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Centre Number

Student Number

SCEGGS Darlinghurst

12th June 2012
HSC Course
Assessment Task 3

Mathematics

Assessment Outcomes: H2, H3, H4, H5, H6, H8, H9
Weighting: 25%

General Instructions

- Time allowed – $1\frac{1}{2}$ hours
- This paper is in **two** sections
- Write your Student Number on the front of each writing booklet
- Attempt **all** questions and show all necessary working
- Marks may be deducted for careless or badly arranged work
- Mathematical templates and geometrical equipment and scientific calculators may be used

Total marks – 50
Assessment Weighting – 25%

Section I

5 marks
- Attempt Questions 1 – 5
- Allow about 10 minutes for this section
- Answer on the separate Multiple Choice Answer Sheet provided

Section II

45 marks
- Attempt Questions 6 – 8
- Allow about 80 minutes for this section
- ALL working must be shown
- Answer in the booklets provided
- Start each question in a **new booklet**

Section	Question	Trig	Log & Exp	Prob.	Coord.	Series	TOTAL
I	1 – 5						/5
II	6						/15
	7						/15
	8						/15
TOTAL							/50

Section I
5 marks
Attempt Questions 1–5
Allow about 10 minutes for this section
Use the multiple-choice answer sheet

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ● C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word ***correct*** and drawing an arrow as follows.

correct

A ● B ● C ○ D ○

1. Which of the following is the correct expression for $\int \cos\frac{x}{3}\,dx$?

(A) $\frac{1}{3}\sin\frac{x}{3} + C$

(B) $3\sin\frac{x}{3} + C$

(C) $-\frac{1}{3}\sin\frac{x}{3} + C$

(D) $-3\sin\frac{x}{3} + C$

2. Differentiate 3^x with respect to x.

(A) $x3^{x-1}$

(B) 3^x

(C) $3^x \ln 3$

(D) $\frac{3^x}{\ln 3}$

3. There are 30 days in June. If a date is picked at random, the probability that it is divisible by 3 or divisible by 5 is:

(A) $\frac{1}{15}$

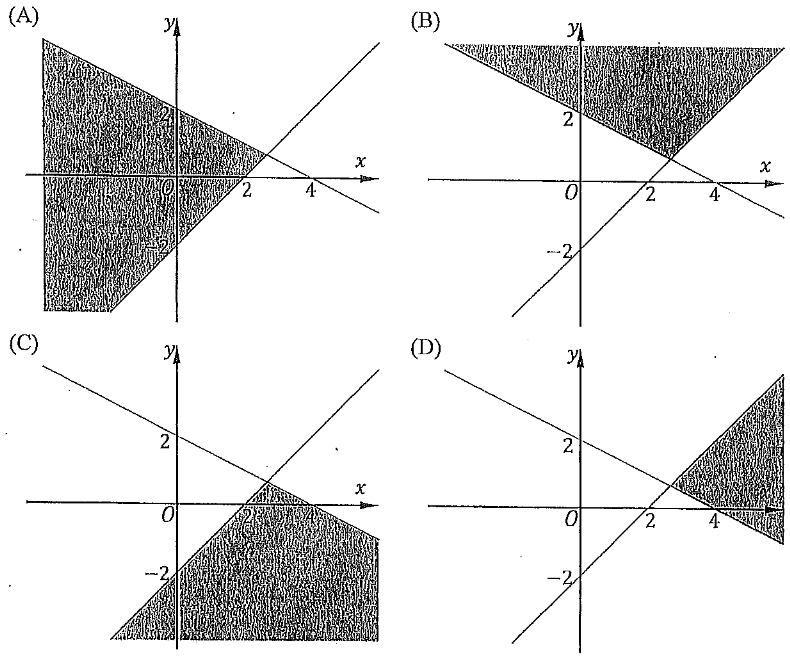
(B) $\frac{6}{15}$

(C) $\frac{7}{15}$

(D) $\frac{8}{15}$

4. Which of the following shadings best represents:

$$x + 2y - 4 \geq 0 \text{ and } x - y - 2 \geq 0?$$



5. After the announcement of her son's engagement on New Year's Eve last year, Chris decides to invest \$2500 at the beginning of each month, starting January 1st 2012, in order to contribute to the cost of the wedding. The account accrues interest at a rate of 5.4% p.a., compounded monthly.

Which of the following is the correct expression for the total value of her investment, in dollars, on March 31st 2013, the day of her son's wedding?

(A) $2500(1.054 + 1.054^2 + \cdots + 1.054^{15})$

(B) $2500(1 + 1.054 + 1.054^2 + \cdots + 1.054^{15})$

(C) $2500(1.0045 + 1.0045^2 + \cdots + 1.0045^{15})$

(D) $2500(1 + 1.0045 + 1.0045^2 + \cdots + 1.0045^{15})$

End of Section I

Section II

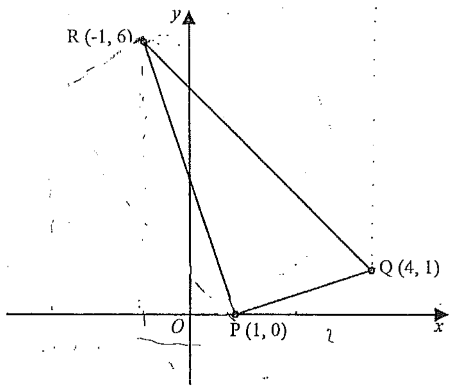
45 marks
Attempt Questions 6–8
Allow about 80 minutes for this section

Write your answers in the booklets provided.
Start each question in a NEW BOOKLET.
Write your STUDENT NUMBER on the front of each booklet.

Marks

Question 6 (15 marks)

(a)



The diagram shows points $P(1,0)$, $Q(4,1)$ and $R(-1,6)$.

(i) Show that $PR \perp PQ$. **2**

(ii) Find the coordinates of the point S such that $PQRS$ forms a parallelogram. **1**

(iii) The line RQ has equation $x + y - 5 = 0$. **1**
Find the perpendicular distance from point P to the line RQ.

(iv) Hence, or otherwise, find the area of $PQRS$. **2**

Question 6 continues on page 6

Question 6 (continued) **Marks**

(b) Differentiate the following with respect to x:

(i) $(e^x + 1)^3$ **2**

(ii) $x \tan 2x$ **2**

(c) The gradient of a curve is given by $\frac{dy}{dx} = \frac{2}{x}$. The curve passes through the point $(\frac{1}{e}, 1)$. **2**
Find the equation of the curve.

(d) Solve $\log_7(x+5) + \log_7 x = \log_7 6$. **3**

End of Question 6

■ Start a new writing booklet

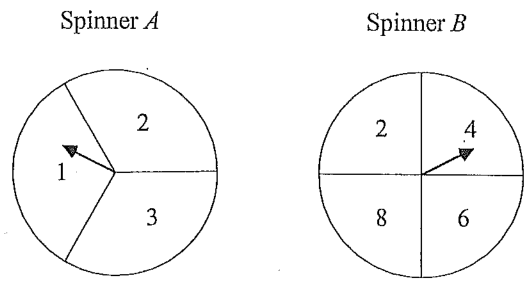
Question 7 (15 marks) **Marks**

(a) Use at least one-third of a page to sketch the graph of $y = \log_2(x+4)$. **2**

Clearly show any asymptotes and intercepts with the axes.

(b) The diagram shows two spinners. Spinner A is divided into 3 equal areas and Spinner B is divided into 4 equal areas, as shown.

The sum of the numbers on the two spinners is calculated.

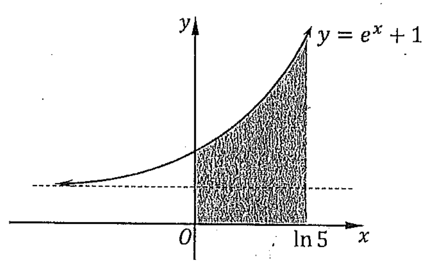


(i) Find the probability of rolling a sum of 5. **1**

(ii) Comment on the validity of this statement: **1**

The sum of the two numbers spun can only be even or odd.
So the probability of spinning a sum which is even is $\frac{1}{2}$.

(c) **3**

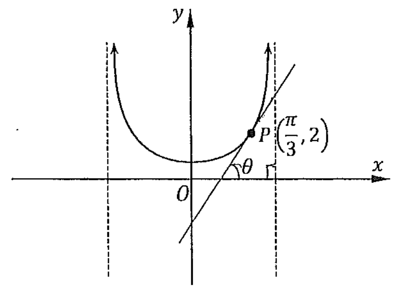


Find the exact area bounded by the curve $y = e^x + 1$ and the x-axis from $x = 0$ to $x = \ln 5$.

Question 7 continues on page 8

Question 7 (continued) **Marks**

(d) The diagram below shows the tangent to the curve $y = \sec x$ at the point $P\left(\frac{\pi}{3}, 2\right)$.



(i) Show that the gradient of the tangent at P is $2\sqrt{3}$. **2**

(ii) Hence find the angle θ that the tangent makes with the positive x-axis. Give your answer in radians to 2 decimal places. **1**

(e) Consider the function $f(x) = x - 3\ln x$.

(i) There is one stationary point on the curve $y = f(x)$. **3**

Find the coordinates of the stationary point and determine its nature.

(ii) Sketch the curve $y = f(x)$ for $x \geq 1$, clearly showing the location of the stationary point. **2**

There is no need to label any intercepts with the axes.

End of Question 7

Start a new writing booklet

Question 8 (15 marks) **Marks**

(a) The Maths department has watched many Year 12 netball games between SCEGGS and Sydney Grammar. They have calculated that the Sydney Grammar netball team has a probability of 0.82 of losing or drawing any match and a probability of 0.18 of winning any match.

(i) Calculate the probability of the Sydney Grammar team losing or drawing three consecutive matches. **1**

(ii) Hence, calculate the probability of the Sydney Grammar team winning at least one of three consecutive matches. **1**

(iii) What is the least number of consecutive matches the Sydney Grammar team must play to be 95% certain of winning at least one match? **2**

(b) Aaron borrows \$400 000 to buy a new apartment. The bank lends him the money at 0.5% per month, monthly reducible to be paid back after 25 years.

Aaron makes monthly repayments of $\$M$.

Let A_n be the amount owing on the loan after n monthly repayments, so that after 1 repayment, the amount owing is given by:

$$A_1 = 400\,000 \times 1.005 - M.$$

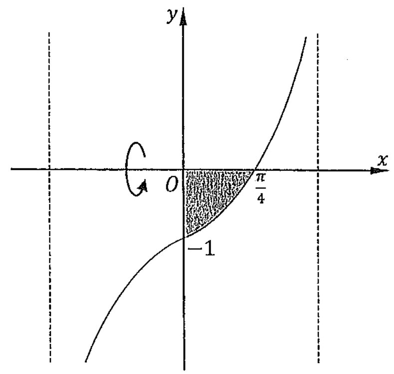
(i) Show that $A_n = 400\,000 \times 1.005^n - 200M(1.005^n - 1)$. **2**

(ii) If Aaron pays off the loan in 25 years, calculate his monthly repayments. **2**

(iii) What percentage of the loan would Aaron still owe after 12.5 years? Give your answer to the nearest percent. **2**

Question 8 continues on page 10

(c) The shaded area below is bounded by the curve $y = \tan x - 1$ and the co-ordinate axes.



The area is rotated about the x-axis to form a solid.

(i) Show that the volume of the solid is given by **2**

$$\pi \int_0^{\frac{\pi}{4}} \sec^2 x + 2\left(\frac{-\sin x}{\cos x}\right) dx.$$

(ii) Hence show that the volume of the solid formed is given by **3**

$$\pi(1 - \ln 2) \text{ units}^3.$$

End of paper

Mathematics Assessment Task 3 2012 - Solutions

M/C

1. $\int \cos\frac{x}{3}\,dx = \frac{-1}{\frac{1}{3}}\sin\frac{x}{3} + c$

$= -3\sin\frac{x}{3} + c$

B

Trig

77% of students got this correct

13% - A
3% - C
7% - D

2. $\frac{d}{dx}(3^x) = \frac{d}{dx}(e^{\ln 3x})$

$= \ln 3\, e^{\ln 3x}$

$= \ln 3\; 3^x$

C

L&E

53% of students got this correct

20% - A
13% - B
13% - D

3. Div by 3   3 6 9 12 15 18 21 24 27 30
Div by 5   5 10 15 20 25 30

$P(\text{div 3 or div 5}) = \frac{10+6-2}{30}$

$= \frac{7}{15}$

C

Prob

67% of students got this correct

3% - A
3% - B
27% - D

4. test $(0,0)$

$x + 2y - 4 \geq 0$ $x - y - 2 \geq 0$

LHS $= 0 + 2\times 0 - 4$ LHS $= 0 - 0 - 2$

$= -4 \not\geq 0$ $= -2 \not\geq 0$

$\therefore$ D

Coord

40% of students got this correct

3% - A
55% - B
2% - C

5. At the end of March 5.4% p.a. $= 0.45\%$ pm

1st instalment value $= \$2500 \times 1.0045^{15}$

2nd   "   " $= \$2500 \times 1.0045^{14}$

⋮   ⋮

last   "   " $= \$2500 \times 1.0045$

Total $= 2500(1.0045 + 1.0045^2 + \ldots + 1.0045^{15})$ C

62% of students got this correct

5% - A
3% - B
30% - D

Q6

a) i) $m_{PR} = \frac{6-0}{-1-1}$ $m_{PQ} = \frac{1-0}{4-1}$

$= -3$ $= \frac{1}{3}$

$m_{PR} \times m_{PQ} = -3 \times \frac{1}{3}$

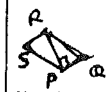
$= -1$

$\therefore PR \perp PQ$ 2 - Coord

This part was well done.
Your conclusion should be
Since $m_1 \times m_2 = -1$
$\therefore PR \perp PQ$

$\boxed{\perp \text{ means is perpendicular}}$

ii) $(-4, 5)$ 1 - Coord.



Note PQRS
The order of the letters is important

iii) $d = \frac{|1 + 0 - 5|}{\sqrt{1^2 + 1^2}}$

$= \frac{4}{\sqrt{2}}$ units   1 - Coord

Some mistakes with tidying up numerator which are avoidable.

iv) $d_{RQ} = \sqrt{(4 - -1)^2 + (1-6)^2}$

$= \sqrt{5^2 + (-5)^2}$

$= \sqrt{50}$

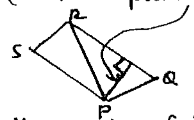
$= 5\sqrt{2}$ units

Area $= b \times h$

$= 5\sqrt{2} \times \frac{4}{\sqrt{2}}$

$= 20$ units^2 2 - Coord.

Area of a parallelogram $= bh$.
(Link to part iii)



You need to find RQ to have perpendiculars.

$\boxed{\text{Also } PR \perp PQ \text{ so you could do } 2\times(\tfrac{1}{2}\times PQ \times PR)}$

b) i) $\frac{d}{dx}(e^x + 1)^3 = 3(e^x + 1)^2 \times e^x$

$= 3e^x(e^x + 1)^2$

2 - L&E

Well done.
Chain Rule $y = f(x)^n$
$y' = n\, f(x)^{n-1} \times f'(x)$

ii) $\frac{d}{dx}(x\tan 2x) = x \times 2\sec^2 2x + 1 \times \tan 2x$

$= 2x\sec^2 2x + \tan 2x$

2 - Trig.

product rule
$u = x$ $v = \tan 2x$
$u' = 1$ $v' = 2\sec^2 2x$
$y' = vu' + uv'$

c) $\frac{dy}{dx} = \frac{2}{x}$

$y = 2\ln x + c$

when $x = \frac{1}{e}$ $y = 1$

$\therefore 1 = 2\ln\frac{1}{e} + c$

$1 = -2 + c$

$c = 3$

$\therefore y = 2\ln x + 3$ 3 – LdE

This is a standard style of question.
It does not mean get the equation of the tangent using $y - y_1 = m(x - x_1)$

It involves integration because you are given the gradient function.

d) $\log_7(x+5) + \log_7 x = \log_7 6$

$\log_7[(x+5)\times x] = \log_7 6$

$\therefore x^2 + 5x = 6$

$x^2 + 5x - 6 = 0$

$(x+6)(x-1) = 0$

$x = -6$ $x = 1$

since $x > 0$ $x = 1$ is only solution

3 – LdE

Log. Laws are used in this question.

$\log a + \log b = \log c$

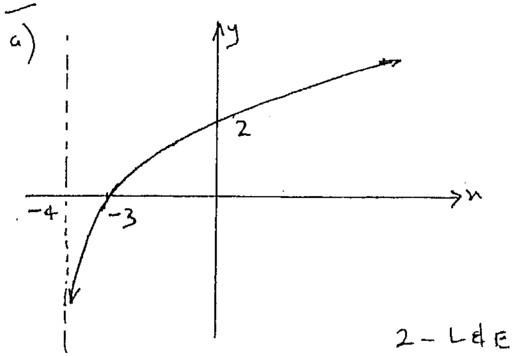
$\log ab = \log c$

$\therefore ab = c.$

You must give a clear conclusion because both answers don't work.

Q7

a)



2 – LdE

Many girls were confused about the base of 2.
You need to plot some points !!
eg. for $x = 0$

$\log_2 4 = ?$

ie. $2^? = 4$

$? = 2$ etc

b) i) Sample space

1 2	2 2	3 2
1 4	2 4	3 4
1 6	2 6	3 6
1 8	2 8	3 8

$P(\text{sum of } 5) = \frac{2}{12} = \frac{1}{6}$

most girls drew a sample space, and could then simply count outcomes which satisfied the events in the question.
Those who didn't had problems.

ii) $P(\text{odd}) = \frac{8}{12} = \frac{2}{3}$ $P(\text{even}) = \frac{4}{12} = \frac{1}{3}$

$\therefore$ the statement is false

2 – Prob.

c) $A = \int_0^{\ln 5} e^x + 1\, dx$

$= \left[e^x + x\right]_0^{\ln 5}$

$= (e^{\ln 5} + \ln 5) - (e^0 + 0)$

$= 5 + \ln 5 - 1$

$= 4 + \ln 5$ units^2 3 – LdE

Generally well done!
A few girls forgot to integrate the 1.

d) i) $y = \sec x$

$y = (\cos x)^{-1}$

$\frac{dy}{dx} = -1(\cos x)^{-2} \times -\sin x$

$= \frac{\sin x}{\cos^2 x}$

$= \tan x \sec x$

when $x = \frac{\pi}{3}$

$\frac{dy}{dx} = \tan\frac{\pi}{3}\sec\frac{\pi}{3}$

$= \tan\frac{\pi}{3} \times \frac{1}{\cos\pi/3}$

$= \sqrt{3} \times \frac{1}{1/2}$

$= 2\sqrt{3}$

$\therefore M_{tang} = 2\sqrt{3}$ at P

2 – Trig

Again quite well done, but REMEMBER the quotient rule !!
(or the chain rule !!

Well done by most.

ii) $\tan\theta = 2\sqrt{3}$

$\theta = 1.29^c$

Make sure you finish off the question. NOT JUST $\theta = \tan^{-1}(2\sqrt{3})$

e) i) $y = x - 3\ln x$

$\frac{dy}{dx} = 1 - 3 \times \frac{1}{x}$

$= 1 - \frac{3}{x}$

stat points occur when $\frac{dy}{dx} = 0$

$1 - \frac{3}{x} = 0$

$\frac{3}{x} = 1$

$x = 3 \quad y = 3 - 3\ln 3$

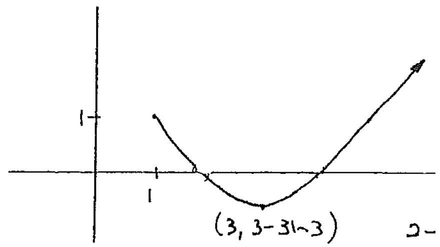
$\frac{d^2y}{dx^2} = \frac{3}{x^2}$

when $x = 3$ $\frac{d^2y}{dx^2} = \frac{1}{3} > 0$

$\therefore (3, 3-3\ln 3)$ is a minimum turning point. 3 − LdE

Quite well done

ii)



2 − LdE

and this as well

NOTE: If a minimum point, then ⌣ maximum ⌢

A number of girls found it to be a minimum, then drew a maximum.
AND evaluate numbers like $3 - 3\ln 3$!! It's negative !!

Q8

a) i) $P(LLL) = (0.82)^3$

$= 0.551368$ 1 − Prob

ii) $P(\text{at least one win}) = 1 - (0.82)^3$

$= 0.448632$ 1 − Prob

iii) $P(\text{at least one win}) = 1 - (0.82)^n$

$\therefore 1 - (0.82)^n \geq 0.95$

$(0.82)^n \leq 0.05$

$n \geq \frac{\ln(0.05)}{\ln(0.82)}$

$\geq 15.09\ldots$ 2 − Prob

Important things to note:
- always give a numerical expression, not just a calculated answer.
- never round a terminating decimal. If you wouldn't round 0.25 to 0.3 (1dp), don't round 0.551368 to 0.55 (2dp).

Generally well done.

b) i) $A_2 = A_1 \times 1.005 - M$

$= [400\,000 \times 1.005 - M] \times 1.005 - M$

$= 400000 \times 1.005^2 - 1.005M - M$

$= 400000 \times 1.005^2 - M(1 + 1.005)$

continuing the pattern

$A_n = 400000 \times 1.005^n - M\underbrace{(1 + 1.005 + \cdots + 1.005^{n-1})}_{G.P.\ a=1\ r=1.005}$

$A_n = 400000 \times 1.005^n - M\left[\frac{1(1.005^n - 1)}{1.005 - 1}\right]$

$= 400000 \times 1.005^n - 200M(1.005^n - 1)$

(since $\frac{1}{1.005-1} = 200$) 2 − Series

students who jumped directly to a summed expression for A_n did not receive full marks. You were required to show the expression for A_n – this includes showing the GP you are summing!

Many lost marks writing the GP as $1 + 1.005 + \cdots + 1.005^n$

ii) when $n = 300$ $A_n = 0$

$\therefore 400000 \times 1.005^{300} - 200M(1.005^{300} - 1) = 0$

$400000 \times 1.005^{300} = 200(1.005^{300} - 1)M$

$M = \frac{400000 \times 1.005^{300}}{200(1.005^{300} - 1)}$

$= \$2577.21$

2 − Series

Many students incorrectly solved $A_{25} = 0$

iii) $12\frac{1}{2}$ years $\rightarrow n = 150$

find $A_{150} = 400000 \times 1.005^{150} - 200 \times 2577.21(1.005^{150} - 1)$

$= \$271\,508.55$

$\therefore \frac{271\,508.55}{400\,000} \times 100\%$

$= 68\%$.

2 − Series

You need to understand t[he] definition of A_n – that i[s] the amount owing after n months (not the amount paid). Also the loan is $400000, not how much is paid in total or any othe[r] calculation.

Some students substituted n=150 and calculated all sor[ts] of random amounts & percentages not understand[ing] what they were actually finding!

c) i) $V = \pi \int_0^{\frac{\pi}{4}} (\tan x - 1)^2 \, dx$

$= \pi \int_0^{\frac{\pi}{4}} \tan^2 x - 2\tan x + 1 \, dx$

$= \pi \int_0^{\frac{\pi}{4}} \sec^2 x - 2\tan x \, dx$ as $1 + \tan^2 x = \sec^2 x$

$= \pi \int_0^{\frac{\pi}{4}} \sec^2 x + 2\left(\frac{-\sin x}{\cos x}\right) dx$ as $\tan x = \frac{\sin x}{\cos x}$

2 - Trig

Students who simply had a formula for volume on their page & who had copied the equation of the curve elsewhere on the page did not score marks.

Proofs need to be clear & logical, proceeding from line to line with clear reasoning – not all over the page!

ii) $V = \pi \left[\tan x + 2\ln(\cos x) \right]_0^{\frac{\pi}{4}}$

$= \pi \left[\left(\tan\frac{\pi}{4} + 2\ln\left(\cos\frac{\pi}{4}\right) \right) - \left(\tan 0 + 2\ln(\cos 0) \right) \right]$

$= \pi \left[\left(1 + 2\ln\frac{1}{\sqrt{2}} \right) - (0 + 0) \right]$

$= \pi \left(1 + 2\ln 2^{-\frac{1}{2}} \right)$

$= \pi (1 - \ln 2)$ units^3

3 - Trig.

Students had to correctly integrate the entire expression to be awarded the first mark, then correctly substitute and evaluate trig exact values for the second mark.

Clear logical steps, using index / log laws needed to be shown to get the final mark.

For example, the following final step was not sufficient:

$\pi\left(1 + \ln\frac{1}{2}\right) = \pi(1 - \ln 2)$

this requires either knowledge that

$\ln\frac{1}{2} = \ln 1 - \ln 2$

& $\ln 1 = 0$

or $\ln\frac{1}{2} = \ln 2^{-1} = -\ln 2$