Student Number

SCEGGS Darlinghurst

2008

Preliminary Course

Assessment Task 3

Extension 1 Mathematics

Outcomes Assessed: P3 and P4

Task Weighting: 20%

General Instructions

Total marks – ~~40~~ 39

- Time allowed – 1 hour
- This paper has ~~four~~ three questions
- Write your Student Number at the top of each page
- Attempt all questions and show all necessary working
- Write using blue or black pen
- Answer all questions on the pad paper provided
- **Begin each question on a new page**
- Marks will be deducted for careless or badly arranged work
- Approved scientific calculators and mathematical templates may be used

- Attempt Questions 1 – 3

Question	Reasoning	Communication	Marks
1			
2			
3			
TOTAL			

QUESTION 1 (10 marks)

1. (a) The remainder when $P(x) = x^4 + 13x + k$ is divided by $(x + 3)$ is 20. Find the value of k. 1

(b) Determine the zeros of $P(x) = x(x^3 - 3x^2 - 4x)$ and hence sketch its graph. What is the multiplicity of the root at $x = 0$? 3

(c) If α, β, and γ are the roots of $3x^3 + 8x^2 - 7 = 0$, find the value of:

(i) $\alpha + \beta + \gamma$ 1

(ii) $\dfrac{1}{\alpha\beta} + \dfrac{1}{\beta\gamma} + \dfrac{1}{\alpha\gamma}$ 2

(d) (i) $P(x) = x^3 + 7x^2 - bx - b$ is divided by $A(x) = x - 1$.

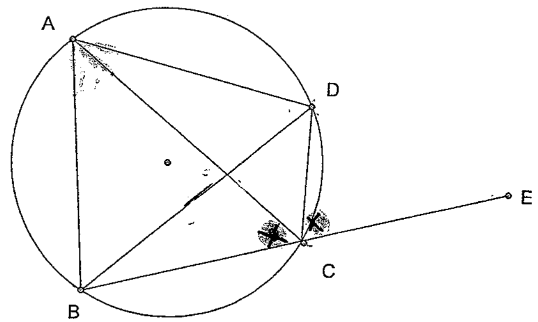
Show by long division that

$P(x) = (x-1)(x^2 + 8x - b + 8) + (-2b + 8)$. 2

(ii) Find the value of the constant 'b' so that P(x) is divisible by A(x) 1

QUESTION 2 (15 marks)

(a)

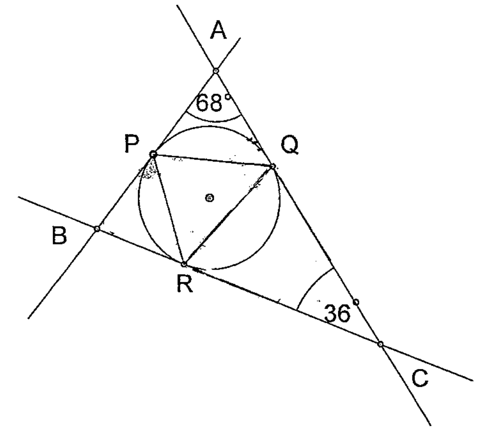


In the diagram above, ABCD is a cyclic quadrilateral. BC is produced to E. $\angle BCA = \angle DCE$.

(i) Give a reason why $\angle BDA = \angle BCA$

(ii) Hence show that BA = BD.

(b).



(i) Find $\angle BPR$ giving reasons. 2

(ii) Hence or otherwise find $\angle PQR$ giving reasons. 1

(c) If $\cos\theta = \frac{2}{3}$ and θ is acute, find the exact values of

(i) $\cos 2\theta$ 2

(ii) $\tan\frac{\theta}{2}$ 2

(d) Solve $\sin^2\theta = 9\sin\theta + \cos^2\theta + 4$ for $0^\circ \le \theta \le 360^\circ$ 3

(e) Write in general form the solution to the equation

$\sin(\theta + 30^\circ) = \frac{1}{2}$. 3

QUESTION 3 (15 marks)

(a) Express in factorial form:

(i) ${}^5C_2$ 1

(ii) ${}^6P_2$ 1

(b) Four married couples are to be seated at a round table for dinner.

(i) In how many different ways can the people be seated around a table? 1

(ii) If each married couple is to be seated together, in how many ways can this be done? 2

(c). The letters of the word MATHEMATICS are to be arranged in a straight line. How many ways are there if:

(i) There are no restrictions at all? 1

(ii) The word is to commence and end with the same letter? 2

EXAMINATION CONTINUES ON NEXT PAGE

(d) (i) In how many ways can 8 stories be arranged in order to form a book? 1

(ii) What is the probability that the longest story will be first and the shortest story will be last? 1

(iii) What is the probability that the longest and shortest stories will be next to each other? 2

(e) In an examination paper, there are 5 questions on calculus and 6 on the other sections of the course. In how many ways can 8 questions be chosen if

(i) exactly 4 questions are to be Calculus questions. 1

(ii) at most 4 questions are to be Calculus questions. 1

END OF EXAMINATION

Question 1

(a) $P(x) = x^4 + 13x + k$ when divided by $(x+3)$ remainder $= 20$

$P(-3) = (-3)^4 + 13(-3) + k = 20$

$k = 20 - 42$

$= -22$

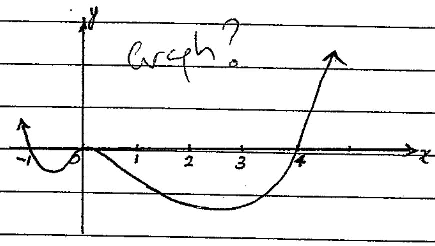
(b) $P(x) = x(x^3 - 3x^2 - 4x)$

$p(x) = x^4 - 3x^3 - 4x^2$

trial $P(-1) = 0$

$\therefore$ $x+1$ is a ~~root~~ factor

$$\begin{array}{r} x^3 - 4x^2 \\ x+1 \overline{) \, x^4 - 3x^3 - 4x^2} \\ \underline{x^4 + x^3} \\ -4x^3 - 4x^2 \\ \underline{-4x^3 - 4x^2} \\ 0 \end{array}$$



$P(x) = (x+1)(x^3 - 4x^2)$

$= x^2(x+1)(x^{\cancel{3}} - 4)$

zeros $\longrightarrow$ 0, -1 and 4

at $x=0$, multiplicity is 2

(c) α, β, γ roots $3x^3 + 8x^2 - 7 = 0$

i. $\alpha + \beta + \gamma = -\frac{b}{a}$

$= -\frac{8}{3}$

ii. $\frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\alpha\gamma}$

$\alpha\beta\gamma = -\frac{d}{a}$

$= \frac{7}{3}$ ~~$\frac{4}{8}$~~

$= \frac{\alpha\beta\gamma^2 + \alpha^2\beta\gamma + \alpha\gamma\beta^2}{\alpha^2\beta^2\gamma^2}$

$= \frac{\alpha\beta\gamma(\gamma + \alpha + \beta)}{(\alpha\beta\gamma)^2} = \frac{\frac{7}{3} \times -\frac{8}{3}}{(-\frac{8}{3})^2} = -\frac{7}{8}$

(d)

i. $P(x) = x^3 + 7x^2 - bx - b$ divided by $A(x) = x - 1$

$$\begin{array}{r} x^2 + 8x - b + 8 \\ x-1 \overline{) \, x^3 + 7x^2 - bx - b} \\ \underline{x^3 - x^2} \\ 8x^2 - bx - b \\ \underline{8x^2 - 8x} \\ \cancel{8x - bx - b} \\ -bx + 8x - b \\ \underline{-bx + b} \\ 8x - 2b \\ \underline{8x - 8} \\ -2b + 8 \end{array}$$

$\therefore x^3 + 7x^2 - bx - b = (x-1)(x^2 + 8x - b + 8) + (-2b + 8)$

ii. If $p(x)$ is divisible by $A(x) \rightarrow x - 1$

$p(1) = 0$

$P(1) = 1^3 + 7(1)^2 - b(1) - b$

$0 = 1 + 7 - b - b$

$0 = 8 - 2b$

$-2b = -8$

$b = \frac{8}{2}$

$b = 4$

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Question 2

(a)

i. $\angle BDA = \angle BCA$ (angles in the same segment of a circle are equal) ✓

ii. ~~$\angle BDA = \angle BCA$ (from i)~~
~~$= \angle AEB$~~

$\angle BCA = \angle DCE$ (given)

$\angle DCE = \angle BAD$ (exterior angle of the vertex of a cyclic quad. is equal to the interior oposite angle)

$\angle BCA = \angle BDA$ (qn i)

$\therefore \angle BAD = \angle BDA = \angle BCA = \angle DCE$

$\therefore \Delta BAD$ is isoceles as base angles $\angle BAD$ and $\angle BDA$ are equal.

$\therefore BA = BD$ as sides opposite equal angles in isoceles triangles are equal. ✓

(b) i. $\angle BPR = \angle RQP$ (angle between the tangent and the chord through the point of contact is equal to the angle in the alternate segment)

$\angle PBR = 180 - 68 - 36 =$
$= 76°$ (angle sum in triangle)

$\angle CRQ = \frac{180 - 36}{2}$
$= 72°$

a) 2
b) 3
c) 3 R/1
d) 3
e) 1

12

See next sheet

ii. $\angle PQR = \angle BPR =$ (angle between the tangent and the chord through the point of contact is equal to the angle in the alternate segment). 3

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Question 2

(b)

i. $\angle BPR = ?$

In ΔBPR,

$BP = BR$ (tangents to a circle from an exterior point are equal) ✓

$\therefore \Delta BPR$ is isoceles

$\therefore \angle BPR = \angle BRP$ (angles opposite equal sides in isoceles triangles are equal) ✓

$\therefore \angle BPR = \frac{180 - 76}{2}$
$= 52°$

$\angle PBR = 180 - 68 - 36$ (angle sum in triangle)
$= 76°$

ii. $\angle PQR = \angle BPR = 52°$ (angle between the tangent and the chord through point of contact is equal to the angle in the alternate segment) ✓

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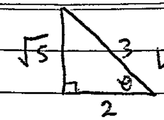
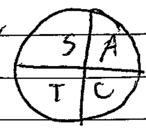
(c) $\cos\theta = \frac{2}{3}$ acute. $\therefore$ in 1^{st} quad.



$\sin\theta = \sqrt{5}/3$

i. $\cos 2\theta = \cos^2 x - \sin^2 x$

$= \left(\frac{2}{3}\right)^2 - \left(\frac{\sqrt{5}}{3}\right)^2$



~~$\tan 2/3$~~ $\frac{4}{9} - \frac{5}{9} = -\frac{1}{9}$

ii. $\tan\frac{\theta}{2} = t$

$\cos\theta = \frac{1-t^2}{1+t^2} = \frac{2}{3}$

$3(1-t^2) = 2(1+t^2)$

$3 - 3t^2 = 2 + 2t^2$

$1 = 5t^2$

$t^2 = \frac{1}{5}$

$t = \pm\sqrt{\frac{1}{5}}$

$= \pm\frac{1}{\sqrt{5}}$ But θ is acute.

$\therefore \tan\frac{\theta}{2} = \pm\frac{1}{\sqrt{5}}$ X

(d) $\sin^2\theta = 9\sin\theta + \cos^2\theta + 4$ $0 \le \theta \le 360°$

$\sin^2\theta = 9\sin\theta + (1 - \cancel{\cos^2\theta}\ \sin^2\theta) + 4$

$\sin^2\theta = 9\sin\theta - \sin^2\theta + 5$

$2\sin^2\theta - 9\sin\theta - 5 = 0$

let $\sin\theta = X$

$2X^2 - 9X - 5 = 0$ $x - 10$, $+ -9$ $-10, 1$

$(2X+1)(X-5) = 0$

$2X + 1 = 0$ $X = 5$

$X = -\frac{1}{2}$ $\sin\theta = 5$ ← no solution as ~~equation is~~ undefined when $\sin\theta = 5$.

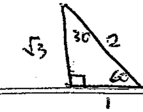
$\sin\theta = -\frac{1}{2}$

related angle

$\theta = 30°$

$\sin\theta$ is $-ve$ in 3^{rd} & 4^{th}

$\therefore \theta = 210°$ and $330°$



(e) $\sin(\theta + 30) = \frac{1}{2}$

$\cos 30 = \frac{\sqrt{3}}{2}$

$\sin 30 = \frac{1}{2}$.

~~$\sin\theta\cos 30 + \cos\theta\sin$~~

~~if sin~~

If $\sin x = k$

$x = (-1)^n \alpha + 180n$

$\alpha = \sin^{-1} k$

$\sin\theta\cos 30 + \cos\theta\sin 30 = \frac{1}{2}$

$\sin\theta \times \frac{\sqrt{3}}{2} + \cos\theta \times \frac{1}{2} = \frac{1}{2}$

$\theta + 30 = (-1)^n 30 + 180n$ $= 30°$

$\frac{\sqrt{3}\sin\theta}{2} + \frac{\cos\theta}{2} = \frac{1}{2}$

$\theta = (-1)^n 30 + 180n - 30$

$\sqrt{3}\sin\theta + \cos\theta = 1$

$\sin\theta + \cos\theta = 1 - \sqrt{3}$

$\sin(\theta + 30) = \frac{1}{2}$ $\Rightarrow$ $\theta = -30 + n\pi + (-1)^n\frac{\pi}{6}$

$= -\frac{\pi}{6} + n\pi + (-1)^n\frac{\pi}{6}$

$\theta + 30 = \sin^{-1}\frac{1}{2}$

$\theta + 30° = 30°$

$\theta = 30 - 30$

$= 0$. X

$\sin\theta = \sin\alpha$

$\theta = 180°n + (-1)^n \times 0$ X

$= 180°n$

when n is an integer.

Question 3

(a)

i. $^5C_2$

$= \dfrac{^nP_r}{r!}$

$= \dfrac{5!}{(5-2)!\,2!}$

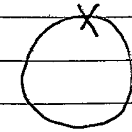
$= \dfrac{5!}{3! \times 2!}$

ii. $^6P_2$

$= \dfrac{6!}{(6-2)!}$

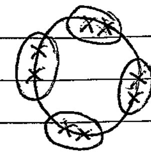
$= \dfrac{6!}{4!}$

(b)

i. 

$4 \times 2 = 8$ people at table.

$7! = 5040$

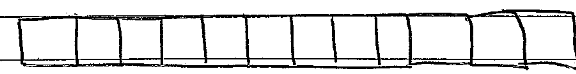
ii. 

$3! \times 2! \times 4$

$= 48$ ways.

(c)

i. $\dfrac{11!}{2!\,2!\,2!} = 4989600$

ii. 

a M 2A's, 2T's

b A 2M's, 2T's

c T 2M's, 2A's

M
A
T

$a = 1 \times \dfrac{9!}{2!\,2!} = 90720$

$b. = 1 \times \dfrac{9!}{2!\,2!} = 90720 +$

$c. = 1 \times \dfrac{9!}{2!\,2!} = 90720$

$\boxed{272160}$ ways.

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(d)

i. $8! = 40320$ ways

ii.

1	6	5	4	3	2	1	1

$= 6! \times 1 \times 1$

$= 720$

P(longest story 1st, Shortest last) $= \frac{720}{40320}$

$= \frac{1}{56}$

~~(e)~~

iii.

L	S	6	5	4	3	2	1

or SL

$7! \times 2! = 10080$ ways

P(nxt to each other) $= \frac{10080}{40320}$

$= \frac{1}{4}$

(e) Total no. of questions = 11

i. ${}^5C_4 \times {}^6C_1 = 30$ ways

add (separate events)

ii. three questions $\longrightarrow$ ${}^5C_3 \times {}^6C_2 = 150$

${}^5C_2 \times {}^6C_3 = 200$

${}^5C_1 \times {}^6C_4 = 75$

${}^5C_0 \times {}^6C_5 = 6$

Total questions – 5 Calculus Questions

OR $= {}^5C_4 \cdot {}^6C_4 + {}^5C_3 \cdot {}^6C_5$

$+ {}^5C_2 \cdot {}^6C_6$

$= 75 + 60 + 10 = \underline{145}$ ways

at most 4 questions on calculus $= 30 + 150 + 200 + 75 + 6$

$= 461$ ways