y	0	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	0

Area $\doteqdot \frac{b-a}{6}\left\{f(a) + 4f\left(\frac{a+b}{2}\right)\right.$

Since symmetrical,

Area $\doteqdot 2 \times \frac{2-0}{}\left\{f(0) + 4f(1) +\right.$