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Centre Number

Student Number

SCEGGS Darlinghurst

2010

HSC Assessment 2

16 March 2010

Mathematics

Assessment Outcomes: P2-P5, P8, H2, H4, H5, H8 and H9

Weighting: 20%

General Instructions

- Time allowed – 1 hour
- This paper has **four** questions
- Answer on the paper provided
- Start each question on a **new page**
- Write your Student Number at the top of each page
- Attempt **all** questions and show all necessary working
- Marks may be deducted for careless or badly arranged work
- Mathematical templates, geometrical equipment and graphics calculators may be used
- A table of standard integrals is provided at the back of this paper

Total marks (42)

- Attempt Questions 1 – 4

Question	Communication	Calculus	Reasoning	Marks
1		/3	/1	/9
2	/2		/1	/10
3		/3	/2	/11
4	/1	/6		/12
TOTAL				/42

Answer each question on the paper provided

Write your Student Number at the top of each page

Start each question on a new page

QUESTION 1 (9 marks)

(a) The third term and the tenth term of an arithmetic series are 7 and 42 respectively.

Find the:

(i) first term and the common difference **2**

(ii) sum of the first 10 terms of the series **1**

(b) (i) Find a primitive function for $x^4 + 4$. **1**

(ii) Find $\int_1^5 \frac{x^3 - 3x^2}{x}\,dx$ **2**

(c) The decimal $0.\dot{3}\dot{4}$ (i.e. 0.34343434.....) can be considered as the sum of a geometric sequence.

(i) What is the value of the first term, a, and the common ratio, r? **1**

(ii) Hence express $0.\dot{3}\dot{4}$ as a fraction. **2**

Start a new Page

QUESTION 2 (10 marks)

(a) The nth term of a sequence is $4n-3$.

(i) Determine the 50^{th} term of the sequence? 1

(ii) Prove that 107 does not belong to the sequence. 1

(iii) How many terms must be taken for the sum to exceed 300? 3

(b) The graphs of $y = x^2 - 2$ and $y = x$ bound a closed region.

(i) Sketch the two graphs and solve for their points of intersection. 3

(ii) Hence find the area of the region. 2

Start a new Page

QUESTION 3 (11 marks)

(a) (i) Show that $\sqrt{7} + \sqrt{28} + \sqrt{63} + \ldots\ldots$ is an arithmetic series 1

(ii) $\sqrt{7} + \sqrt{28} + \sqrt{63} + \ldots\ldots = 36\sqrt{7}$.

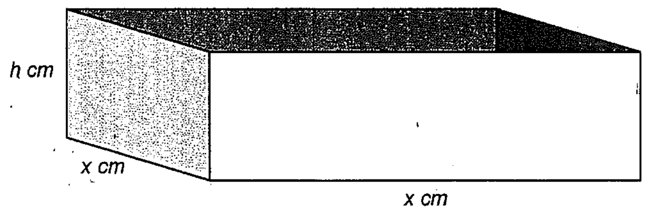
How many terms are in the sequence? 2

(b) A golf ball is dropped from a height of one metre. Each time it hits the ground it bounces to two thirds of its previous height.

Calculate the distance that the golf ball travels before it comes to rest. 3

QUESTION 3 (Continued)

(c). A metal tray, in the shape of a rectangular prism with a square base, is made out of 108 square centimetres of sheet metal. The tray is open at the top.



Let x cm be the side length of the base, and let h cm be the height.

(i) Show that $h = \frac{27}{x} - \frac{x}{4}$. 1

(ii) Show that the volume V of the tray is given by:

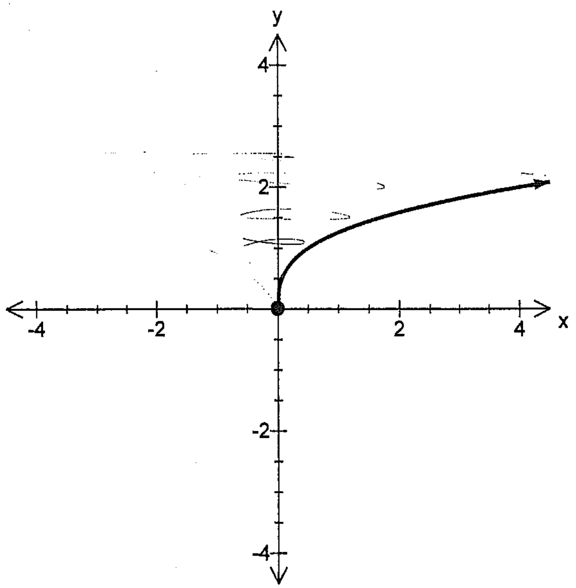
$$V = 27x - \frac{x^3}{4}$$ 1

(iii) Find the maximum volume of the tray. 3

Start a new Page

QUESTION 4 (11 marks)

(a) The diagram shows the graph of $y = \sqrt[3]{2x}$ for $x \geq 0$.

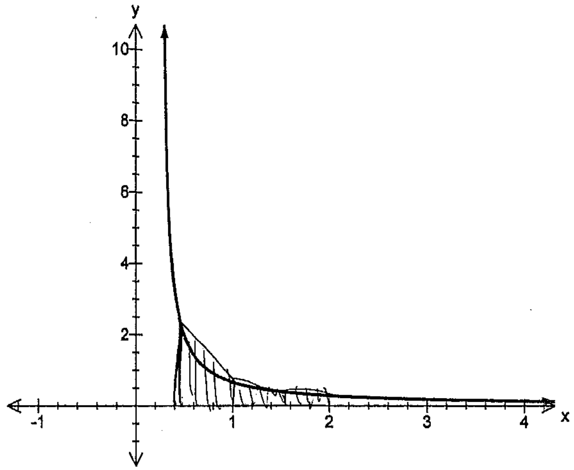


Calculate the volume of the solid formed when the section of the curve $y = \sqrt[3]{2x}$ between $y = 1$ and $y = 3$ is rotated about the y axis. 3

Give your answer correct to 2 decimal places.

QUESTION 4 (continued)

(b) The diagram shows part of the graph of the function $y = \dfrac{2}{4x-1}$.



(i) Copy and complete the table of values for $y = \dfrac{2}{4x-1}$ with each value correct to 3 decimal places. 1

x	0.5	1	1.5	2
y				

(ii) Use the trapezoidal rule with the four y values in the table to approximate $\displaystyle\int_{0.5}^{2} \frac{2}{4x-1}\,dx$ to one decimal place. 2

(iii) Is the value obtained using the trapezoidal rule greater than or less than the exact value of the integral? Use a diagram to justify your answer. 1

(e) The quantity of fuel a delivery truck uses depends on the speed at which it is travelling. When the truck is travelling at a constant speed of v kilometres per hour, the fuel consumed can be represented by the following equation:

$$U = \frac{1}{125}\left(\frac{1100}{v} + \frac{v}{7}\right) \text{ litres per kilometre.}$$

(i) The cost of fuel is 112.5 cents per litre. Show that the total cost, C dollars, of fuel for the truck to travel 1000 kilometres is given by

$$C = \frac{9900}{v} + \frac{9v}{7}.$$ 2

(ii) At what speed (to the nearest whole number) is the cost of fuel for the truck a minimum? 3

End of Paper

Solutions HSC Ass 2 2010 mathematics

Question 1 /9

(a) (i) $T_3 = 7 = a+2d$

$T_{10} = 42 = a+9d$

$\therefore 7d = 35$

$d = 5$

$a + 10 = 7$

$a = -3$

First term $= -3$ ✓

Common difference $= 5$ ✓

"Start of exam" nerves caused Algebraic errors. Substitution errors were also fairly common.

(ii) $S_{10} = \frac{10}{2}[-6+9(5)]$

$= 195$ ✓

(b) (i) $\frac{x^5}{5} + 4x + C$ ✓

A few students differentiated instead of integrated.

Calc ③ (ii) $\int_1^5 x^2 - 3x \, dx$

$= \left[\frac{x^3}{3} - \frac{3x^2}{2}\right]_1^5$ ✓

$= \frac{5^3}{3} - \frac{3(25)}{2} - \left(\frac{1}{3} - \frac{3}{2}\right)$

$= \frac{16}{3}$ ✓

The negative was poorly handled. Distribution of a negative is a basic technique which students should not get wrong

(c) $0.\dot{3}\dot{4} = 0.343434\ldots$

$= 0.34 + 0.0034 + 0.000034 + \ldots$

Reas ① (i) $a = 0.34$, $r = 0.01$ ✓

(ii) $S_\infty = \frac{0.34}{0.99} = \frac{34}{99}$ ✓

A few students could not write the repeating decimal as an infinite series.

Some students used earlier techniques $x = .34$, $100x = 34.34$, $99x = 34$ } etc but did not receive marks for this approach.

Question 2 /10

(a) $T_n = 4n-3$ 1, 5, 9 ...

(i) $T_{50} = 4(50) - 3$

$= 197$ ✓

(ii) $107 = 4n - 3$ Reas ①

$110 = 4n$

$n = 27\frac{1}{2}$

n is not an integer > 0

∴ 107 is not a member. ✓

Parts (i) & (ii) of this question were very well answered.

(iii) $S_n = \frac{n}{2}[2 + (n-1)4] > 300$ ✓

$n(2 + 4n - 4) > 300 \times 2$

$n(4n-2) > 600$

$4n^2 - 2n - 600 > 0$

$2n^2 - n - 300 > 0$ ✓

Use the quadratic formula to solve

$n > \frac{1 \pm \sqrt{1 - 4(2)(-300)}}{4}$

$n > 12\frac{1}{2}$

∴ $n = 13$ ✓

- most students were able to get the first mark in this question by writing $S_n > 300$.
- Difficulties arose when solving $2n^2 - n - 300 > 0$ and many students did not know how to correctly solve this
- The quadratic formula gave the solution $n < -12$, $n > 12\frac{1}{2}$ students then needed to state that $n = 13$ as it must be a positive integer.

(b) Com ②

$x^2 - 2 = x$

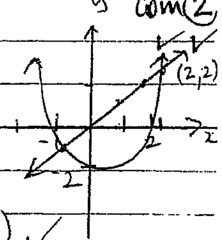
$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2$ and -1

Points of intersection

$(2, 2)$ and $(-1, -1)$ ✓



$A = \int_{-1}^{2} x - x^2 + 2 \, dx = \left[\frac{x^2}{2} - \frac{x^3}{3} + 2x\right]_{-1}^{2}$ ✓

$= \frac{4}{2} - \frac{8}{3} + 4 - \left(\frac{1}{2} + \frac{1}{3} - 2\right)$

$= 9/2$ ✓

- the shape of these graphs were generally well drawn. Many students did not indicate important points on their sketches and this should always be shown.
- Many students only found the x coordinates of the points of intersection but the y-values were also needed
- Area was mostly well done although there were a number of errors with the minus sign and calculation errors.

Question 3 /11

(a) (i) $\sqrt{28} = \sqrt{4\times7} = 2\sqrt{7}$

$\sqrt{63} = \sqrt{9\times7} = 3\sqrt{7}$

$\therefore$ series $= \sqrt{7} + 2\sqrt{7} + 3\sqrt{7} + \ldots$

$\therefore a = \sqrt{7} \quad d = \sqrt{7}$ ✓

$(2\sqrt{7} - \sqrt{7}) = (3\sqrt{7} - 2\sqrt{7})$

$T_2 - T_1 = T_3 - T_2$

(ii) $S_n = 36\sqrt{7} = \frac{n}{2}\left[2\sqrt{7} + (n-1)\sqrt{7}\right]$

$72\sqrt{7} = n\left[2\sqrt{7} + n\sqrt{7} - \sqrt{7}\right]$

$= n\left[\sqrt{7} + n\sqrt{7}\right]$

$= n\sqrt{7} + n^2\sqrt{7}$

$= \sqrt{7}(n + n^2)$ ✓

$\therefore n^2 + n = 72$

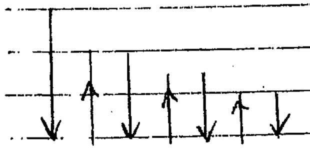
$n^2 + n - 72 = 0$

$(n+9)(n-8) = 0$

$n = 8 \quad n \neq -9$

There are 8 terms in this sequence. ✓

(b)



✓

$1 + \frac{2}{3}\times1 + \frac{2}{3}\times1 + \frac{2}{3}\times\frac{2}{3}\times1 + \ldots$

$= 1 + 2\times\left(\frac{2}{3} + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^3 + \ldots\right)$ ✓

$= 1 + 2\times S_\infty$

where $S_\infty = \dfrac{\frac{2}{3}}{1-\frac{2}{3}} = 2$

$\therefore$ total distance travelled

$= 1 + 2\times2$

$= 5$ m ✓

The amount of students that think $2\sqrt{7} - \sqrt{7} = 1$ is worrying.

Alot of students simplified $\sqrt{28}$ with their calculator. You needed to identify that $a = \sqrt{7}$ and $d = \sqrt{7}$ to help you in the next part

Students could not handle the $\sqrt{7}$.

Poorly done Q. Learn formulas

Alot of students did not realise $S_n = 36\sqrt{7}$.

1mk awarded for S_∞

1mk for doubling

1mk for $1 + 2\times S_\infty$

(c) (i) $SA = 108\text{ cm}^2 = hx\times4 + x^2$

$108 = x^2 + 4hx$

$\therefore h = \frac{108 - x^2}{4x} = \frac{108}{4x} - \frac{x^2}{4x}$

$h = \frac{27}{x} - \frac{x}{4}$ ✓

(ii) $V = hx^2$ (Reas 2)

$= x^2\left(\frac{27}{x} - \frac{x}{4}\right)$ ✓

$= 27x - \frac{x^3}{4}$

(iii) For max/min $V' = 0$

$V = 27x - \frac{x^3}{4}$

$V' = 27 - \frac{3x^2}{4} = 0$ ✓

$3x^2 = 4\times27$

$x^2 = \frac{4\times27}{3} = 36$

$x = 6$ cm ✓

Confirm that $x = 6$ provides max. vol.

$V'' = -\frac{6x}{4}$

At $x = 6 \quad V'' = -9 \Rightarrow$ maximum

$\therefore$ the maximum volume occurs when $x = 6$

ie $V = 27(6) - \frac{6^3}{4}$ ✓

$= 108\text{ cm}^3$ (Calc 3)

When asked to "show" please don't "fudge". Students needed to be aware that $SA = 108\text{ cm}^2$.

vol = A × H again show this substitution.

Careless errors made with finding V' and solving $V' = 0$.

Students correctly showed $x = 6$ was a max, however substitution -9 into vol.

3rd mark for finding the max vol of the tray.

Question 4 /12

(a) $y = \sqrt[3]{2x} = (2x)^{1/3}$

make sure it is the y axis that you are finding

Calc ③ For rotation around y axis

$V = \pi \int x^2 \, dy$

$y = (2x)^{1/3}$

$y^3 = 2x$

$\frac{1}{2} y^3 = x$

$\frac{1}{4} y^6 = x^2$ ✓

$\therefore V = \pi \int_1^3 \frac{y^6}{4} \, dy$

$= \frac{\pi}{4} \int_1^3 y^6 \, dy$

$= \frac{\pi}{4} \left[\frac{y^7}{7} \right]_1^3$ ✓

$= \frac{\pi}{4} \left[\frac{3^7}{7} - \frac{1}{7} \right]$ ✓

$= 245.27$ units^3

Be careful of your Algebra

$(\frac{1}{2}y^3)^2$ is NOT $\frac{1}{4}y^5$!!

It is often a good idea to take the 4 out like this.

Read the question for decimal places etc.

(b)

x	0.5	1	1.5	2
y	2	2/3	2/5	2/7
Fact	1	2	2	1
Prod	2	4/3	4/5	2/7

✓

$\Sigma = \frac{464}{105}$ $h = 0.5$ ✓

$\doteqdot \frac{0.5}{2} \times \Sigma$ ✓

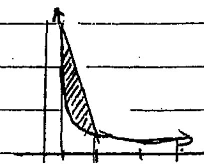
$\doteqdot 1.1$ (1 dp)

An integral was requested not an Area.

Be careful when finding h.

Decimal places again.

(iii) The value obtained by the trapezoidal rule is an over estimate. The diagram below contains shaded surpluses



Comm ① ✓

Very badly done.

make sure you understand the Trapezoidal Rule.

(d) (i) $\text{Cost} = 11 \times 1000 \times 1.125$ ✓

$= \frac{1}{125} \left(\frac{1100}{v} + \frac{v}{7} \right) 1.125 \times 1000$

$= \frac{9900}{v} + \frac{9v}{7}$ ✓

must multiply by 1000 and 1.125 to obtain dollars

(ii) $C = 9900v^{-1} + \frac{9v}{7}$

For max/min $C' = 0$

$C' = -\frac{9900}{v^2} + \frac{9}{7} = 0$ ✓

$\frac{9900}{v^2} = \frac{9}{7}$ Calc ③

$9v^2 = 9900 \times 7$

$v^2 = 1100 \times 7$

$= 7700$

$v = \sqrt{7700}$ ✓

To confirm that cost is a minimum find C''

$C'' = \frac{19800}{v^3}$: when $v = \sqrt{7700}$

$C'' = 0.02$

$C'' > 0$, the cost has been minimised at $v = \sqrt{7700}$ or 88 km/h ✓

Poor differentiation here.

In another question it is good to explain why $-\sqrt{7700}$ has been discarded.

Must test.

Be careful of the correct final statement.