SCEGGS Darlinghurst

2012
Year 10
Semester 2 Examination

Mathematics (Pathway 5.3E)

Task Weighting: 40%
Outcomes tested: MS 5.3.2, NS 5.1.3, NS 5.3.1, NS 5.3.2, PAS 5.1.2, PAS 5.2.2, PAS 5.2.3, PAS 5.3.2, PAS 5.3.3, PAS 5.3.4, PAS 5.3.5, PAS 5.3.6, WMS 5.3.1, WMS 5.3.2, WMS 5.3.3, WMS 5.3.4, WMS 5.3.5, DS 5.2.1

General Instructions

- Working time – 2 hours
- Write using black or blue pen
- This paper has **two** sections
- Answer Section I on the Multiple Choice Answer Sheet provided
- Answer each of the four questions in Section II in a **separate** writing booklet
- Attempt **all** questions and show all necessary working
- Marks will be deducted for careless or badly arranged work
- Mathematical templates, geometrical equipment and scientific calculators may be used

Total marks – 67

Section I

7 marks
- Attempt all parts of Section I
- Allow about 15 minutes for this section

Section II

60 marks
- Attempt Questions 8 – 11
- Allow about 1 hour and 45 minutes for this section

Section I -- Multiple Choice

7 marks
Attempt Questions 1–7
Allow about 15 minutes for this section

Use the multiple-choice answer sheet provided.

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word ***correct*** and drawing an arrow as follows.

1 The gradient of the line $3x-4y+5=0$ is

(A) 3

(B) $\frac{3}{4}$

(C) $-\frac{3}{4}$

(D) $\frac{4}{3}$

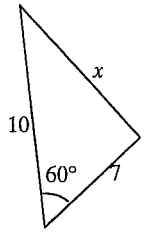
2 Which equation should be used to obtain the value of x in this triangle?

(A) $\frac{x}{\sin 60^\circ}=\frac{7}{\sin 70^\circ}$

(B) $x^2=10^2+7^2-2\times10\times7\cos60^\circ$

(C) $\cos60^\circ=\frac{x^2+10^2-7^2}{2\times10\times7}$

(D) $x^2=10^2-7^2$



NOT TO SCALE

3 What is the formula for q as the subject of $4p=5t+2q^2$?

(A) $q=\pm\sqrt{\frac{4p-5t}{2}}$

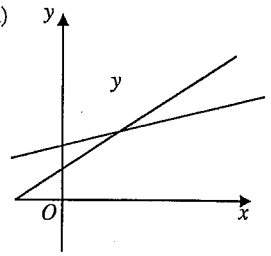
(B) $q=\pm\frac{\sqrt{4p-5t}}{2}$

(C) $q=\pm\sqrt{\frac{5t-4p}{2}}$

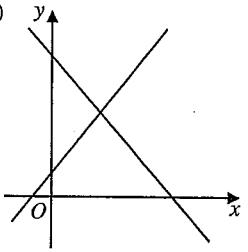
(D) $q=\pm\frac{\sqrt{5t-4p}}{2}$

4 George drew a correct diagram that gave the solution to the simultaneous equations $y=2x-5$ and $y=x+6$. Which diagram did he draw?

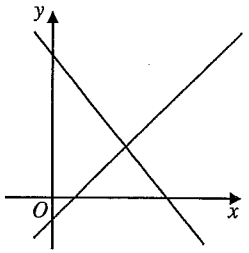
(A)



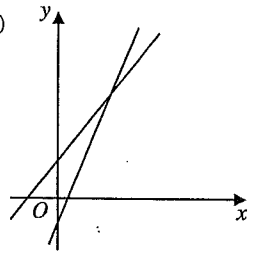
(B)



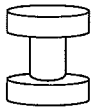
(C)



(D)

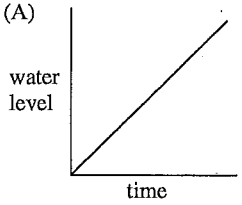


5 Water is poured into the following container at a constant rate.

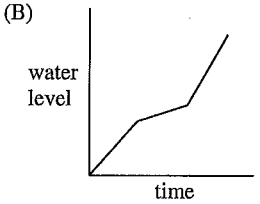


Which graph shows the height of the water level in the container as the water is being poured into the container?

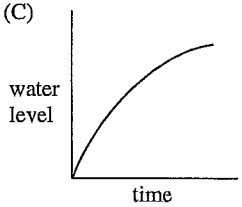
(A)



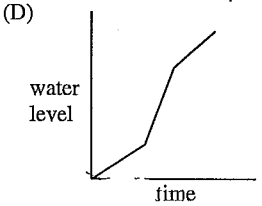
(B)



(C)



(D)



6 In a guessing competition, a jar of jellybeans contains x green jellybeans and y red jellybeans.
The probability of selecting a green jellybean at random from the jar is

(A) $\frac{y}{x-y}$

(B) $\frac{x}{y}$

(C) $\frac{x}{x+y}$

(D) $\frac{x}{xy}$

7 This set of data is arranged in order from smallest to largest.

5, 6, 11, x, 13, 18, 25

The range is six less than twice the value of x.

Which one of the following is true?

(A) The median is 13 and the interquartile range is 12.

(B) The median is 12 and the interquartile range is 12.

(C) The median is 13 and the interquartile range is 7.

(D) The median is 12 and the interquartile range is 7.

End of Section I

Section II

60 marks
Attempt Questions 8 – 11
Allow about 1 hour and 45 minutes for this section

Answer each question in the writing booklet provided. Extra writing booklets are available.
Start each question in a **new** booklet. Write your name on each writing booklet

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Question 8 (15 marks)

(a) Simply fully $\sqrt{28} + \sqrt{63}$ **1**

(b) Solve for x

(i) $\frac{x+3}{5} - \frac{x+1}{4} = 3$ **2**

(ii) $x^2 - 5x = 6$ **2**

(c) For the parabola $y = 2x^2 - 5x + 2$, find:

(i) the equation of the axis of symmetry **1**

(ii) the coordinates of the vertex **1**

(iii) the x-intercepts **2**

(iv) draw a neat sketch ($\frac{1}{3}$ of a page) showing all important features. **2**

Question 8 continues on the next page

(d) A bag of lollies contains 8 jellybabies, 5 snakes and 3 minties. Georgia selects two lollies at random from the bag.
Find the probability that the lollies selected are

(i) both snakes **1**

(ii) a jellybaby and a snake **2**

(iii) not minties **1**

End of Question 8

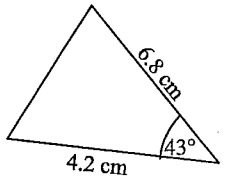
▪ **Start a new booklet**

Question 9 (15 marks)

(a) Rationalise the denominator and simplify **2**

$$\frac{10}{\sqrt{7}+\sqrt{2}}$$

(b) Find the area of the triangle correct to one decimal place. **2**



(c) Solve $3x^2 + 2x - 10 = 0$. **3**
Give your answers correct to two decimal places.

(d) Sketch the graphs of the following, showing all important features

(i) $y = x^3$ **1**

(ii) $y = (x-2)^2 + 3$ **2**

(iii) $y = 2^x + 1$ **2**

Question 9 continues on the next page

Question 9 (continued)

(e) Each week at Athletics training, Claudia and Veronica race 100 metres.
Their results, in seconds, are shown in the table below.

Claudia	14.7	14.2	14.3	13.5	13.5	13.2	13.7	13.0
Veronica	15.2	15.1	14.2	14.2	13.9	13.8	13.5	13.2

(i) Find the mean and standard deviation for each runner. **2**

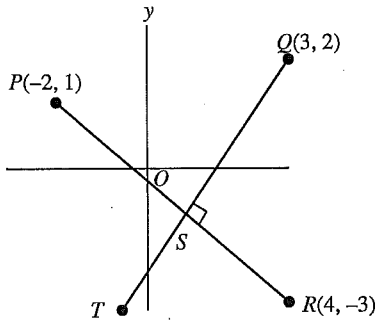
(ii) Which runner is more consistent? Give a reason **1**

End of Question 9

- **Start a new booklet**

Question 10 (15 marks)

(a)



NOT TO SCALE

Copy the diagram into your booklet.

(i) Find the exact length of the interval of PR. **1**

(ii) Find the gradient of PR. **1**

(iii) Show that the equation of the line through Q, perpendicular to PR is given by $3x - 2y - 5 = 0$. **2**

(iv) The line PR has equation $2x + 3y + 1 = 0$.
Find the coordinates of point S where the line PR meets the line QT. **2**

(v) Find the area of ΔPQR. **2**

(b) (i) Use completing the square method to find the centre and radius of circle given by **3**

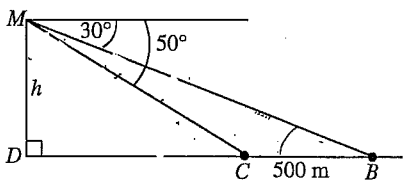
$$x^2 - 2x + y^2 + 6y + 6 = 0$$

(ii) State the y intercepts. Answer in exact form. **1**

Question 10 continues on the next page

Question 10 (continued)

(c) Mathew is whale watching from the top of a cliff when he spots two humpback whales out to sea. The angles of depression of the two whales are 50° and 30° when the whales are 500 metres apart.



Let h be the height of the cliff.

(i) Show that the height, h is given by **2**

$$h = \frac{500\sin 30^\circ \sin 50^\circ}{\sin 20^\circ}$$

(ii) Hence, calculate the height of the cliff to the nearest metre. **1**

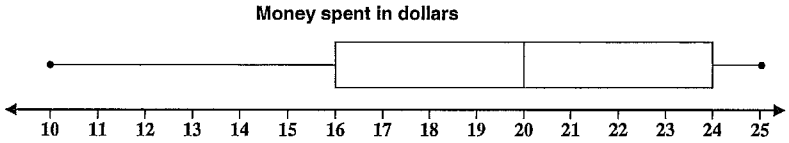
End of Question 10

- **Start a new booklet**

Question 11 (15 marks)

(a) Thirty Year 10 students were surveyed during one week to find out how much money they spent at the school canteen.

The results are summarised in the box-and-whisker plot shown below.



(i) What is the median amount spent? **1**

(ii) Find the interquartile range. **2**

(iii) Find the percentage of students who spent more than \$16 in one week. **1**

(iv) Is the data symmetrical or positively or negatively skewed? **1**

(b) (i) Amy buys tickets in a raffle in which 200 tickets are sold. She has a 7% chance of winning first prize. How many tickets did Amy buy? **1**

(ii) If there are two prizes in the raffle, what is the chance of Amy winning both prizes? **1**

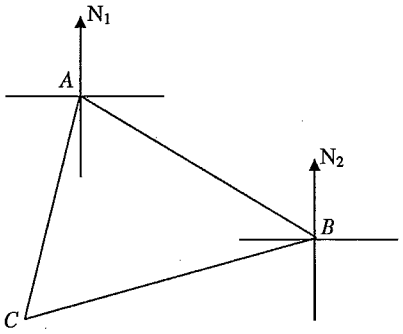
Question 11 continues on the next page

Question 11 (continued)

(c) The Flying Doctor Service based in outback Northern Territory flies to remote properties to deliver medical supplies.

A plane leaves Town A and flies to Town B on a bearing of 130° for 600 km. It then changes direction at Town B and flies on a bearing of 250° until it reaches a cattle station, C. The distance between Town A and the cattle station, C is 1200 km.

(i) Copy and complete the following diagram, marking on it all the given information. **1**



(ii) Give reasons why $\angle ABC = 60°$ **1**

(iii) Find the bearing of Town A from the cattle station, C, correct to the nearest degree. **2**

(d) (i) Solve simultaneously **3**

$$y = 2x^2 - x$$
$$y = 7x - 8$$

(ii) What is the geometrical significance of your result in part (i)? **1**

End of Paper

Name: .....

Section I
7 marks

Multiple Choice Answer Sheet

Write your name at the top of this page

Question								
	1	A ○	B ●	C ○	D ○	✓
	2	A ○	B ●	C ○	D ○	✓
	3	A ●	B ○	C ○	D ○	✓
	4	A ○	B ○	C ○	D ●	✓
	5	A ○	B ○	C ○	D ●	✓
	6	A ○	B ○	C ●	D ○	✓
	7	A ●	B ○	C ○	D ○	✓

7/7

Year 10 Semester 2 Exam. 2012 — 5.3E Pathway

Section I

① $3x-4y+5=0$

$4y=3x+5$

$y=\frac{3}{4}x+\frac{5}{4}$

gradient $m=\frac{3}{4}$ (B)

② (B)

③ $4p=5t+2q^2$

$2q^2=4p-5t$

$q^2=\frac{4p-5t}{2}$

$q=\pm\sqrt{\frac{4p-5t}{2}}$ (A)

④ $y=2x-5$ (positive slope, y-intercept at −5)

$y=x+6$ (positive slope, y-intercept at 6.)

(D)

⑤ (D)

slow constant slope.
faster constant slope
slower constant slope.

⑥ $P(\text{green})=\frac{\text{green}}{\text{total}}=\frac{x}{x+y}$ (C)

⑦ Range $=25-5=20$

Range $=2x-6$

$20=2x-6$

$2x=26$

$x=13$

Q_1 med. Q_3

5, (6), 11, (13), 13, (18), 25

median $=13$

$Q_1=6$

$Q_3=18$

IQR $=18-6=12$ (A)

Section II

⑧ a) $\sqrt{28}+\sqrt{63}$

$=\sqrt{4\times7}+\sqrt{9\times7}$

$=2\sqrt{7}+3\sqrt{7}$

$=5\sqrt{7}$ ✓ Eqn/surds 1

b) i) $\frac{x+3}{5}-\frac{x+1}{4}=3$

$\frac{4(x+3)-5(x+1)}{20}=3$ ✓

$4x+12-5x-5=60$

$-x+7=60$

$-x=53$

$x=-53$ ✓ Eqn/surds 2

ii) $x^2-5x=6$

$x^2-5x-6=0$ ✓

$(x-6)(x+1)=0$ ✓

$x=6,\ x=-1$ ✓

Quad 2

c) $y=2x^2-5x+2$

i) Axis

$x=-\frac{b}{2a}$

$=-\frac{-5}{2\times2}$

$=\frac{5}{4}$ ✓

ii) Vertex

$x=\frac{5}{4}$

$y=2(\frac{5}{4})^2-5(\frac{5}{4})+2$

$=-1\frac{1}{8}$

$(1\frac{1}{4},-1\frac{1}{8})$ ✓

(iii) x-intercepts $y=0$ ($p=4$, $s=-5$)

$2x^2-5x+2=0$

$2x^2-4x-1x+2=0$

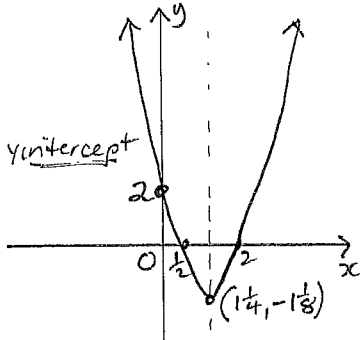
$2x(x-2)-1(x-2)=0$

$(2x-1)(x-2)=0$ ✓

$2x-1=0$ or $x=2$ ✓

$2x=1$

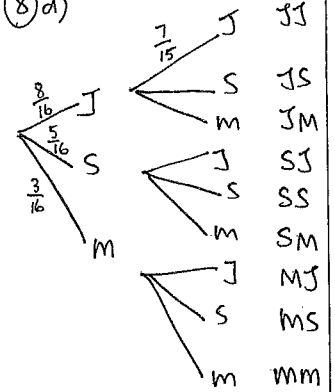
$x=\frac{1}{2}$



Show all features including y-intercept ✓ ✓

Quad 6

⑧ d)



i) $P(SS)=\frac{5}{16}\times\frac{4}{15}$ ✓

$=\frac{1}{12}$ ✓

ii) $P(JS)+P(SJ)$

$=\frac{8}{16}\times\frac{5}{15}+\frac{5}{16}\times\frac{8}{15}$ ✓

$=\frac{1}{3}$ ✓

iii) $P(\text{not minties})$

$=P(JJ)+\underbrace{P(SS)}_{(i)}+\underbrace{P(JS)+P(SJ)}_{(ii)}$

$=\frac{8}{16}\times\frac{7}{15}+\frac{1}{12}+\frac{1}{3}$

$=\frac{13}{20}$ ✓

Prob. 4

Question 9

a) $\frac{10}{\sqrt{7}+\sqrt{2}}=\frac{10}{\sqrt{7}+\sqrt{2}}\times\frac{\sqrt{7}-\sqrt{2}}{\sqrt{7}-\sqrt{2}}$

$=\frac{10(\sqrt{7}-\sqrt{2})}{7-2}$ ✓

$=\frac{10(\sqrt{7}-\sqrt{2})}{5}$

$=2(\sqrt{7}-\sqrt{2})$ ✓

Surds 2

b) Area $=\frac{1}{2}ab\sin C$ ✓

$=\frac{1}{2}\times6.8\times4.2\times\sin43^\circ$

$=9.7\ cm^2$ (1dp) ✓

Trig 2

c) $3x^2+2x-10=0$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ ✓

$=\frac{-2\pm\sqrt{(2)^2-4.3.-10}}{2\times3}$

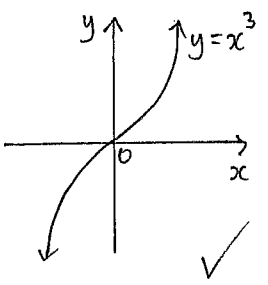
$=\frac{-2\pm\sqrt{124}}{6}$ ✓

$x=\frac{-2-\sqrt{124}}{6}$, $x=\frac{-2+\sqrt{124}}{6}$

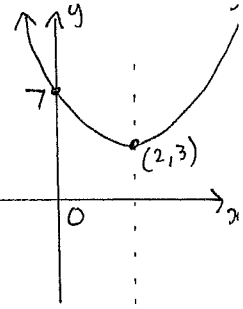
$x\doteq-2.19$ ✓ $x\doteq1.52$ (2dp) ✓

Quad 3

d)(i)



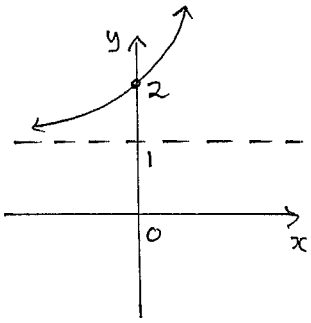
(ii) $y=(x-2)^2+3$



Vertex (2,3) ✓

shape and y-intercept ✓

(iii) $y=2^x+1$



Shape & y-intercept ✓

horizontal asymptote $y=1$ ✓

Graphs 5

Q9

e) i)

Claudia

$\bar{x} = 13.7625$

$\sigma x = 0.55$ ✓

Veronica

$\bar{x} = 14.1375$

$\sigma x = 0.66$ ✓

ii) Claudia is more consistent because a lower standard deviation shows less spread in her results. ✓

Statistics 3

Question 10.

i) $P(-2,1)$ $R(4,-3)$

$PR = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$= \sqrt{(4+2)^2 + (-3-1)^2}$

$= \sqrt{6^2 + (-4)^2}$

$= \sqrt{52}$

$= 2\sqrt{13}$ units. ✓

---

ii) $m_{PR} = \frac{y_2-y_1}{x_2-x_1}$

$= \frac{-3-1}{4+2}$

$= \frac{-4}{6}$

$= -\frac{2}{3}$ ✓

iii) gradient $m_2 = -\frac{1}{m_1}$

$MQT = -\frac{1}{-\frac{2}{3}}$

$= \frac{3}{2}$ ✓

Equation QT

$y - y_1 = m(x-x_1)$

$y - 2 = \frac{3}{2}(x-3)$

$2y - 4 = 3x - 9$

$3x - 2y - 5 = 0$ ✓

---

iv) Solve simultaneously

$3x - 2y - 5 = 0$ ①

$2x + 3y + 1 = 0$ ②

① × 2 $6x - 4y - 10 = 0$ ③

② × 3 $6x + 9y + 3 = 0$ ④

subtract $-13y - 13 = 0$

$13y = -13$

$y = -1$ ✓

sub. into ①

$3x + 2 - 5 = 0$

$3x = 3$

$x = 1$ ✓

$\therefore S(1,-1)$

---

v) $SQ = \sqrt{(1-3)^2 + (-1-2)^2}$

$= \sqrt{(-2)^2 + (-3)^2}$

$= \sqrt{13}$ ✓

Area $\Delta PQR = \frac{1}{2} \times PR \times SQ$

$= \frac{1}{2} \times 2\sqrt{13} \times \sqrt{13}$

$= 13 \ u^2$ ✓

Coord. geom 8

---

b) i) $x^2 - 2x + y^2 + 6y + 6 = 0$

complete the squares

$x^2 - 2x + 1 + y^2 + 6y + 9 = -6 + 1 + 9$

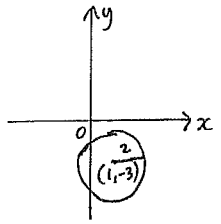
$(x-1)^2 + (y+3)^2 = 4$ ✓

Centre $(1,-3)$ ✓

radius $= \sqrt{4}$

$= 2$ units ✓

ii)



cuts y axis when $x = 0$

$(-1)^2 + (y+3)^2 = 4$

$(y+3)^2 = 3$

$y + 3 = \pm\sqrt{3}$

$y = -3 \pm \sqrt{3}$ ✓

Graphs 4

Q10

c) $\angle MBC = 30°$

(alternate angles are equal on parallel lines)

$\angle CMB = 50° - 30°$

$= 20°$

In ΔMCB

$\frac{b}{\sin B} = \frac{m}{\sin M}$

$\frac{x}{\sin 30°} = \frac{500}{\sin 20°}$

$MC = \frac{500 \cdot \sin 30°}{\sin 20°}$ ✓

In ΔMDC

$\sin \angle MCD = \frac{MD}{MC}$

$\sin 50° = \frac{h}{MC}$

$h = MC \times \sin 50°$

$= \frac{500 \sin 30° \sin 50°}{\sin 20°}$ ✓

---

ii) $\therefore h = 560$ m ✓

Trig 3

Question 11.

a) i) median = \$20 ✓

ii) $Q_3 = 24$

$Q_1 = 16$

$IQR = Q_3 - Q_1$

$= 24 - 16$

$= \$8$ ✓ ✓

iii) 75% ✓

iv) Negatively skewed ✓

Stats

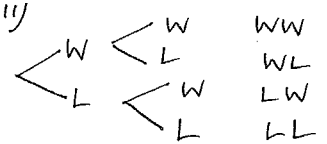
---

b) i) $7\% \times 200$

$= \frac{7}{100} \times 200$

$= 14$ tickets ✓

ii)

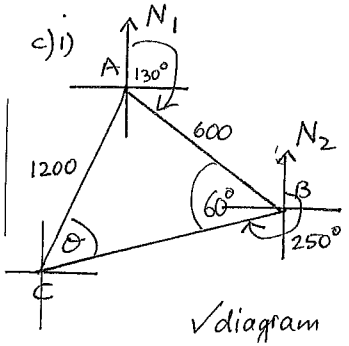


$P(WW) = \frac{14}{200} \times \frac{13}{199}$

$= \frac{91}{19900}$ ✓

---

c) i)



✓ diagram

ii) To get this mark, you must use correct wording for your reasons as well as naming the angle in some way as you find each step.

Here's one way ...

$\angle N_2BA = 180° - 130°$

$= 50°$

(corresponding angles are equal on parallel lines)

$\angle ABC = 360° - (250° + 50°)$

$= 360° - 300°$

$= 60°$

(Angle sum of a revolution is 360°)

---

iii) $\frac{\sin\theta}{600} = \frac{\sin 60°}{1200}$

$\sin\theta = 600 \times \frac{\sin 60°}{1200}$

$\sin\theta = 0.433...$ ✓

$\theta = 26°$

$\angle BCE_3 = 20°$ (alternate angles are.

Bearing $= 90° - (26 + 20°$

$= 44°$

$\therefore = 044°$ T

---

d) i) $y = 2x^2 - x$

$y = 7x - 8$

$2x^2 - x = 7x - 8$

$2x^2 - 8x + 8 = 0$ ✓

$2(x^2 - 4x + 4) = 0$

$2(x-2)^2 = 0$

$x = 2$ ✓

$y = 14 - 8$

$= 6$ $(2,6)$ ✓

ii) The line is a tangent since only one solution. ✓